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ASYMPTOTIC ANALYSIS OF A VISCOUS FLOW IN A CURVED PIPE WITH ELASTIC WALLS

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This short communication is devoted to the presentation of our recent results regarding the asymptotic analysis of models for the blood flow in an artery. With this aim, we consider the dynamic problem of the incompressible flow of a viscous fluid through a curved pipe with a smooth central curve. Our analysis leads to the obtention of an one dimensional model via singular perturbation of the Navier-Stokes system as ϵ, a non dimensional parameter related to the radius of cross-section of the vessel, tends to zero. We allow the radius depend on tangential direction and time, so a coupling with an elastic or viscoelastic law on the wall of the vessel is possible.

To perform the asymptotic analysis, we do a change of variable to a reference domain where we assume the existence of asymptotic expansions on ϵ for both velocity and pressure which, upon substitution on Navier-Stokes equations, leads to the characterization of various terms. First order term of velocity asymptotic expansion depends only on the tangential direction, while the second order term contains the effects of curvature of central curve of the pipe. Finally, the third order term contains effects of curvature and torsion, and depends on transitional effects as well through the pressure terms. We also detail boundary conditions for our problem to be well posed. We end by showing some particular examples and results of numerical simulations, including some plots of velocity on cross section exhibiting dependence on curve characteristics of the pipe.

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