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CONTROL OF A URBAN HEAT ISLAND (UHI) BY NUMERICAL SIMULATION

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Abstract. *The Urban Heat Island (UHI) effect is a very usual environmental phenomenon where the metropolitan areas present a significantly warmer temperature than its surrounding areas, mainly due to the consequences of human activities. At the present time, urban heat island is considered as one of the major environmental problems in the 21st century as an undesired result of urbanization and industrialization of human civilization. In this work we combine optimization techniques, numerical simulation and optimal control theory of partial differential equations in order to mitigate the UHI effect in an urban domain. We introduce a well-posed mathematical formulation of the environmental problem (related to the optimal location of green zones in metropolitan areas), we propose a numerical algorithm for its resolution, and finally we present several numerical results.*

1 INTRODUCTION

The Urban Heat Island (UHI) effect is a very usual environmental phenomenon, where the metropolitan areas present a significantly warmer temperature than its surrounding areas, mainly due to the consequences of human activities. Growing population and industrial development of large cities have caused (and continue causing) an increased thermal energy generated during the day. In addition, the proliferation of materials such as asphalt, concrete or brick, promotes the absorption of the thermal energy, which later is released overnight. Because of this, the temperature differences between urban areas and the surrounding suburban or rural areas are larger at night than during the day, and is most strongly marked when winds are very weak (for example, in the metropolitan areas of Barcelona, Mexico City, Tokyo or New York, the differences have come to be nearly ten degrees [1]).

At the current time, urban heat island is considered as one of the major environmental problems in the 21st century as an undesired result of urbanization and industrialization of human civilization. Due to the importance of the problem, a vast research effort has been dedicated and a wide range of scientific literature is available for the subject, mainly from a engineering point of view (the interested reader can see, for instance, the review articles [2, 3, 4] and the references therein). However, as far as we know, the mathematical approach to the problem has been much more poorly attended [5, 6].

The present study was carried out in order to introduce and develop a mathematical framework to deal with this environmental problem. So, in section 2 we propose a mathematical model to simulate air temperature and velocity in an urban 2D or 3D domain, where there may be (or not) green zones. In section 3, in order to mitigate the effect of UHI in this domain, we deal with the problem of the optimal location of parklands. Finally, in section 4, we present some numerical results for a 2D simplified domain.

2 NUMERICAL SIMULATION

We consider a domain $\Omega \subset \mathbb{R}^n$ (with $n = 2$ or 3), corresponding to the air layer over a city, possibly including some green zones $\Omega_k \subset \Omega$, $k = 1, \ldots, N_{GZ}$. We suppose that the domain boundary $\partial\Omega$ is divided into $\partial\Omega = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_s \cup \Gamma_r \cup \Gamma_w$, where the different parts of the boundary (as can be seen in Fig. 1 for a schematic representation of the two-dimensional case, and in Fig. 2 for the three-dimensional one) are defined by:

- Γ_1 represents the inflow open boundary,
- Γ_2 represents the top open boundary,
- Γ_3 represents the outflow open boundary,
- $\Gamma_s = \cup_{i=1}^{M} \Gamma_{s_i}$ represents the bottom boundary. It includes the bottom boundary of green zones (Γ_{g_k}, $k = 1, \ldots, N_{GZ}$) and the paved area ($\Gamma_s - \cup_{k=1}^{N_{GZ}} \Gamma_{g_k}$). Without any loss of generality, we can assume that $\Gamma_s \subset \{\mathbf{x} \in \mathbb{R}^n \ : \ x_n = 0\}$.

- Γ_r represents the bottom boundary associated to the roofs of buildings, and
- Γ_w represents the bottom boundary associated to the walls of buildings.

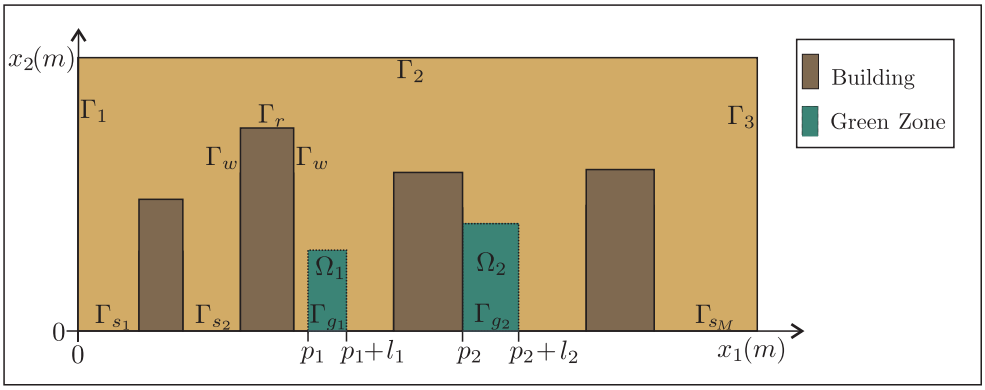

Figure 1: Schematic representation of the air domain over a city in the 2D case, showing the different elements of the problem. In this configuration four buildings ($M = 5$) and two green zones ($N_{GZ} = 2$) are considered.

We need to simulate the air temperature in the domain Ω and, in order to do it, we propose a microscale climatic model [7] with the following state variables:

- Air velocity $\mathbf{u}(\mathbf{x}, t)$ (m/s) and pressure $p(\mathbf{x}, t)$ (m^2/s^2), solutions of the system of partial differential equations:

$$\begin{cases} \dfrac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} - \nabla \cdot (K_m \nabla \mathbf{u}) + \nabla p = \dfrac{\theta}{\theta_{ref}} \mathbf{g} \\ \qquad\qquad\qquad\qquad -c_{df} \displaystyle\sum_{k=1}^{N_{GZ}} LAD_k 1_{\Omega_k} \|\mathbf{u}\| \mathbf{u} \quad \text{in } \Omega \times (0, T), \\ \nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega \times (0, T), \\ \mathbf{u} \cdot \mathbf{n} = 0 \quad \text{on } (\Gamma_r \cup \Gamma_w \cup \Gamma_s) \times (0, T), \\ \mathbf{u} \cdot \mathbf{n} = -u^* \quad \text{on } \Gamma_1 \times (0, T), \\ \mathbf{u} \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_2 \times (0, T), \\ \mathbf{u} \cdot \mathbf{n} = u^* \quad \text{on } \Gamma_3 \times (0, T), \\ \mathbf{u}(0) = \mathbf{u}_0 \quad \text{in } \Omega, \end{cases} \tag{1}$$

with θ_{ref} giving a reference temperature, $\mathbf{g}$ representing the gravity acceleration, $LAD_k(x_n)$ (expressed in m^2/m^3) giving the leaf area density of vegetation at height x_n, 1_{Ω_k} denoting the indicator function of the green zone Ω_k, and u^* and $\mathbf{u}_0$ corresponding to the given boundary and initial conditions.

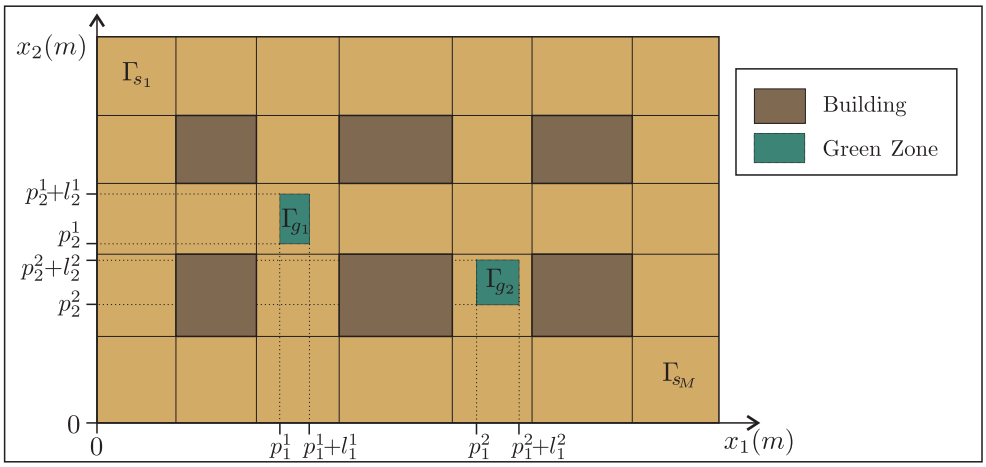

Figure 2: Schematic representation of the air domain over a city in the 3D case, showing the different elements of the problem. In this configuration six buildings ($M = 29$) and two green zones ($N_{GZ} = 2$) are considered.

- Air temperature $\theta(\mathbf{x}, t)$ (K), solution of the system:

$$\begin{cases} \dfrac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta - \nabla \cdot (K_h \nabla \theta) = \displaystyle\sum_{k=1}^{N_{GZ}} LAD_k 1_{\Omega_k} \frac{\theta_f^k - \theta}{r} \quad \text{in } \Omega \times (0,T), \\ \theta = \theta_{in} \quad \text{on } \Gamma_1 \times (0,T), \\ \nabla \theta \cdot \mathbf{n} = 0 \quad \text{on } (\Gamma_2 \cup \Gamma_3) \times (0,T), \\ K_h \nabla \theta \cdot \mathbf{n} = \gamma_1 (T_{r_w}^4 - \theta^4) \quad \text{on } \Gamma_w \times (0,T), \\ K_h \nabla \theta \cdot \mathbf{n} = \gamma_1 (T_{r_r}^4 - \theta^4) \quad \text{on } \Gamma_r \times (0,T), \\ K_h \nabla \theta \cdot \mathbf{n} = \displaystyle\sum_{k=1}^{N_{GZ}} \left(\sigma_1^k \gamma_1 (T_{r_p}^4 - \theta^4) + \sigma_2^k \gamma_2 (\overline{\theta_f^k}^4 - \theta^4) \right) 1_{\Gamma_{g_k}} \\ \qquad\qquad + \gamma_1 (T_r^4 - \theta^4) \left(1 - \displaystyle\sum_{k=1}^{N_{GZ}} \Gamma_{g_k} \right) \quad \text{on } \Gamma_s \times (0,T), \\ \theta(0) = \theta_0 \quad \text{in } \Omega, \end{cases} \tag{2}$$

  with θ_{in} and θ_0 corresponding to the known boundary and initial conditions.

- Foliage temperature $\theta_f^k(\mathbf{x}, t)$ (K) at parkland Ω_k, $k = 1, \ldots, N_{GZ}$, given by the algebraic equation:

$$\frac{\theta_f^k - \theta}{r} = \sigma_1^k \gamma_1 (T_{r_f}^4 - \theta_f^{k\,4}) + \sigma_2^k \gamma_2 (\overline{\theta^k}^4 - \theta_f^{k\,4}) \quad \text{in } \Omega_k \times (0,T), \tag{3}$$

where the definition of the different coefficients can be seen in Table 1.

Symbol	Definition	Units
ρ	air density	g/m^3
c_p	specific heat capacity	Ws/gK
K_m K_h	diffusion coefficients	m^2/s m^2/s
c_{df}	mechanical drag coefficient at plant elements	-
r	aerodynamic resistance (depending on leaf geometry and wind velocity): $r = \frac{A}{1.1}\sqrt{\frac{D}{\lVert \mathbf{u} \rVert}}$	s/m
A D	empirical parameters related to leaf geometry	$s^{1/2}/m$ m
γ_1 γ_2	$\gamma_1 = \frac{\epsilon_w \sigma_B}{\rho c_p}$ $\gamma_2 = \frac{\epsilon_w \epsilon_s}{\epsilon_w + \epsilon_s - \epsilon_w \epsilon_s} \frac{\sigma_B}{\rho c_p}$ (see full details in [7])	$m/K^3 s$ $m/K^3 s$
σ_1^k σ_2^k	attenuation coefficients related to vegetal mass density: $\sigma_1^k(y) = \exp(-F \int_y^{Z_k} LAD_k(z)\, dz)$ $\sigma_2^k(y) = \exp(-F \int_0^y LAD_k(z)\, dz)$ (see [7])	- -
$\overline{\theta^k}$	mean temperature of the parkland	K
$\overline{\theta_f^k}$	mean temperature of the foliage	K
T_r T_{r_w} T_{r_r} T_{r_a} T_{r_p} T_{r_f}	generic radiation temperature (computed from solar radiation, albedo and emissivity). In particular: radiation temperature on the walls radiation temperature on the roofs radiation temperature on the paved areas radiation temperature on the parklands radiation temperature on the vegetation	K K K K K K

Table 1: Definitions, symbols and units corresponding to the physical coefficients appearing in the microscale climatic model (1) − (3).

To solve the mathematical model given by system (1) − (3), we propose a numerical algorithm consisting of combining a time semi-discretization using the method of characteristics, with a space discretization by the finite element method. This algorithm, which details can be seen in [8], has been completely implemented in the scientific software FreeFem++ [9], and the resultant software is a useful tool to simulate the air temperature and velocity in a given domain $\Omega \subset \mathbb{R}^n,\ n = 2$ or 3.

3 OPTIMAL CONTROL

In this section, we center our attention into an urban domain Ω without parklands, suffering the urban heat island effect. In order to mitigate this effect, N_{GZ} green zones are going to be built in Ω. The vegetation in each zone is already known (a datum of the problem) and, consequently, for $k = 1, \ldots, N_{GZ}$, we know the value of Z_k (expressed in units m), giving the characteristic height of vegetation in Ω_k, and the corresponding positive real function $LAD_k(x_n)$ defined in $[0, Z_k]$. In this situation, $\Omega_k = \Gamma_{g_k} \times [0, Z_k]$, and the problem consist of locating $\Gamma_{g_1}, \ldots, \Gamma_{g_{N_{GZ}}} \subset \Gamma_s$ where the green zones will be built. Moreover, we assume that the total joint measure of the parklands can be assumed fixed, in such a way that there exists a positive number $L > 0$ verifying $\sum_{k=1}^{N_{GZ}} \mu(\Gamma_{g_k}) = L$, where $\mu(B)$ denotes the Euclidean measure of a generic set B (i.e., $\mu(B) = \int_B 1\, d\mathbf{x}$). As we detail in following remarks, with these hypotheses the problem can be formulated as a discrete problem for the two-dimensional case (see Remark 1) and, if rectangular green zones are assumed, also for the three-dimensional case (see Remark 2). For the sake of simplicity, in the following we will assume that $N_{GZ} = 2$, however the extension to a higher number of green zones is straightforward.

Remark 1 *In the particular two-dimensional case* ($n = 2$)*, the subsets* $\Gamma_{g_1}, \Gamma_{g_2} \subset \mathbb{R} \times \{0\} \equiv \mathbb{R}$*. Thus, the location of the green zones (represented by intervals) will be determined in the following way:*

$$\begin{array}{rcl} \Gamma_{g_1} & = & [p_1, p_1 + l_1] \\ \Gamma_{g_2} & = & [p_2, p_2 + L - l_1] \end{array} \tag{4}$$

where, p_1 *and* p_2 *denote, respectively, the initial points of both intervals, and* l_1 *is the length of the first interval. (It is worthwhile remarking here that the total joint measure* L *of green zones is fixed, so the length of the second one is necessarily* $l_2 = L - l_1$*). A graphical explanation of these details can be found in Fig. 1.*

In this case, the location of both green zones is completely characterized by the vector $\mathbf{b} = (p_1, p_2, l_1) \in \mathbb{R}^3$*. We denote by* Γ_{s_k}*,* $k = 1, \ldots, M$*, the parts of the boundary* Γ_s *where a green zone can be located.*

Remark 2 *In the three-dimensional case* ($n = 3$)*, the subsets* $\Gamma_{g_1}, \Gamma_{g_2} \subset \mathbb{R}^2 \times \{0\} \equiv \mathbb{R}^2$*. So, the location of the green zones (represented in this case by rectangles, as shown in*

Fig. 2) will be determined by:

$$\begin{array}{rcl} \Gamma_{g_1} & = & [p_1^1, p_1^1 + l_1^1] \times [p_2^1, p_2^1 + l_2^1] \\ \Gamma_{g_2} & = & [p_1^2, p_1^2 + l_1^2] \times [p_2^2, p_2^2 + (L - l_1^1 l_2^1)/l_1^2] \end{array} \tag{5}$$

where, for $k = 1, 2$, $(p_1^k, p_2^k) \in \mathbb{R}^2$ is the point showing the origin of the corresponding green zone, l_1^1 is the length of the first green zone, l_2^1 represents its width, and l_1^2 is the length of the second green zone. We must note that the width of this second green zone remains fixed, once known the previous dimensions, due to the constraint on the total measure L of the parkland area:

$$l_1^1 l_2^1 + l_1^2 l_2^2 = L \implies l_2^2 = \frac{L - l_1^1 l_2^1}{l_1^2} \tag{6}$$

The location of the green zones is determined by the vector $\mathbf{b} = (p_1^1, p_2^1, p_1^2, p_2^2, l_1^1, l_2^1, l_1^2) \in \mathbb{R}^7$. In a similar way to the two-dimensional case, the subsets Γ_{s_k}, $k = 1, \ldots, M$, represent the parts of the boundary Γ_s where the parks can be located. (A graphical representation of this situation is given in Fig. 2, where all these elements can be found).

New green zones are going to be built in order to improve the comfort of the pedestrians. Then, the location of these green zones should be chosen in such a way that the following objective function (representing the average temperature of air, along a given time interval $[0, T]$, inside a suitable region of the city where pedestrians can walk) be minimized:

$$J(b) = \frac{1}{T(b-a)\mu\left(\Gamma_s \setminus (\Gamma_{g_1} \cup \Gamma_{g_2})\right)} \int_0^T \int_{\left(\Gamma_s \setminus (\Gamma_{g_1} \cup \Gamma_{g_2})\right) \times [a,b]} \theta(\mathbf{x}, t)\, d\mathbf{x}\, dt, \tag{7}$$

where $[a, b]$ represents the typical range for pedestrians (for instance, we can consider that the main part of pedestrians' heads usually lie between the heights $a = 1$ meter and $b = 2$ meters).

We also have to take into account some constraints: we will allow the presence of only one green zone in each area enabled to it, and we will require the existence of two green zones, avoiding the degenerate case with only one parkland of measure L. Then, we suppose that there exist two given thresholds $0 < \mu_{min} < \mu_{max} < L$, and we define the set of admissible controls in the following way:

$$\begin{array}{c} \mathcal{U}_{ad} = \{\mathbf{b} \in \mathbb{R}^{4(n-1)-1} : \ \forall k = 1, 2, \ \Gamma_{g_k} \subset \Gamma_{s_{j_k}}, \text{ for any } j_k \in \{1, \ldots, M\}, \\ \text{with } \Gamma_{s_{j_1}} \cap \Gamma_{s_{j_2}} = \emptyset \ \text{ and } \ \mu_{min} \leq \mu(\Gamma_{g_k}) \leq \mu_{max}\}. \end{array} \tag{8}$$

Thus, the problem of the optimal location of the two green zones can be formulated as the following optimal control problem:

$$\min_{\mathbf{b} \in \mathcal{U}_{ad}} J(b) \tag{9}$$

To solve this problem, first of all, we have to take into account the time-space discretization used to solve the state system (1) − (3). If $J_h^{\Delta t}(b)$ denotes the corresponding discretization of the objective function (7), then the fully discretized control problem

$$\min_{\mathbf{b} \in \mathcal{U}_{ad}} J_h^{\Delta t}(b) \tag{10}$$

can be reformulated as a problem of type MINLP (Mixed Integer Nonlinear Programming). To solve this problem we propose to develop an exhaustive search in the integer variable and use the IPOPT code (an interior-point filter line-search algorithm for large-scale nonlinear programming recently developed by Wachter and Biegler [10]) for solving the associated NLP problems (see [8] for more details).

4 NUMERICAL RESULTS

In order to show the goodness of the methods proposed in previous sections, we present here the numerical results that we have obtained for a 2D domain of $75\,m \times 25\,m$ with four buildings.

4.1 Numerical simulation without green zones

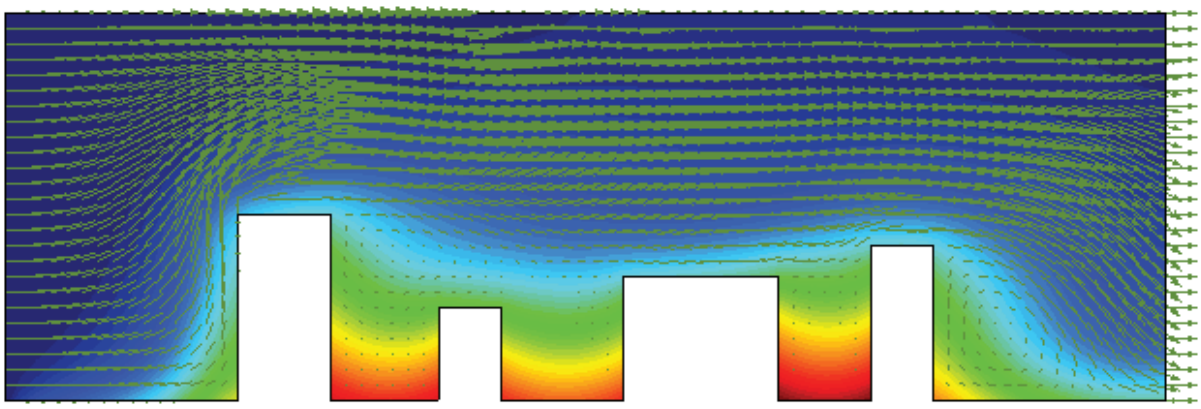

Figure 3: Air temperature and velocity profile, at final time of simulation, before parklands have been built

First, we suppose that there are not any parkland in the domain and we simulate air temperature and velocity, by solving the state system (1)-(3) with the following radiation temperatures: $T_{r_w} = 352.60\,K$, $T_{r_r} = T_{r_a} = 368.28\,K$, $T_{r_p} = 358.90\,K$, and $T_{r_f} = 309.71\,K$. These radiation temperatures have been computed, using the incident solar radiation, the albedo and the emissivity of the material, by the following standard formula:

$$R_{sw,net}(1 - a_m) + R_{lw,dow} - \epsilon_m\,\sigma_B\,T_r^4 = 0, \tag{11}$$

where ϵ_m is the emissivity of the material, σ_B (W/m^2K^4) is the Stefan-Boltzmann constant, $R_{sw,net}$ (W/m^2) is the net shortwave solar radiation incident to the surface, $R_{lw,dow}$ (W/m^2) is the incident longwave solar radiation, and a_m is the albedo of the material.

The results obtained (air temperature and velocity profile), at the final time of simulation, can be seen in Figure 3.

4.2 Optimal location of two green zones

Now, we suppose that two green zones are going to be built in this area, and we solve the problem (9) to obtain their optimal locations. We take same heights of vegetation in both parklands ($Z_1 = Z_2 = 6\,m$), same leaf area density functions ($LAD_1(z) = LAD_1(z)$), and technical bounds $L = 8\,m$, $\mu_{min} = 1\,m$, and $\mu_{max} = 5\,m$.

We obtain that one of the parklands should be located between the second building and the third one, and the other should be on the right side of the last building. The air temperature and velocity, at the final time of simulation, corresponding to this optimal situation, can be seen in Figure 4.

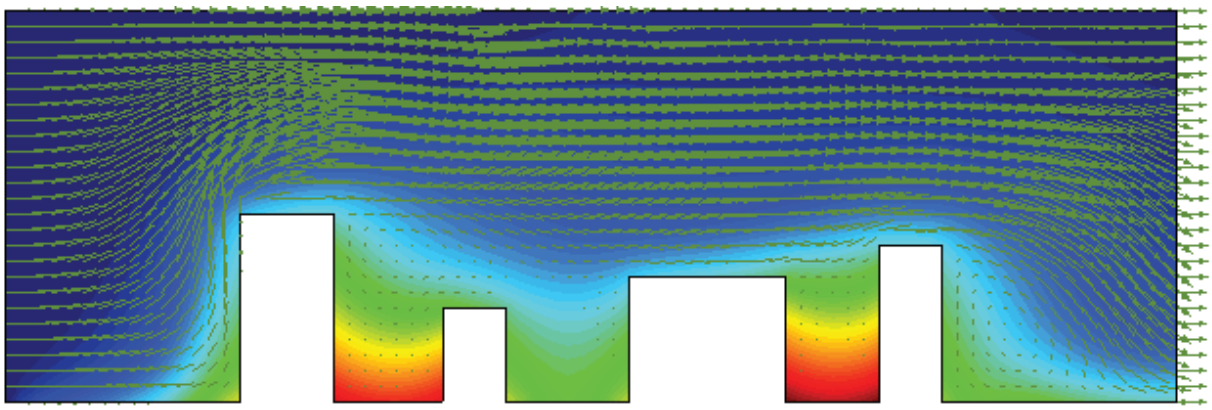

Figure 4: Air temperature and velocity profile, at final time of simulation, corresponding to the optimal location of two parklands

Comparing Figures 3 and 4 we observe that a suitable choice of the parklands locations can help to mitigate, in a significant way, the urban heat island effect in the region under study.

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