NUMERICAL SIMULATION OF LINEAL CONSOLIDATION IN LAYERED SOILS UNDER STEP LOADS

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Abstract Consolidation processes in layered soils are numerically simulated using the electric analogy and the code Pspice. Loads that produce the settlements may be constant or a step function. The proposed model, that makes good use of the powerful algorithms implemented in the circuits’ simulation codes, is applied to heterogeneous layered soils.

1. INTRODUCTION

Consolidation process in soils is governed by a diffusion equation, where the dependent variable is the excess pore pressure, with boundary conditions of first and second class [1,2]. Assuming a constant consolidation coefficient the solution of the process can be obtained analytically. In this work, we present an efficient and reliable model based on electrical analogy and solved by a standard circuit simulation code providing the numerical solution of this engineering processes in layered soils. The design of the model follows the rules of the network simulation method [3], a procedure already applied, successfully, in other engineering fields such as tribology, elasticity, electrochemical, coupled fluid and flow problems, etc. [4,5,6]. Network method takes advantages of the powerful computational algorithms that are implemented in the simulation codes of electric circuits, which are capable of solving sharp non-linear processes with relatively small computational times.

The design of the model starts from the finite difference equation that derives from the spatial discretization of the partial differential equation of consolidation. As all the terms of the equation are linear, they are implemented by simple electrical components (resistors or capacitors) allowing a direct numerical calculation. By using a suitable analogy between geotechnical and electric variables (effective pressure and electric potential), the terms of the equation are considered as currents that converge in the only node of the network where they are balanced according to the topology of the equation. The potential in the node adjusts providing the pressure solution in order that the balance is fulfilled. Heterogeneities are
assumed in the model by defining successive layers of different physical properties while loads are introduced by software in the electric components that implement the boundary conditions. Numerical simulation provides (time dependent) effective pressures with errors below 1% in each cell for grids of more than 50 volume elements [3].

2. THE GOVERNING EQUATION AND THE DESIGN OF THE MODEL

2.1. Consolidation equation

Consolidation process is ruled by the following equation (1) and boundary (2) an initial (3) conditions:

\[ \frac{C_v}{\Delta z^2} \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \]  
(1)

\[ \frac{\partial u}{\partial z(z=0)} = 0, \quad u(z=H) = 0 \]  
(2)

\[ u(t = 0) = u_0 \]  
(3)

where \( u \) is the excess pore pressure, \( z \) and \( t \) are the independent location and time coordinate, \( C_v \) the consolidation coefficient, \( H \) the height of the domain and \( u_0 \) the initial excess pore pressure. For heterogeneous multi-layered soils, \( C_v \) has different value for each layer. As regards initial conditions, each incremental load carries the excess pore pressure to a new value from which consolidation process starts newly.

2.2. The network model

For the design of the model, the nomenclature of Figure 1 is used. The spatial discretization of governing equation is written as:

\[ \frac{C_v}{\Delta z} \left( \frac{\Delta u}{\Delta z(z+\Delta z/2)} - \frac{\Delta u}{\Delta z(z-\Delta z/2)} \right) = \frac{\partial u}{\partial t} \]  
(4)

Re-ordering this equation:

\[ \frac{u(z + \Delta z/2) - u(z)}{(\Delta z)^2/2C_v} - \frac{u(z) - u(z - \Delta z/2)}{(\Delta z)^2/2C_v} = \frac{\partial u}{\partial t} \]  
(5)
Figure 1. Nomenclature of the volume element or cell

The terms at the left of this equation are lineal and, according to the Ohm’s law, may be implemented by simple resistors electrically connected between the ends of the cell and the central node, Figure 2. The value of these resistors is

$$R = \frac{(\Delta z)^2}{2C_p}$$

(6)

As regards the term of the right side, it is implemented by a capacitor of capacity unity since in this component the current is proportional to the change in the voltage at its ends. The capacitor is connected between the central node and the common reference node (earth), Figure 2.
3. APPLICATIONS

Firstly, a comparison between a soil composed of a single layer and a soil composed of two layers is performed, in the case of a uniform load applied on the ground surface. In both cases the total thickness of the model is the same (H = 6 m). The soil of the second sample consists of two layers, an upper layer of 2 m. thick and a lower layer of thickness 4 m. The value of the applied load is 1 N/m². For the case of one layer the consolidation coefficient $C_v$ that governs the process has a value of 0.1 cm²/s. In the case of two layers, the upper one has a value 5 times higher (0.5 cm²/s) than the lower layer.

The model on which the calculations are made is made of a mesh of 120 cells. The results of the simulations are shown in Figure 3, where it is represented the excess pore pressure ($u$) as a function of the depth ($z$), for different times during consolidation. A summary of the main data is shown in Tables 1 and 2, where it is also expressed the value of the average degree of consolidation ($\bar{U}$) existing in the soil column.

![Figure 3. Excess pore pressure during consolidation. One layer (up) and two layers (down) soils](image-url)
For the case of a uniform load, the degree of consolidation to any depth is obtained from the expression

\[ U = 1 - \frac{u}{u_o} \]  \hspace{1cm} (7)

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>(u) (z=1)</th>
<th>(u) (z=2)</th>
<th>(u) (z=4)</th>
<th>(u) (z=6)</th>
<th>(U) (%)</th>
</tr>
</thead>
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<tr>
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<td>1</td>
<td>1</td>
<td>1</td>
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<tr>
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<tr>
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<tr>
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<tr>
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<td>0.022</td>
<td>0.043</td>
<td>0.075</td>
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</tr>
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</table>

Table 1. Excess pore pressure and average degree of consolidation for different times. One layer soil.

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>(u) (z=1)</th>
<th>(u) (z=2)</th>
<th>(u) (z=4)</th>
<th>(u) (z=6)</th>
<th>(U) (%)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
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<td>0.0008</td>
<td>0.0018</td>
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<td>0.011</td>
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</tbody>
</table>

Table 2. Excess pore pressure and average degree of consolidation for different times. Two layers soil.

Figure 4. Step load change at \(t=100\) days. Excess pore pressure in three typical locations: \(z=1, 3.5\) and 6 m.

Secondly, it is analyzed the case of a soil formed by the same two layers, as in the previous example, in which a second additional load is applied once some time has elapsed (100 and 1000 days) since the initial load without having finished the total consolidation of the soil. This makes the consolidation process to start again. The value of this second additional load is 3 N/m². The results of this simulation are shown in Figures 4 and 5 that come from the output.
graphic ambient of Pspice. Figure 4 is a case in which step load change occurs at time $t = 100$ days, while for Figure 5 $t = 1000$ days. Vertical scales of these figures has been modified in such a way that final excess pore pressure is zero.

4. CONCLUSIONS
An efficient model using the network method has been developed for simulating soils consolidation process. The program allows to know the time dependence of the excess pore pressure at any depth in heterogeneous soils with layers of different properties. The network method is presented as a very powerful calculation tool, with a great potential in the study of nonlinear consolidation.

5. REFERENCES