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LARGE SCALE ANALYSIS OF THE RIVER FLOW FLUCTUATIONS IN BRAZIL

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Abstract. *We report on a large scale characterization of the flow in 141 different stations of 51 Brazilian rivers via daily time series obtained from the period of 1931 to 2012. We show that it is possible to associate a period to the natural seasonality of these time series by analyzing the permutation entropy and complexity. We investigate the possibility of this period displaying evolutive facets by estimating its value within different time intervals. Linear regressions reveal that these periods show no statistically significant increasing/decreasing linear trend. We further study the normalized and detrended fluctuations of the river flows. We show that the overall probability distributions of these fluctuations exhibit the same profile for all rivers in our dataset. Also, these distributions can be roughly described by extreme value distributions, which may connect these fluctuations with the theory of extreme values. Finally, we have studied the temporal correlations within the elements of the time series. By employing detrended fluctuation analysis, we have verified the river flow fluctuations actually display long-range and persistent correlations characterized by an averaged Hurst exponent* $h = 1.2$*, but that the* h *value displays a range of variation (standard deviation of* 0.2*) that depends on the river and on the year under analysis. This last finding can be eventually used for promoting a classification of rivers and also prompt other questions on possible relationships between* h *and particularities of the river and/or of the year under analysis. Our findings thus contribute to consolidated previous results and also shed new possibilities for investigating these series, which may find implications for modeling and forecasting river flows.*

1 Introduction

The study of earth-related systems is an important topic that have been addressed by engineers and physicists via methods of statistical physics. Earthquakes [1], geomagnetic activities [2], climate [3] and weather-related [4, 5] systems are just a few examples of systems that researchers have tackled in these pages. Regarding weather-related systems, important systems are the rivers and their discharges, which have an large impact on human activities and that may also suffer huge influence from these activities. A river flow results from complicated interactions between the climate systems (such as rainfall, temperature and evaporation), the landscape (such as basin area and land relief) and human activity (such as pollution and power generation). These many features makes river flow rates (river discharges) a complex process that has attracted the attention scholars over the last six decades. For instance, the seminal work of Hurst about the long-range dependence of runoff records from several rivers [6] has fostered several discussions on the fractal/multifractal and scaling properties of the temporal evolution of river flows [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25].

Despite this considerable attention towards the investigation of river flows, most of these works are based on data from small sets of rivers and we still lack of a large scale characterization of time series related to river flows. In this context, Brazil has the largest river basin (the Amazon) and one of world's most complex and extensive system of rivers; despite that, to the best of our knowledge, a large scale investigation on the Brazilian rivers has not been reported yet. Here we fill this gap by studying the flow in 141 different stations (in the vicinity of hydroelectric power plants) of 53 Brazilian rivers via daily time series obtained from the period of 1931 to 2012. We start by investigating the natural periodicity exhibited by these time series via the permutation entropy and statistical complexity. By analyzing these two measures as a function of a time delay between the elements of the time series (the so-called permutation spectrum), we identify peaks/valleys (approximately) spaced in one year in these two measures as a function of the time delay. We thus define the periods of time intervals of each time series and investigated the possibility that they evolutive trends. Linear regressions reveal that the period of each time series shows no statistically significant increasing/decreasing trend. Next, we focus our attention on the normalized and detrended fluctuations of the river flows. Similar to previous works, we find that the overall probability distributions of the normalized fluctuations exhibit the same profile for all rivers in our dataset. Also, these distributions can be roughly described by extreme value distributions. Finally, we study the temporal correlations in our time series via detrended fluctuation analysis (DFA) and the results (in agreement with previous-reported works) show that the river flow fluctuations display long-range and persistent correlations.

2 Data presentation and analysis

The data we analyzed consist of daily time series of the natural river flow rate (river discharges) measured in 141 different stations in the vicinity of hydroelectric power plants. These data cover 53 Brazilian rivers, spanning the period of 1931 to 2012 and are made freely available by the Operador Nacional do Sistema Elétrico — ONS — (a federal organ that controls the power system in Brazil) [26]. For matter of convenience, let's denote the flow rates by $x(t)$, where $t = 1, 2, \ldots, 365$ is a discrete time variable indexing the days of the year. In our analysis, we have removed the last datapoint of all time series from leap years, ensuring that all time series have the same length by year. The flow rate $x(t)$ has units of m^3/s and Fig. 1 shows examples of its temporal evolution for three different stations in four years.

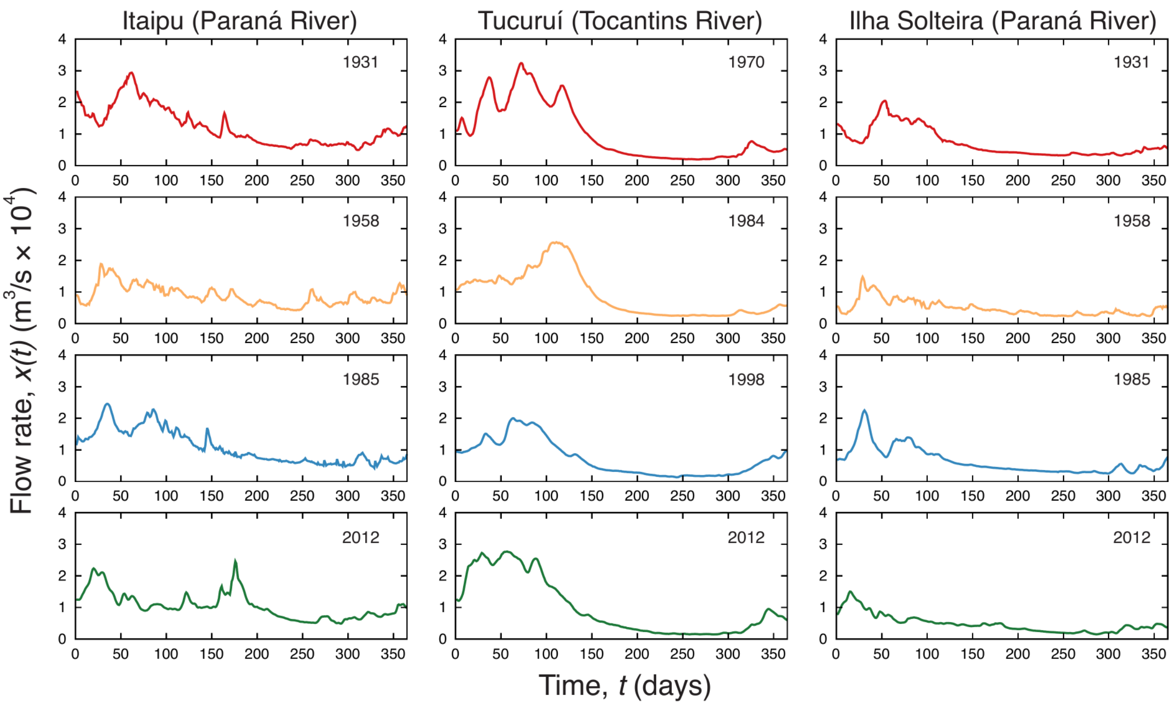

Figure 1. Time series examples of the natural flow rates in Brazilian rivers. Each plot shows the natural flow rate $x(t)$ in units of m^3/s along the year (indicated in the plots) with sampling rate of one day. The leftmost panel shows $x(t)$ in the Paraná River measured in the vicinity of the Itaipu Dam; the center panel shows data from the Tocantins River obtained nearby the Tucuruí Dam; finally, the rightmost panel also show data from the Paraná River measured close to the Ilha Solteira Dam. In these plots, we observe the natural seasonality of the flow rates $x(t)$, which are characterized by greater values in the first and last days of the year (summertime).

One of most striking feature displayed in Fig. 1 is the natural seasonality of the evolution of $x(t)$, which reflects the coupling of river discharges with the climate system. It is clear from this figure that large discharges occurs during the first and last months of the year (summertime) and that low discharges appears during the winter (around mid-year). In this context, a natural question is how one can define the period T associated with these time series and whether T has some evolutive aspects over the years.

In order to answer these questions, we can apply the permutation spectrum technique based on the idea of Bandt and Pompe [27] of associating symbolic sequences to the segments of the time series. For a time series $x(t)$ composed of N elements, we consider partitions s of length d ($d > 1$) defined by d-dimensional vectors of the form

$$s \mapsto \{x(s),\, x(s+\tau),\, \ldots,\, x(s+(d-2)\tau),\, x(s+(d-1)\tau)\}\,, \tag{1}$$

where $s = 1, 2, \ldots, N-(d-1)\tau$. The parameter d is called embedding dimension and τ is the embedding delay. Next, for each one of these $[N-(d-1)\tau]$ vectors, we evaluate the permutations $\pi = (r_0, r_1, \ldots, r_{d-1})$ of $(0, 1, \ldots, d-1)$ defined by

$$x(s-r_0\,\tau) \geq x(s-r_1\,\tau) \geq \cdots \geq x(s-r_{d-2}\,\tau) \geq x(s-r_{d-1}\,\tau)\,. \tag{2}$$

The $d!$ possible permutations of π are the so-called accessible states of the system, and for each one, we estimate the probability of finding a given permutation π_i by evaluating the relative frequency of its occurrences along the time series, that is,

$$p(\pi_i) = \frac{\#[s|s \leq N-(d-1)\tau\,;\ s \text{ has type } \pi_i]}{N-(d-1)\tau} \tag{3}$$

where the symbol # stands for the number of occurrences of the permutation π_i. We further define the ordinal patterns probability $P = \{p(\pi_i)\}$ (with $i = 1, 2, \ldots, d!$) as the set of probabilities associated the each one of $d!$ possible permutations. In order to help clarifying this concept, consider an example where $x(t) = \{8,7,4,9,2,5,1\}$ ($N = 7$) and supposed that we want to evaluate the permutations π considering the embedding dimension $d = 3$ and the embedding delay $\tau = 2$. Thus, we have $(s = 1) \mapsto \{8,4,2\}$, $(s = 2) \mapsto \{7,9,5\}$ and $(s = 3) \mapsto \{4,2,1\}$ leading (respectively) to $\pi_{(s=1)} = (2,1,0)$, $\pi_{(s=2)} = (2,0,1)$ and $\pi_{(s=3)} = (2,1,0)$. In this manner, $p(\pi = (2,1,0)) = {}^2/_3$, $p(\pi = (2,0,1)) = {}^1/_3$ and all the other probabilities are zero ($p(\pi = (0,1,2)) = 0$, $p(\pi = (0,2,1)) = 0$, $p(\pi = (1,0,2)) = 0$ and $p(\pi = (1,2,0)) = 0$), yielding $P = \{{}^2/_3, {}^1/_3, 0, 0, 0, 0\}$.

The ordinal patterns probability $P = \{p(\pi_i)\}$ thus retain information of the ordering dynamics of the elements of the time series $x(t)$; clearly, the larger the value of d the more information regarding this dynamics is available. However, large values of d requires N to be much larger in order to obtain reliable statistics on $p(\pi_i)$, in general, the rule of thumbs where $d! \ll N$ works for most practical purposes and academics usually follow the Bandt and Pompe recommendation of $d = 3, \ldots, 7$. The embedding delay τ has other proposed: for $\tau > 1$, the values of $x(t)$ are taken non-consecutively and thus mapping the ordering dynamics of the time series at different temporal resolutions.

Once we have obtained the ordinal patterns probability $P = \{p(\pi_i)\}$, we can evaluate the normalized Shannon's entropy defined by

$$H[P] = -\frac{1}{S_{\max}} \sum_{i=1}^{d!} p(\pi_i) \log p(\pi_i) \quad (4)$$

where $S_{\max} = \log d!$ is obtained when all permutations π_i are equiprobable, that is, $P = P_e = \{{}^1/_{d!}\}$. By definition, $0 < H < 1$, where the upper bound occurs for a completely random series and one expect $H < 1$ for series exhibiting some kind of correlated ordering dynamics.

The other quantity we evaluate from $P = \{p(\pi_i)\}$ is the statistical complexity measure of López-Ruiz *et al.* [28] defined by

$$C[P] = Q[P, P_e]\, H[P], \quad (5)$$

where $Q[P, P_e]$ is a relative entropic metric between the empirical ordinal probability $P = \{p(\pi)\}$ and the equiprobable state $P_e = 1/d!$. The quantity $Q[P, P_e]$ is also known as disequilibrium and it is defined in terms of the Jensen-Shannon divergence [29] (or also in terms of a symmetrized Kullback-Leibler divergence [30]):

$$Q[P, P_e] = \frac{S[(P + P_e)/2] - S[P]/2 - S[P_e]/2}{Q_{\max}}, \quad (6)$$

where

$$Q_{\max} = -\frac{1}{2}\left\{\frac{d!+1}{d!}\log(d!+1) - 2\log(2d!) + \log(d!)\right\}$$

is the maximum possible value of $Q[P, P_e]$, obtained when one of the components of P is equal to one and all the other vanish. The disequilibrium C quantifies the degree of correlational structures, providing important additional information that may not be carried only by the permutation entropy. In addition, for a given $H[P]$ value there exists a range of possible values for $C[P]$ [31]. This is the main reason why Rosso *et al.* [32] proposed to employ a diagram of $C[P]$ versus $H[P]$ as a diagnostic tool, building up the complexity-entropy causality plane.

Here, as previously mentioned, we focus on the so-called permutation spectrum technique that consist in evaluating H and C as function of τ in a particular range. Similar to what occurs for the embedding dimension, large values of the embedding delay τ require long time series and, for good statistics, we chose the maximum value of τ around 20% of length N of the time series. This permutation spectrum has proved to be successful in the identification of delay phenomena and feedback mechanisms in time series [33, 34]. In general, the relationships H versus τ and C versus τ display peaks or valleys that correspond to harmonics and subharmonics associated with the time series. In order identify the periods T associated with the river discharges, we have grouped the times series $x(t)$ in time intervals of 20 years, and for each set, we calculated the permutation spectrum associated with H and C. Figure 2A shows an example of permutation spectrum of the Paraná River discharges measured at the Itaipu station, both for H (top panel) and C (middle panel). We observe that the entropy H presents peaks while the complexity C has valleys spaced in about 365 days. For estimating the period T associated with the time series, we have analyzed the difference between H and C as shown in the bottom panel of Fig. 2A. This ensures that we are obtaining more information from the time series and may help increasing the accuracy in the estimated value of T. In this relationship between $(H - C)$ and τ, we numerically identify the location, that is, the values of $\tau = \tau_i^*$ where the peaks occurs, by imposing that a peak must be largest value surrounded by 180 smaller values on the left and on the right. The performance of this simple procedure was manually checked and we find no misidentifications and that the accuracy is quite good. The period T associated with a particular time interval of a time series is thus defined average value of the difference between two consecutive τ_i^*. The bottom panel of Fig. 2A shows the identified peaks in relationship of $H - C$ versus τ and also illustrates the definition of T.

We estimate the value of T associated to each time interval of 20 years from all time series. Figure 2B shows the values of T estimated the four time intervals from the Paraná River discharges at the Itaipu station. We note that the estimated T is around 365 days, but that it also displays small fluctuations. In particular, from the example of Fig. 2B, one could imagine that T has been increasing. In order to access, in an systematic way, whether T may present evolutive facets, we fit the linear model

$$T = A + \beta n \tag{7}$$

to the estimated values of T for all time series with minimum length of 80 years. Here A and β are constants, and n stands for the center of the time interval used for estimating T. We thus verify the statistical significance of β via its p-value. Figure 2C shows all p-values where it is clear that β has no statistical meaning, and therefore, T does not present a linear tendency of increasing or decreasing. Naturally, this result does not rule out other possibilities for the evolution T such as an oscillatory behavior or more complicated temporal dependencies. However, four datapoints are not enough to answer these questions in an objective way. In addition to that, another important question (probably an important one) is regarding the fluctuations of T, for instance, are these fluctuations increasing? This hypothesis is much harder to test because the fluctuations of τ_i does not have any reason to be directed associated with the fluctuations in T.

We now focus our attention on the normalized version of the time series $x(t)$. For its definition, we evaluate the average of $x(t)$ grouped by year, that is,

$$\mu(t) = \frac{1}{Y} \sum_{k=1}^{Y} x_k(t) \,, \tag{8}$$

where x_k stands for the part of $x(t)$ within the year k and Y is number of years in $x(t)$.

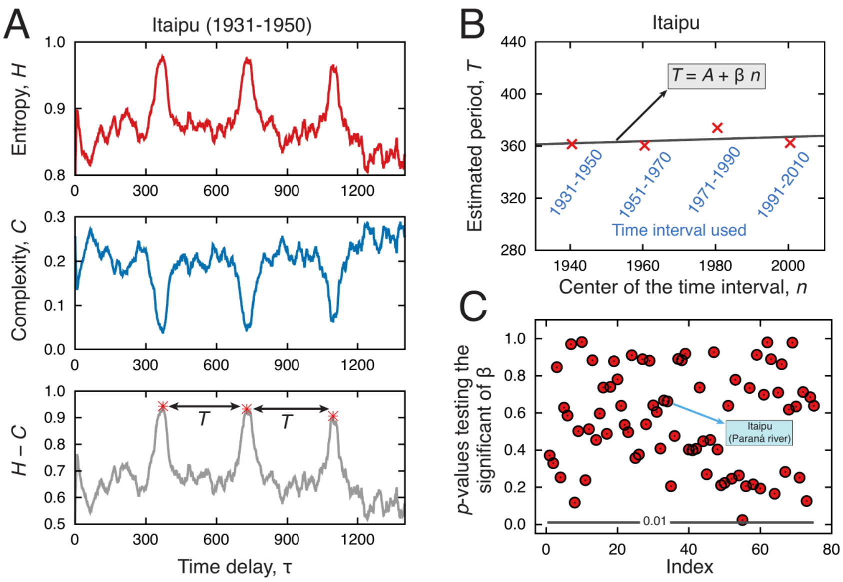

Figure 2. Obtaining the period of time series via the permutation entropy and statistical complexity. **(A)** Values of the permutation entropy H, statistical complexity C and the difference $H-C$ as a function of the time delay τ. In this example, we have used data of the Itaipu Dam (Paraná River) in the time interval of 1931 to 1950. Notice that the entropy H displays peaks while the complexity C presents valleys spaced in about 365 days. In order to simultaneously use H and C for defining the period T associated with the time series, we have focused on the difference $H-C$. By employing this quantity, we numerically identify the location of peaks (as shown in bottom plot of panel A) for defining the period T as the averaged value of the temporal distance between these peaks. **(B)** Estimated value for period T in four time intervals from data of the Itaipu station. We note that the values of T are close to one year (365 days) and that they display fluctuations. We run the linear model $T = A + \beta n$, where A and β are fitting parameters and n stands for the center of the time interval used for estimating T. The continuous line shows the adjusted model in which the linear coefficient β was found to be nonsignificant ($\beta = 0.08 \pm 0.16$; p-value = 0.62), suggesting that a constant value function is a better description for these data. **(C)** Statistical significance (as measured by the p-value) of linear coefficient β obtained for all time series. Note that all p-values are larger than 0.01 (horizontal line in the plot) indicating that we can reject the null hypothesis that the values of β are statistically significant at a confidence level of 99%. Thus, no increasing or decreasing linear tendency can be associated with the evolution of the period T.

We also calculate the standard deviation

$$\sigma(t) = \sqrt{\frac{1}{Y-1}\sum_{k=1}^{Y}[x_k(t)-\mu(t)]^2}\,. \tag{9}$$

By using these quantities, the normalized time series is thus defined as

$$z(t) = \frac{x(t)-\mu(t)}{\sigma(t)}\,. \tag{10}$$

Figure 3 illustrate this definition and shows that this procedure remove the main seasonality present in $x(t)$.

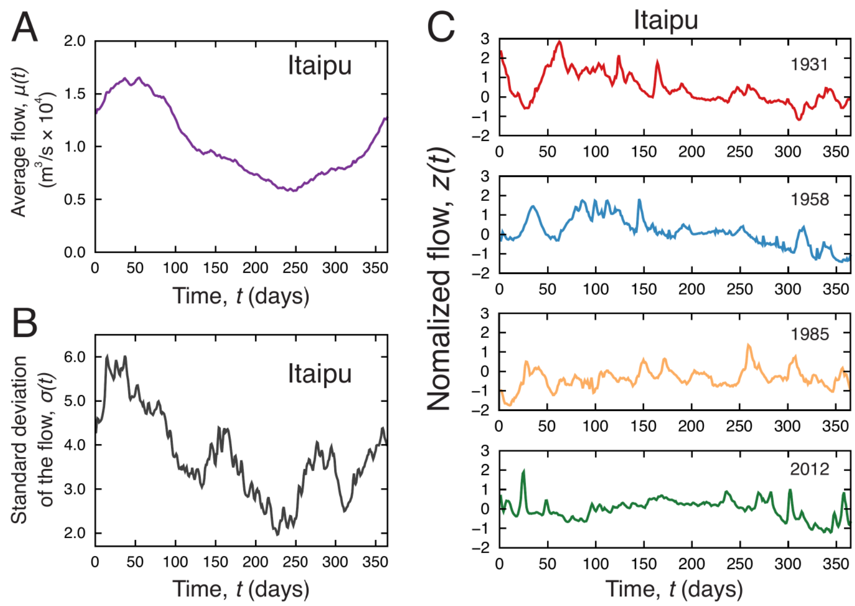

Figure 3. Definition of the normalized time series. **(A)** Average value of the flow rate, $\mu(t)$, as a function of the day of the year t at the Paraná River close to the Itaipu Dam. Notice that $\mu(t)$ makes clear the natural seasonality of the flow rates. **(B)** Standard deviation of flow rate, $\sigma(t)$, as a function of the day of the year t at the same location. We further observe that large fluctuations are observed when the average $\mu(t)$ is also large. **(C)** Examples of normalized time series, $z(t) = [x(t) - \mu(t)]/\sigma(t)$, for flow the rates at the Paraná River close to the Itaipu Dam in four years (as indicated in the plots). We note that the normalization practically removes the trends associated with the natural seasonality.

An important question regarding $z(t)$ is related to its probability distribution. Several works have pointed out that an universal distribution may describe the empirical distribution of $z(t)$ independent of particularities of the river [12, 15, 21]. The large number of time series in our dataset may provide a definitive answer to this question. To do so, we evaluate the aggregated distribution of $z(t)$ for all time series. Figure 4 shows this distributions where a good collapse is observed, further corroborating the hypothesis that the empirical distribution of $z(t)$ is universal among different rivers. Another questions is whether this empirical universal distribution could be described by a some functional form. In this context, the Bramwell-Holdsworth-Pinton (BHP) model [35], which describes the magnetic fluctuations in the classical XY-model close to criticality, and the first-order extreme value distribution (or the Gumbel distribution [36]),

$$P(z) = \frac{1}{\theta} e^{\frac{\lambda - z}{\theta} - e^{\frac{\lambda - z}{\theta}}} , \tag{11}$$

where θ and λ are fitting parameters, have been used to fit these empirical distributions. In Fig. 4, we compare the empirical distributions with the Gumbel form and a good agreement is in fact observed (similar agreement is obtained for the BHP model). The Gumbel distribution is related to the distribution of the maximum of a set of $n \to \infty$ random numbers drawn from a distribution that asymptotically decays faster than any power law. The agreement with this distribution thus suggests that the normalized fluctuations $z(t)$ may be also modeled as an extreme value process.

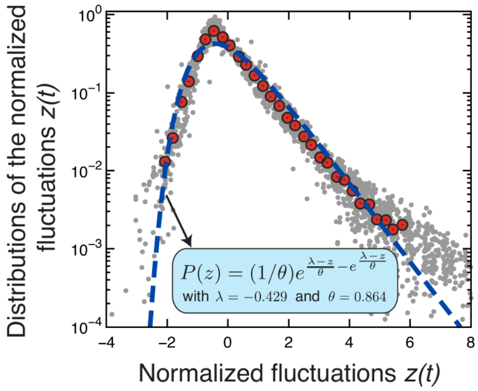

Figure 4. Universal behavior of the probability distributions of the normalized flow rates. The gray dots show the distributions of the aggregated values of the normalized fluctuations $z(t)$ for each time series in our data set. We note the good collapsed of these distributions, suggesting that distributions of $z(t)$ may have a universal form. The red circles are the average values over all distributions and the dashed line is the Gumbel distribution (Eq. 11) adjusted to the average values via least square method (the best fitting parameters are shown in the plot).

As our last concern, we study whether there is long-term memory in the time series of river discharges. To investigate this hypothesis, we have considered the normalized time series $z(t)$ grouped by year and employed the detrended fluctuation analysis (DFA) [37, 11], which can summarized in four steps: i) We define the integrated profile

$$Z(t) = \sum_{t'=1}^{t} z(t')\,; \tag{12}$$

ii) We split $Z(t)$ into $N_m = N/m$ non-overlapping segments of size m, $Z_{\nu,m}(t)$, where N is the length of the series and ν is an index for the segments; iii) For each segment a local polynomial trend (here we have used a first order polynomial, but higher orders do not alter our results) is calculated and subtracted from $Z_{\nu,m}(t)$, defining

$$(\Delta Z_{\nu,m})^2 = \frac{1}{m-1}\sum_{t=1}^{m}[Z_{\nu,m}(t) - p_\nu(t)]^2 \tag{13}$$

where $p_\nu(t)$ represents the local trend in the ν-th segment; iv) Finally, the fluctuation function

$$F(m) = \left[\frac{1}{N_m}\sum_{\nu=1}^{N_m}(\Delta Z_{\nu,m})^2\right]^{1/2} \tag{14}$$

is calculated. If $z(t)$ is self-similar, $F(m)$ presents a power-law dependence on the time scale m, that is, $F(m) \sim m^h$, where h is the Hurst exponent. For $h > 0.5$ or $h < 0.5$ the series is long-range correlated, whereas for $h = 0.5$ its is uncorrelated or present short-range correlations (exponential-like). The value of h must also be around 0.5 for random shuffled versions of $z(t)$, otherwise the DFA may lead to false correlations associated with a possible scaling free nature of the distribution of $z(t)$ (which not happens in our case). Figure 5A shows an working example of DFA for the normalized fluctuations $z(t)$ of the Paraná River at the Itaipu station in the year of 1937. We note that this log-log plot linearizes the relationship between $F(m)$ and m, so that the slope of this curve is the value of h. We have thus estimated the value of h through linear regression on this log-log relationship, and for this case, $h = 1.3$ for the original series and $h = 0.5$ for a shuffled version. By employing the same procedure, we estimate h for all series grouped by year and Fig. 5B shows the probability distribution of h. We verify that h is normally distributed around 1.16 with standard deviation of 0.19. We further note that the shuffled versions of $z(t)$ display h values distributed around 0.50 with standard deviation of 0.05. Thus, our results agree with previous works [17, 7, 11, 16, 19, 20, 23, 18] regarding the existence of long-range correlations in the normalized time series associated with the river discharges. However, the values of h show important fluctuations and this range of variation may be useful in a classification of rivers and could prompt other investigations trying to related h to particularities of the river and/or of the year under analysis.

Summary and conclusions

We have studied a large set of time series of river discharges covering 53 Brazilian rivers and spanning more than eighty years. By using permutation entropy and complexity, we showed that it is possible to associate a period T to the natural seasonality of these time series. We investigated the possibility of T displaying evolutive facets by estimating its value within different time intervals of the time series. Linear regressions showed that T does not present a linear tendency of increasing or decreasing. We have also studied normalized versions of the time series and a universal distribution was found regarding all complexity and differences of rivers under analysis. We argued that a Gumbel distribution can be adjusted to the empirical data, which somehow may connect the normalized fluctuations with extreme value processes. Finally, we analyzed the long-term memory present in these time series via DFA. We showed the Hurst exponent actually confirm the existence of long-range correlations, but that its value displays a range of variation depending on the river and on the year under analysis. This last finding can be eventually used for promoting a classification of rivers and also prompt other questions on possible relationships between h and particularities of the river and/or of the year under analysis. Our findings thus contribute to consolidated previous results (regarding the distributions of the normalized fluctuations [12, 15, 21] and long-range correlations [17, 7, 11, 16, 19, 20, 23, 18]) and also shed new possibilities for investigating these series which may find implications for modeling and forecasting river flows.

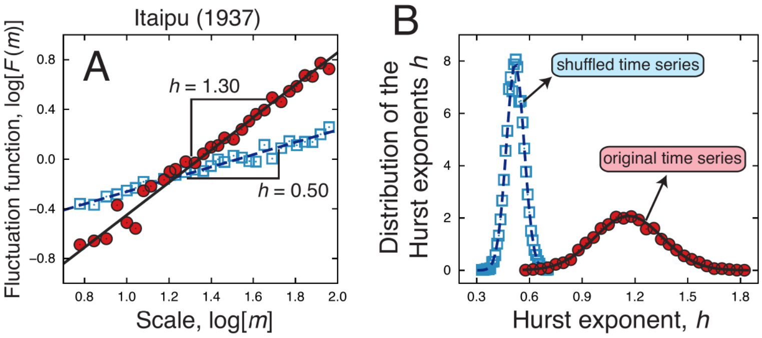

Figure 5. Longe-range correlations in the normalized flow rates. **(A)** An example of the detrended fluctuation analysis (DFA) employing the normalized flow rate $z(t)$ at the Paraná River close to the Itaipu Dam in year of 1937. This plot shows the logarithm of the fluctuation function $F(m)$ as a function of the logarithm of the temporal scale m for the original series (red circles) and for a random shuffled version (blue squares). We note that $F(m) \sim m^h$, where h is the Hurst exponent, which is $h = 1.30$ for the original series and $h = 0.50$ for shuffled version (as would be expected). This result indicates the presence of longe-range correlations in the temporal evolution of $z(t)$. **(B)** We have applied DFA to all time series in our dataset and obtained the value of Hurst exponent h for each one and for each year. In this plot, we show probability distribution of h for the original time series (red circles) in comparison with a Gaussian distribution of mean 1.16 and standard deviation 0.19 (continuous line). We further show the distribution of h for the shuffled versions of the series (blue squares) in comparison with a Gaussian distribution of mean 0.50 and standard deviation 0.05 (dashed line). We have thus confirmed that the longe-range correlations are present in all time series of our dataset.

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