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NUMERICAL SCHEME BASED ON CONVOLUTION INTEGRALS TO STUDY THERMOELECTRIC NON-EQUILIBRIUM INTERACTIONS WITH RELAXATION TIMES

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Classically, thermoelectric materials couple thermal and electric fields through four non-equilibrium interactions: Fourier, Ohm, Peltier and Seebeck. Recently, these interactions have been expanded by the Extended Non-Equilibrium Thermodynamic theory to include relaxation times. Consequently, four new interactions appear: Cattaneo, Drude, visco-Peltier and visco-Seebeck, resulting in a set of two transport equations that depend on time.

The aim of the present work is to develop a numerical technique based on the Finite Element Method (FEM) to study the thermoelectric interactions in presence of relaxation times. For this purpose, the energy and electric charge balances are written in a variational form using the divergence theorem to obtain two residuals related with two degrees of freedom: temperature and voltage. The energy balance is non-linear since, in addition, includes Joule's effect that quadratically depends on voltage; the problem must be solved using Newton-Rhapson techniques, defining consistent stiffness and capacity matrices.

The transport equations must be time-discretized, in a first approximation with finite differences. However, a more sophisticated and thermodynamically consistent approximation is obtained through convolution integrals as often done in viscoelastic algorithms. The resulting FEM formulation is implemented into the research code FEAP from the University of California at Berkeley.

The numerical algorithm is validated using simple analytical solutions for the four visco-interactions, developing important visco-coupled interactions to be used in new and advanced technological applications.