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# FAILURE ENVELOPE DETERMINATION IN FIBER REINFORCED COMPOSITES USING ASYMPTOTIC HOMOGENIZATION TECHNIQUES

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**Abstract.** *The objective of this work is to apply asymptotic homogenization techniques to predict elastic properties and strengths in unidirectional fiber reinforced composites (Glass/Epoxy and Carbon/Epoxy). Considering the composite's microstructure, the homogenization permits the prediction of elastic properties. The homogenized elastic properties were compared to experimental data and to properties generated through RVE (representative volume element) analysis. The results from the numerical methods (asymptotic homogenization and RVE) showed good agreement for some elastic properties, but some large discrepancies were also found for some cases. Moreover, considering loads applied to the macro level, the asymptotic homogenization allows to obtain stresses at the micro level. Due to the micromechanical model employed, different failure criteria can be applied for fiber and matrix, and the component that fails for an applied external load can be detected. A modified von Mises failure criterion was applied for matrix and maximum stress failure criterion was used to predict fiber failure. A methodology to predict strength properties using asymptotic homogenization is presented in this paper. A comparison was made between strength properties calculated through asymptotic homogenization and RVE. Good agreement was found between them. Composite's strengths calculated through homogenization were compared to strengths experimentally obtained and some discrepancies were found between them. Also, numerical failure envelopes were compared to Puck & Schürmann failure envelopes.*

## 1 INTRODUCTION

Fiber reinforced composites [1] are widely used in aeronautic and aerospace industry due to its low mass and high strength properties. Several other applications are also seen: tennis rackets, baseball bats, boats, wind turbines, etc. It is crucial in projects that use composite materials to know its elastic properties and to predict when failure is going to happen. With the increase of computers' capabilities, numerical techniques that are able to perform predictions of elastic properties and failure envelopes of composites are sought by researchers. These techniques can be also faster and cheaper compared to experimental tests.

An example of analysis at micro level is found in [2], where elastic and strength properties are calculated using a representative volume element (RVE) of a polymer composite reinforced by fibers, whose matrix is modelled as elasto-plastic with isotropic damage law. In [3], failure envelopes are generated through micro analysis where a RVE is used to model a plain weave textile composite using finite element method. Also, Tsai-Wu [4] and maximum principal stress failure criteria [3] are applied in fibers and matrix regions, respectively, in order to evaluate failure. In [5], the constituents' strength are calculated in a micro analysis whose inputs are the strengths of a laminate ply. In this study, it is assumed that failure can happen also at the interface between fiber and matrix. In [6], it is proposed a correction for matrix stresses, calculated assuming linear behaviour. A methodology to evaluate failure in unidirectional fiber reinforced composites is shown in [7] and comparisons of failure envelopes using different failure criteria for the constituents are performed.

In the present work, the asymptotic homogenization technique [8] is applied to unidirectional fiber reinforced composites (Glass/Epoxy and Carbon/Epoxy) to determine its homogenized equivalent elastic properties. A comparison is performed between elastic properties calculated through asymptotic homogenization and representative volume element from [2]. Also a comparison between experimental elastic properties of a composite from [9] and its homogenized elastic properties is made.

In addition, stresses at micro level obtained through asymptotic homogenization are used to evaluate failure of the composite. Given strength properties of the constituents, failure criteria are applied to each region of the mesh: generalized von Mises criterion for matrix, Eq.(8), and maximum stress for fiber, Eq.(4). A methodology that calculates failure stresses in linear elastic materials is shown. Composite's strengths calculated using asymptotic homogenization and RVE are compared. Moreover, failure envelopes generated by asymptotic homogenization are compared to the macroscopic envelopes obtained using Puck & Schürmann [10] failure criterion.

## 2 ASYMPTOTIC HOMOGENIZATION

Asymptotic homogenization techniques allow to obtain micromechanics elastic properties of a composite considering its microstructure. In this technique, two levels are considered:

the macro ($\mathbf{x}$) and the micro ($\mathbf{y}$). The micro level is considered to be periodic and it is represented by an unit cell formed by matrix and fiber. Its geometry can be changed in order to vary fiber volume fraction and cross section shape. Inclusions and holes can also be inserted in it. In the present work, a hexagonal unit cell (Fig.1) is used to perform the homogenization.

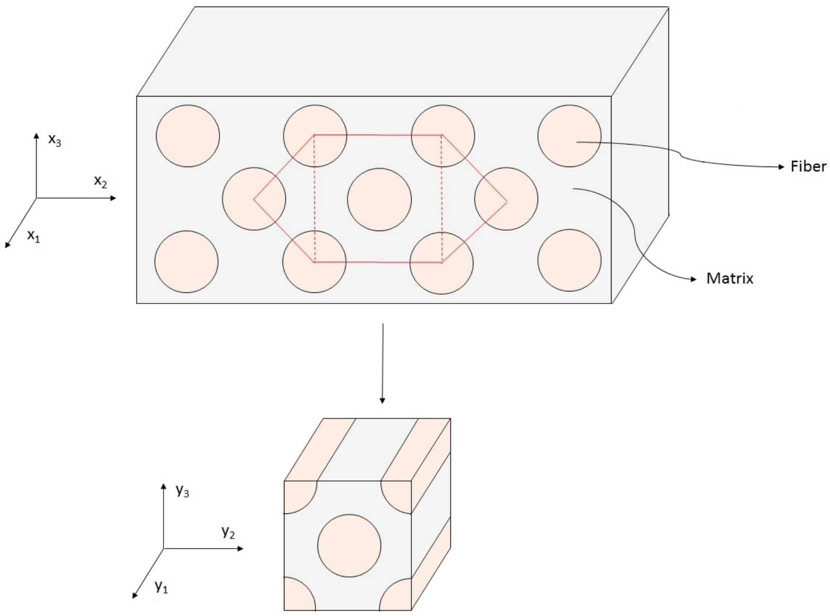


Figure 1: Periodical fiber distribution in the composite and the hexagonal unit cell.

### 2.1 Homogenized elastic properties

The homogenized elastic properties of the composite are obtained at micro scale by:

$$E^H_{ijkl} = \frac{1}{Y}\int_Y \left(E_{ijkl} - E_{ijkm}\frac{\partial \chi^{kl}_m}{\partial y_n}\right) dY. \tag{1}$$

In Eq.(1), tensorial notation is used, with the indexes vary from 1 to 3. $Y$ is the volume of the unit cell, $E_{ijkl}$ are the elastic properties of each point within the microstructure, $y_n$ are the coordinates in the unit cell, $\chi^{kl}_m$ are the auxiliar displacement fields that are obtained from the following equations,

$$\int_Y E_{ijmn}\frac{\partial \chi^{kl}_m}{\partial y_n}\frac{\partial \delta u_i}{\partial y_j} dY = \int_Y E_{ijop}\epsilon^{kl}_{op}\frac{\partial \delta u_i}{\partial y_j} dY. \tag{2}$$

In Eq.(2), $\epsilon^{kl}_{op}$ are unitary constant strain test fields and $\delta u_i$ are the virtual displacements. Equation (2) is here solved in the software PREMAT [8] using finite element method [11].

### 2.2 Micro stresses

After computing the elastic properties, stresses at micro level can be calculated with the aid of the $\chi_m^{kl}$ fields. Equation (3) allows the determination of the stress tensor $\sigma_{ij}$ for each point of the unit cell:

$$\sigma_{ij} = \left( E_{ijkl} - E_{ijmn} \frac{\partial \chi_m^{kl}}{\partial y_n} \right) \frac{\partial u_k^0}{\partial x_l} = \left( E_{ijkl} - E_{ijmn} \frac{\partial \chi_m^{kl}}{\partial y_n} \right) \epsilon_{kl}^0. \tag{3}$$

In Eq.(3), $\epsilon_{kl}^0$ are the strains at macro level that are obtained from the average macroscopic displacement field $\mathbf{u}^0$ and $x_l$ are the coordinates at macro level. In our case the stress calculations are performed using the software POSTMAT [8].

## 3 FAILURE ANALYSIS

Failure happens in a structure when it can not perform anymore the function for which it was designed. To tell whether a structure fails or not for a given set of loadings is a fundamental question that every designer needs to answer. There are a large number of failure criteria applied to composite materials available in the open literature. Here, two types of failure criteria are shown: those applied to the macro level and those applied to the micro level.

### 3.1 Failure criteria at macro level

Failure criteria at the macro level are mostly conceived to be straightforward to apply once the local stresses at a lamina of composite material are known. Tsai-Wu [4], maximum stress, maximum strain [1], Puck & Schürmann [10] are some examples of failure criteria that uses stresses at macro level. To be able to tell, for an applied load, which constituent (matrix or fiber) fails is an important data for design that a few failure criteria can predict. One example of criterion that reveals this information is the Puck & Schürmann [10]. In this paper, it will be used as a benchmark to validate the failure envelopes obtained using asymptotic homogenization.

### 3.2 Failure criteria at micro level

For an applied load to the structure at macro level, the distribution of stress at micro level (unit cell) can be obtained using the asymptotic homogenization technique. As said before, the micro domain is discretized in order to apply the finite element method. Then the stresses at micro level are obtained for each single node of the microstructure mesh and different failure criteria can be applied for matrix and fiber. Because of that, the knowledge of which constituent fails first for an applied load is possible. In this work, the structure is considered to fail when at least one nodal stress state exceeds the failure criterion.

### 3.2.1 Fiber failure criteria

Carbon and glass reinforcing fibers are considered to be transversally isotropic [5]. They have considerably higher elastic modulus and strength in the longitudinal direction than the matrix. In [5], the authors apply a quadratic failure criterion for fiber and show that terms regarding transverse strength can be eliminated from it. The result is a criterion that compares the normal stress in the fiber $\sigma_{11}^f$ to fiber longitudinal strengths:

$$-X_C^f < \sigma_{11}^f < X_T^f. \tag{4}$$

Where $X_C^f$ ($X_C^f > 0$) and $X_T^f$ are the compressive and tensile strength of the fiber in the longitudinal direction, respectively.

### 3.2.2 Matrix failure criteria

Epoxy matrix is considered as an isotropic material with different tensile and compressive strengths. There are several experiments, [5], showing that matrix failure depends on the deviatoric stress invariant $J_2$ and on the volumetric stress invariant $I_1$. Defining these stress invariants:

$$I_1 = \sigma_{11}^m + \sigma_{22}^m + \sigma_{33}^m, \tag{5}$$

$$I_2 = -(\sigma_{11}^m \sigma_{22}^m + \sigma_{11}^m \sigma_{33}^m + \sigma_{33}^m \sigma_{22}^m) + (\tau_{12}^m)^2 + (\tau_{23}^m)^2 + (\tau_{23}^m)^2. \tag{6}$$

In Eq.(5) and (6), $\sigma_{ij}^m$ are the components of the stress tensor of the matrix. The invariant $J_2$ of the deviatoric tensor is given by:

$$J_2 = \frac{I_1^2}{3} + I_2. \tag{7}$$

A failure criterion based on the relationship between the invariants $I_1$ and $J_2$ is presented in [5]. It is known as the modified von Mises criterion for isotropic material that has different compressive and tensile strength, and is given as follows,

$$\frac{3J_2}{T_m C_m} + \frac{I_1 (C_m - T_m)}{T_m C_m} = 1. \tag{8}$$

Where $C_m$ and $T_m$ are respectively the compressive and tensile strengths of matrix.

### 3.2.3 Methodology of failure analysis

A stress $\sigma^{\mathbf{0}}$ is applied at the macro level in a certain point of a lamina, assuming the common hypothesis of plane stress. It has the same direction of $\vec{s}$, as shown in Fig.2, but its magnitude is arbitrary at first.

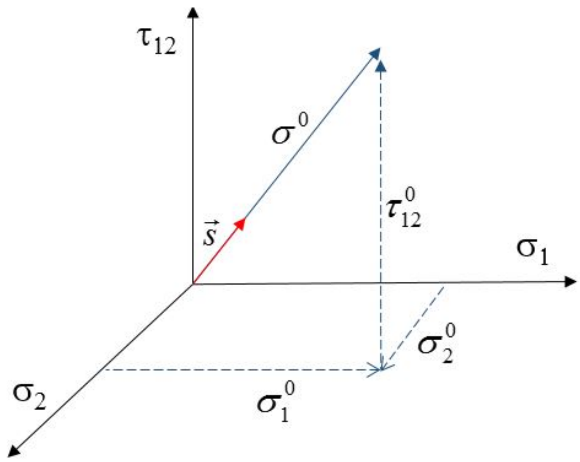


Figure 2: Load applied at a macro level in a macro stress space whose direction is defined by $\vec{s}$.

Based on $\sigma^{\mathbf{0}}$, the micro stresses tensor $\sigma^{\mathbf{1}}$ are then obtained (now a 3D stress state), through homogenization [8], for each node of the mesh. Once the constitutive model assumed in this paper is linear elastic, if the macro load applied is multiplied by a scalar $\beta$, the micro stresses are also multiplied by this scalar:

$$\beta\sigma^{\mathbf{0}} \Rightarrow \beta\sigma^{\mathbf{1}}. \tag{9}$$

To find the macro failure stress in $\vec{s}$ direction means finding the scalar $\beta$ that causes this failure. With this search in mind, the failure criterion applied to fiber turns into applying an arbitrary macro stress with $\vec{s}$ direction that generates a micro stress state with $\sigma_{11}^{f}$ being the normal component in the longitudinal direction. Then, $\beta_f$ is obtained using Eq.(4):

$$\beta_f = \begin{cases} X_T^f/\sigma_{11}^f & \text{if } \sigma_{11}^f > 0 \\ -X_C^f/\sigma_{11}^f & \text{if } \sigma_{11}^f < 0 \end{cases}. \tag{10}$$

To apply the failure criterion in Eq.(8) for matrix, turns into solving a quadratic equation where two values are obtained:

$$\beta_m = \frac{I_1\left(T_m - C_m\right) \pm \sqrt{\Delta}}{6J_2}, \Delta = I_1^2\left(C_m - T_m\right)^2 + 12J_2T_mC_m. \tag{11}$$

As we want to find a failure load in a specific direction of stress space, the value for $\beta_m$ is chosen to be the positive one. Having in hands $\beta_f$ and $\beta_m$ of all mesh nodes, the value $\beta$ that causes failure in the composite lamina is:

$$\beta = \min\left(\beta_f, \beta_m\right). \tag{12}$$

## 4 RESULTS

In this section, composite's elastic properties generated through asymptotic homogenization technique are compared to elastic properties experimentally obtained. In addition, a comparison between elastic properties generated through asymptotic homogenization technique and the RVE technique is made.
Following the methodology presented in Section 3, failure analysis is done to determine composite's strengths and a comparison is made to experimental data. In addition, strengths determined through asymptotic homogenization and RVE are compared. Finally, failure envelopes using Puck & Schürmann [10] criterion are compared to the numerical failure envelopes generated using the failure methodology here described.

### 4.1 Elastic properties - asymptotic homogenization and experimental data

Here, comparisons between experimental elastic properties and homogenized elastic properties are made. The two composites used in this analysis are: E-Glass 21xK43 Gevetex /LY556/HT907/DY063 epoxy and AS4/3501-6 epoxy. Their constituents' properties are found in [9] and can be seen in Tab.1. Properties $E_l$ and $\nu_t$ are, the longitudinal elastic modulus and transverse Poisson's ratio, respectively.

| | $E_l$ (GPa) | $\nu_t$ |
|---|---|---|
| Fiber | | |
| E-Glass 21xK43 Gevetex | 80 | 0.2 |
| AS4(carbon) | 225 | 0.2 |
| Matrix | | |
| LY556/HT907/DY063 epoxy | 3.35 | 0.35 |
| 3501-6 epoxy | 4.2 | 0.34 |

Table 1: Elastic properties of fibers and matrices from [9].

E-Glass 21xK43 Gevetex/LY556/HT907/DY063 epoxy and AS4/3501-6 epoxy experimental elastic properties are also found in [9] and are shown in Tab.2 and 3, respectively. The asymptotic homogenization was applied twice for each composite with different fiber volume fractions. The first volume fraction is the same as considered in the experiment and the second one was chosen in order to reduce the differences between the numerical and experimental $E_1$ property (elastic modulus at fiber direction). This procedure was adopted since the fiber volume fraction in a composite may vary in production. When the fiber volume fraction was changed, good agreement to experimental data was obtained for properties $E_1$ and $\nu_{23}$. The maximum variation found, for both composites, was for $E_2$. Analyses of mesh convergence were done in order to ensure that elastic properties did not diverge with mesh refinement.

| | Experimental data | Asymptotic homogenization | | | |
|---|---|---|---|---|---|
| Elastic properties | $V_f = 62\%$ | $V_f = 62\%$ | $\Delta$ (%) | $V_f = 66\%$ | $\Delta$ (%) |
| $E_1$ (GPa) | 53.480 | 50.658 | 5.3 | 53.722 | 0.5 |
| $E_2$ (GPa) | 17.700 | 12.771 | 27.8 | 14.469 | 18.3 |
| $E_3$ (GPa) | - | 12.620 | - | 14.236 | - |
| $G_{12}$(GPa) | 5.830 | 4.623 | 20.7 | 5.213 | 10.6 |
| $G_{23}$(GPa) | - | 4.511 | - | 5.146 | - |
| $G_{13}$(GPa) | - | 4.588 | - | 5.156 | - |
| $\nu_{23}$ | 0.400 | 0.393 | 1.8 | 0.378 | 5.5 |
| $\nu_{13}$ | - | 0.249 | - | 0.244 | - |
| $\nu_{12}$ | 0.278 | 0.248 | 10.8 | 0.242 | 12.9 |
| | | | $\sum = 66.4$ | | $\sum = 47.8$ |

Table 2: Experimental elastic properties of a E-Glass 21xK43 Gevetex/LY556/HT907/DY063 epoxy composite [9] and from asymptotic homogenization. The difference between experimental data and homogenized is $\Delta = 100\frac{|Exp-AH|}{Exp}$. The sign "-" means that experimental data were not provided.

| | Experimental data | Asymptotic homogenization | | | |
|---|---|---|---|---|---|
| Elastic properties | $V_f = 60\%$ | $V_f = 60\%$ | $\Delta$ (%) | $V_f = 55\%$ | $\Delta$ (%) |
| $E_1$ (GPa) | 126.000 | 136.010 | 7.9 | 124.940 | 0.8 |
| $E_2$ (GPa) | 11.000 | 16.131 | 46.6 | 13.863 | 26.0 |
| $E_3$ (GPa) | - | 15.936 | - | 13.760 | - |
| $G_{12}$(GPa) | 6.600 | 5.919 | 10.3 | 5.128 | 22.3 |
| $G_{23}$(GPa) | - | 5.682 | - | 4.861 | - |
| $G_{13}$(GPa) | - | 5.877 | - | 5.104 | - |
| $\nu_{23}$ | 0.400 | 0.398 | 0.5 | 0.414 | 3.5 |
| $\nu_{13}$ | - | 0.248 | - | 0.254 | - |
| $\nu_{12}$ | 0.280 | 0.246 | 12.1 | 0.253 | 9.6 |
| | | | $\sum = 77.4$ | | $\sum = 62.2$ |

Table 3: Experimental elastic properties of a AS4/3501-6 epoxy composite [9] and from asymptotic homogenization. The difference between experimental data and homogenized is $\Delta = 100\frac{|Exp-AH|}{Exp}$. The sign "-" means that experimental data were not provided.

## 4.2 Elastic properties - asymptotic homogenization and RVE

An AS4/Epoxy composite, whose constituents' elastic properties are found in [2] and shown in Tab.4, has its elastic properties obtained through RVE and asymptotic homogenization. The results from RVE are found in [2].

| | $E_l$ (GPa) | $\nu_t$ |
|---|---|---|
| Fiber | | |
| AS4(carbon) | 225 | 0.2 |
| Matrix | | |
| Epoxy | 3.76 | 0.39 |

Table 4: Elastic properties of AS4 fiber and Epoxy matrix from [2].

The RVE results were based on $V_f = 65\%$ and once more, two volume fractions were used to perform homogenization: $V_f = 65\%$ and $V_f = 61\%$. The first was the same as used in the RVE analysis in [2] and the second was considered because provided the closest $E_1$ elastic modulus (fiber direction) in comparison to the obtained with the RVE. The results from both methods are seen in Tab.5.

| | RVE | Asymptotic homogenization | | | |
|---|---|---|---|---|---|
| Elastic properties | $V_f = 65\%$ | $V_f = 65\%$ | $\Delta$ (%) | $V_f = 61\%$ | $\Delta$ (%) |
| $E_1$ (GPa) | 138.910 | 146.910 | 5.8 | 138.050 | 0.6 |
| $E_2$ (GPa) | 9.380 | 19.271 | 105.4 | 16.666 | 77.7 |
| $E_3$ (GPa) | - | 18.935 | - | 16.461 | - |
| $G_{12}$(GPa) | 5.080 | 6.047 | 19.0 | 5.310 | 4.5 |
| $G_{23}$(GPa) | - | 6.452 | - | 5.268 | - |
| $G_{13}$(GPa) | - | 5.978 | - | 5.539 | - |
| $\nu_{23}$ | 0.35 | 0.464 | 32.6 | 0.483 | 38.0 |
| $\nu_{13}$ | - | 0.259 | - | 0.266 | - |
| $\nu_{12}$ | 0.245 | 0.257 | 4.9 | 0.265 | 8.2 |
| | | | $\sum = 167.7$ | | $\sum = 129.0$ |

Table 5: Elastic properties calculated through RVE [2] and asymptotic homogenization for different values of fiber volume fraction. The difference between RVE and asymptotic homogenization is $\Delta = 100\frac{|RVE-AH|}{RVE}$. The sign "-" means that numerical data from RVE were not provided.

When $V_f = 65\%$, the elastic properties calculated through asymptotic homogenization are very different to those found with the RVE. The minimum difference in the properties is at $\nu_{12}$ and the maximum is at $E_2$. However, when $V_f = 61\%$ the elastic properties $E_1$ and $G_{12}$ are very similar to those found using RVE. Properties $E_2$ and $\nu_{23}$ have significant differences and it is believed to be caused by the random fiber distribution in the RVE cell, since fiber relative positioning in the matrix affects the mechanical properties of a composite. The maximum variation continues to be $E_2$ and the minimum is now at $E_1$.

### 4.3 Composite strength - asymptotic homogenization and experimental data

Following the methodology of failure presented in Section 3, the strengths of an E-Glass 21xK43 Gevetex/LY556/HT907/DY063 epoxy and AS4/3501-6 epoxy composites are obtained numerically. Their constituents' strength properties are found in [9] and shown in Tab.6, where $S_t$ and $S_c$ are the tensile and compressive strength, respectively.

| | $S_t$ (MPa) | $S_c$ (MPa) |
|---|---|---|
| Fiber | | |
| E-Glass 21xK43 Gevetex | 2150 | 1450 |
| AS4(carbon) | 3350 | 2500 |
| Matrix | | |
| LY556/HT907/DY063 | 80 | 120 |
| 3501-6 epoxy | 69 | 250 |

Table 6: Strength properties of fibers and matrices from [9] used in the failure analysis.

The results for each composite are presented in Tab.7 and 8, respectively, and compared to experimental data from [9]. In those tables, properties $X_t$ and $X_c$ are respectively the tensile and compressive strengths in the longitudinal direction. $Y_t$ and $Y_c$ are respectively the tensile and compressive strengths in the transversal direction and $S_{12}$ is the in-plane shear strength.

In Tab.7, there are large discrepancies between numerical and experimental strengths. The highest variation happens at $X_c$. The analysis in Tab.8 shows better agreement to experimental data, and the highest variation happens at $Y_t$. These results indicate that modifications in the failure determination procedure in Section 3 may be necessary to better estimate lamina strengths using asymptotic homogenization.

| | Experimental data | Asymptotic homogenization | |
|---|---|---|---|
| Strength properties | $V_f = 62\%$ | $V_f = 66\%$ | $\Delta$ (%) |
| $X_t$(MPa) | 1140 | 1236.750 | 8.5 |
| $X_c$(MPa) | 570 | 974.135 | 70.9 |
| $Y_t$ (MPa) | 35 | 51.316 | 46.6 |
| $Y_c$ (MPa) | 114 | 135.566 | 18.9 |
| $S_{12}$(MPa) | 72 | 46.059 | 36.0 |

Table 7: Experimental strength properties of a E-Glass 21xK43 Gevetex/LY556/HT907/DY063 epoxy composite [9] and from asymptotic homogenization. The difference between experimental data and homogenized is $\Delta = 100\frac{|Exp-AH|}{Exp}$.

| | Experimental data | Asymptotic homogenization | |
|---|---|---|---|
| Strength properties | $V_f = 60\%$ | $V_f = 55\%$ | $\Delta$ (%) |
| $X_t$(MPa) | 1950 | 1859.920 | 4.6 |
| $X_c$(MPa) | 1480 | 1388.503 | 6.2 |
| $Y_t$ (MPa) | 48 | 36.585 | 23.8 |
| $Y_c$ (MPa) | 200 | 209.022 | 4.5 |
| $S_{12}$(MPa) | 79 | 63.953 | 19.0 |

Table 8: Experimental strength properties of a AS4/3501-6 epoxy composite [9] and from asymptotic homogenization. The difference between experimental data and homogenized is $\Delta = 100\frac{|Exp-AH|}{Exp}$.

### 4.4 Composite strength - asymptotic homogenization and RVE

Strengths of an AS4/Epoxy composite, whose constituents' strengths are found in [2] and shown in Tab.9, are obtained numerically using the methodology of failure prediction presented in Section 3. Those are compared to the strengths obtained through a RVE model [2], as depicted in Tab.10.

| | $S_t$ (MPa) | $S_c$ (MPa) |
|---|---|---|
| Fiber | | |
| AS4(carbon) | 3350 | 2500 |
| Matrix | | |
| Epoxy | 93 | 124 |

Table 9: Strength properties of AS4 fiber and Epoxy matrix [2].

| | RVE | Asymptotic homogenization | |
|---|---|---|---|
| Strength properties | $V_f = 65\%$ | $V_f = 61\%$ | $\Delta$ (%) |
| $X_t$(MPa) | 2056.5 | 2055.845 | 0.0 |
| $X_c$(MPa) | - | 1534.212 | - |
| $Y_t$ (MPa) | 67.7 | 63.632 | 6.0 |
| $Y_c$ (MPa) | 122.5 | 127.773 | 4.3 |
| $S_{12}$ (MPa) | 47.9 | 51.201 | 6.9 |

Table 10: Strength properties calculated through RVE ([2]) and asymptotic homogenization for an AS4/Epoxy composite. The difference between RVE and asymptotic homogenization is $\Delta = 100\frac{|RVE-AH|}{RVE}$. The sign "-" means that numerical data from RVE were not provided.

Good agreement is seen between strength calculated through asymptotic homogenization and RVE. The minimum discrepancy happened at $X_t$ and the maximum at $S_t$.

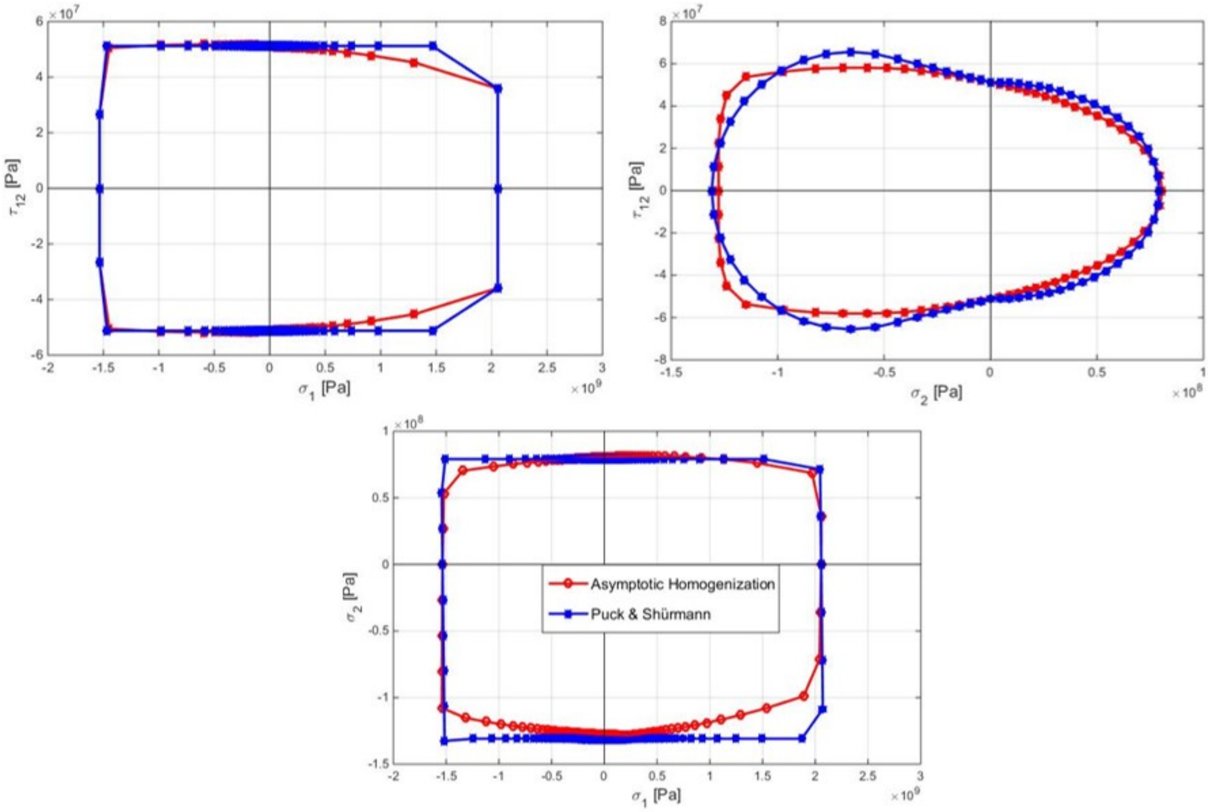


Figure 3: Comparison between Failure envelopes generated through Asymptotic Homogenization and Puck and Schürmann criterion.

## 4.5 Composite strength - asymptotic homogenization and Puck & Schürmann criterion

The methodology of failure presented in Section 3 is applied to the AS4/Epoxy composite of Tab.9 to investigate failure under plane stress loads in order to produce numerical failure envelopes. The composite's strengths obtained through this methodology (Tab.10) are used as Puck & Schürmann [10] failure criterion inputs. In addition, Tab.11 shows additional input parameters required by Puck & Schürmann failure criterion, where $p_{\perp||}$ is the slope of the $(\sigma_n, \tau_{n1})$ envelope for $\sigma_n \leq 0$ at $\sigma_n = 0$, $E_1$ is the composite's elastic modulus in the longitudinal direction, $\nu_{f12}$ is Poisson's ratio of the fibers (strain in tranversal direction caused by a stress in the longitudinal direction), $m_{\sigma f}$ is the mean magnification factor for the fibers and $E_f$ is fiber modulus in the longitudinal direction. Figure 3 shows the numerical results and the Puck & Schürmann criterion.

| Puck terms | Value |
|---|---|
| $p_{\perp\|\|}$ | 0.25 |
| $E_1$ (GPa) | 146.91 |
| $\nu_{f12}$ | 0.2 |
| $m_{\sigma f}$ | 1.1 |
| $E_f$ (GPa) | 225 |

Table 11: Inputs for Puck & Schürmann criterion.

At the $\sigma_1 - \tau_{12}$ plane, a good agreement between the failure envelopes obtained using asymptotic homogenization and the Puck & Schürmann criterion is obtained. At the second and third quadrants, where the maximum difference between asymptotic homogenization and Puck & Schürmann was 0.13%. At the first and fourth quadrants, such difference was 12.84%. At the $\sigma_2 - \tau_{12}$ plane, better correlation is observed in the first and forth quadrants, where the maximum difference was 8.85%. At second and third quadrants, the maximum difference was 12.93%. Finally, at the $\sigma_1 - \sigma_2$ plane, good agreements are seen at the uniaxial loads, where the maximum difference was 4.32%. The maximum difference found over the envelope was 21.40% at the fourth quadrant.

## 5 CONCLUSIONS

The asymptotic homogenization revealed to be a good method to predict elastic properties, once the homogenizations of the composites E-Glass 21xK43 Gevetex/ LY556/ HT907/DY063 epoxy and AS4/3501-6 epoxy showed good agreement to experimental data. The maximum difference to experimental data found on the E-Glass 21xK43 Gevetex/ LY556/HT907/DY063 epoxy was 18% and on the AS4/3501-6 epoxy was 26%, which is considered reasonable. A point worth mentioning that may explain the discrepancies is the fiber misalignment which was not included in the micromechanical model. The composite AS4/Epoxy had elastic properties calculated through asymptotic homogenization and compared to properties obtained with a RVE method. Some predicted properties show good agreement but large discrepancies were also found for some cases. Those are believed to happen because the fiber distribution in those methods are different, and the relative fiber positioning within the unit cell changes the composite's mechanical properties [12].

In terms of assessing the asymptotic homogenization in terms of strength/failure prediction using the methodoly presented in Section 3, strengths of E-Glass 21xK43 Gevetex/ LY556/ HT907/DY063 epoxy and AS4/3501-6 epoxy composites were calculated. The comparison of the strengths obtained numerically and experimentally revealed significant discrepancies. Moreover, strengths of the AS4/Epoxy composite were obtained through asymptotic homogenization and compared to results from RVE technique. These numerically predicted strength properties had good correlation, with a maximum difference

between them of 6.9%.
Finally, failure envelopes generated through asymptotic homogenization and Puck & Schürmann criterion were compared. Good correlations were observed between them. This reveals that if the composite's unidirectional failure strengths ($X_t, X_c, Y_t, Y_c, S_{12}$), predicted through homogenization, show good agreement to experimental data, the numerical failure envelopes are going to be close to the composite's failure envelopes obtained experimentally.
With the objective to obtain numerical failure envelopes close to experimental, future work has been focusing on correcting the methodology of failure to obtain composite's strengths with good agreement to experimental data.

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