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TOPOLOGY OPTIMISATION MODELLING CONSIDERING UNCERTAINTIES

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Abstract The *structural design should ensure suitable working conditions by requiring safety and economic criteria. In most structural designs, the optimal solution is not easily available, as these conditions depend on the bodies' dimensions, materials' strength and structural system configuration. Topology optimisation is a scientific domain that aims to achieve the optimal structural geometry, i.e. the shape that leads to the minimum requirement of material respecting constraints related to the stress state at each structural point. The present study applies an evolutionary approach to determine the optimal geometry of 2D structures by coupling the Boundary Element Method (BEM) and the Level Set Method (LSM).*

The proposed algorithm consists of mechanical model, topology optimisation and structure reconstruction. The mechanical model is composed by singular and hyper-singular BEM algebraic equations. The topology optimisation is performed using LSM. Internal and external geometries are determined by the Level Set function evaluated at its zero level. The reconstruction process concerns the remeshing procedure. As the boundary moves at each iteration, the body's geometry change and a new mesh has to be defined. During the optimisation process, the proposed algorithm introduces automatically internal cavities, according to the intensity of Von Mises stress at the cavity centre.

In spite of the optimal structural geometry be achieved, any information is provided concerning the inherent randomness that exist on all mechanical problem, such as material properties and load intensity, for instance. In this regard, this study aims to evaluate the structural reliability of the optimized structure. The volume target required in the optimisation procedure is analysed by the reliability index, which makes the analysis consistent in the context of safety. The results obtained demonstrate the efficiency of the proposed model.

1 INTRODUCTION

The structural design tries to couple conditions that are normally opposite: safety and economy. In most part of structural designs, the optimal solution, i.e., the solution that couples perfectly such conditions, is not easily available as it depends on the structural dimensions, material strengths and structural system adopted. To solve consistently the optimisation problem some scientific domains have recently emerged [1-3]. Among them, it is worth to mention the topology optimisation, which is a scientific domain aiming to design the geometry of structural components with appropriate safety level using the minimum amount of material. Such type of optimisation aims, therefore, to achieve the optimal structural geometry, i.e. the shape that leads to the minimum requirement of material respecting constraints related to the stress state at each structural point.

The mechanical modelling can be performed by analytical methods based on the knowledge of the theory of elasticity. However, the solutions provided by the theory of elasticity are limited to simplified geometries and boundary conditions. To consider complex geometries, boundary conditions and material constitutive relations numerical methods are required. The present study applies the Boundary Element Method (BEM), which is recognized as a robust and efficient numerical technique capable to handle accurately several types of mechanical problems. Due to its mesh dimension reduction and accuracy in determining stress concentration fields, this numerical method becomes well adapted to solve topology optimisation problems.

Topology optimisation was initially proposed by [4], which analysed the material distribution into a fixed domain. The optimal structural geometry was determined by defining a material density, which may vary from void (no material) until full presence of material. An intermediate material density may also be assumed. [5] introduced the bubble method, which is based on the insertion of new holes in the structure and the subsequent use of a shape optimisation method to determine the optimal size and shape. The simple evolutionary structural method (ESO) presented by [6] progressively removes material from low stress regions based on some acceptation/rejection criteria. This technique was widely applied coupled to finite element models.

[7, 8] used BEM for topology optimisation of two and three dimensional problems. In their ESO approach, the moving geometry of the structure was represented by NURBS explicitly, the spline control points being moved in response to local stress values. The boundary element based topological derivatives concept was used for the first time by [9] for the topology optimisation of thermally conducting solids. The proposed formulation was based on the concept of introducing an iterative material removal procedure in a BEM framework. [10] presented topology optimisation of 2D elastic structures using the BEM with linear elements, inserting small holes in the model around internal points with the lowest values of the topological derivative. [11] presented 3D elastic topology optimisation in a BEM framework with the topological shape sensitivity method for the direct calculation of topological derivatives from stress fields. [12] combined the shape derivatives with topological derivatives to present a level set based optimisation method capable of automatic hole insertion.

An efficient approach in the topology optimisation domain consists of the Level Set Method (LSM). This method was proposed, initially, to model the movement of curves [13, 14]. In the structural context, the desired curve is the body's boundary and its movement or evolution characterizes the new structural geometry. The LSM is based on the solution of Hamilton-Jacobi differential equations, it represents the body's geometry and its evolution by the zero level of an important function denominated Level Set [13-17]. [18] proposed a topology optimisation approach based on coupling BEM-LSM. During the optimisation process, the proposed procedure introduces internal cavities automatically, in order to accelerate the numerical analysis. The zero level set describes the internal and external structural geometry. The initial boundaries as well as its evolutions are represented by NURBS, which smooth the BEM mesh. The internal cavities are punctually included where the von Mises stress is lower than an established threshold.

In the present study, the topology optimisation analysis is developed by the coupling BEM-LSM. The proposed algorithm consists of three parts: mechanical model, topology optimisation procedure and structure reconstruction. The mechanical model is composed by BEM algebraic equations in which singular and hyper-singular integral representations can be applied. The topology optimisation procedure is performed using LSM. The body's geometry is described by the level set function evaluated at its zero level. The velocities of moving boundaries are determined according to the intensity of Von Mises stress at each boundary node. Finally, the reconstruction process concerns the remeshing procedure. As the boundary moves at each iteration, the body's geometry change and a new mesh has to be constructed.

The scientific community has accumulated significant knowledge on mechanical structural behaviour and optimisation solutions. However, most of these developments, as previously presented, consider only deterministic aspects of the phenomena. Actually, the properties related to structural dimensions, material and loading have inherent uncertainties. Such parameters are more accurately represented by random variables with some forms of statistical distributions. When the inherent randomness of these parameters is considered, the structural response has some probabilistic characteristic [19]. Consequently, the problem is properly analysed considering the reliability aspect of the structural problem. In recent years, many efforts have focused on the development of reliability methods and algorithms [20, 21]. Based on such algorithms, the reliability problem is solved more readily.

The present study aims to evaluate the probability of structural failure in structures optimised topologically by coupling reliability algorithms to BEM-LSM model. The probability of structural failure is determined using the classical Monte Carlo simulation and the First Order Reliability Method (FORM). The random variables considered are the material properties and the applied load intensity. The limit state function is assessed by the mechanical responses provided by BEM in some structural geometries defined by BEM-LSM model. The goal of the analysis concerns the assessment of the dependence between the probability of structural failure and the structural volume. As the volume reduces, the growth of the probability of failure is expected. One application is presented in order to illustrate the potential of application of the numerical tool developed.

2 THE BOUNDARY ELEMENT METHOD

The BEM has been widely applied in engineering fields such as contact mechanics and fracture mechanics due to its high accuracy and robustness in modelling strong stress concentration. In addition to this inherent ability of BEM, its mesh dimension reduction makes this numerical method well adapted to solve topology optimisation problems. The integral equations required by BEM can be obtained from the equilibrium equation. Considering a two-dimensional homogeneous elastic domain, Ω, with boundary, Γ, the equilibrium equation, written in terms of displacements, is given by:

$$u_{i,jj} + \frac{1}{1-2\nu} u_{j,ji} + \frac{b_i}{\mu} = 0 \tag{1}$$

in which μ is the shear modulus, ν is the Poisson's ratio, u_i are components of the displacement field, and b_i are body forces.
Using Betti's theorem, the singular integral representation, written in terms of displacements, is obtained (without body forces) as follows:

$$c_{ij}(s,f)u_j(s) + \int_\Gamma P_{ij}{}^*(s,f)u_j(f)d\Gamma = \int_\Gamma U_{ij}{}^*(s,f)p_j(f)d\Gamma \tag{2}$$

in which p_j and u_j are tractions and displacements at the boundary, respectively, the free term c_{ij} is equal to $\delta_{ij}/2$ for smooth boundaries and $P_{ij}{}^*$ and $U_{ij}{}^*$ are the fundamental solutions for tractions and displacements written for the source point s [22, 23].

Differentiating the Eq. (2) with respect to the directions x, y an integral equation written in terms of strains is obtained. Appling, for instance, the generalized Hooke's law the integral representation of stresses for a boundary source point is obtained. Then, the Cauchy formula can be applied to obtain the hyper-singular integral equation which is written in terms of tractions as follows:

$$\frac{1}{2}p_j(s) + \eta_k \int_\Gamma S_{kij}{}^*(s,f)u_k(f)d\Gamma = \eta_k \int_\Gamma D_{kij}{}^*(s,f)p_k(f)d\Gamma \tag{3}$$

in which $S_{kij}{}^*$ and $D_{kij}{}^*$ contain the new kernels computed from $P_{ij}{}^*$ and $U_{ij}{}^*$respectively [22, 23] and η_k is the outward normal vector.

To assemble the system of BEM equations, as usual, Eq. (2) or (3) are transformed into algebraic relations by discretizing the boundary into elements along which displacements and tractions are approximated. In the present study, only linear discontinuous boundary elements are applied. After determining the displacement and tractions fields at the boundary, internal values for displacements, stresses and strains can be achieved. Internal displacements are determined using the integral Eq. (2) with the source point s located at the domain. In such case, the free term c_{ij} becomes δ_{ij}. On the other hand, the stress field at internal nodes is obtained through the integral representation of stresses [22, 23].

3 THE LEVEL SET METHOD

The LSM is a robust technique to simulate and to determine the movement of curves in different physical scenarios. This method represents a particular curve (or surface) Γ and its evolution along time as the zero level (zero level set) of a function ϕ , which is denominated Level Set function. Figure 1 illustrates a classical example of one LS function. This figure presents the evolution of a circle, whose level set function is a cylinder. The LS equation is represented in the following form:

$$\phi_t + \vec{V}.\nabla\phi = 0 \tag{4}$$

in which ϕ_t is the partial time derivative, $\vec{V}$ is the velocity field at the grid points and $\nabla\phi$ is the gradient of the ϕ function which represents the partial derivatives according to x and y directions.

By solving Eq.(4), using finite differences, for instance, the values of ϕ are determined for all points that belong to the analysed domain. Therefore, based on ϕ values, the new structural boundaries are achieved. The LSM is capable to eliminate complexities of movement of curves such as singularities, weak solutions, shocks formation, non-stable conditions of entropy and topology changes that involve interfaces. In the context of numerical analysis, the LSM uses natural and accurate computational procedures. The method represents accurately corners and geometry ends as well as topology discontinuities.

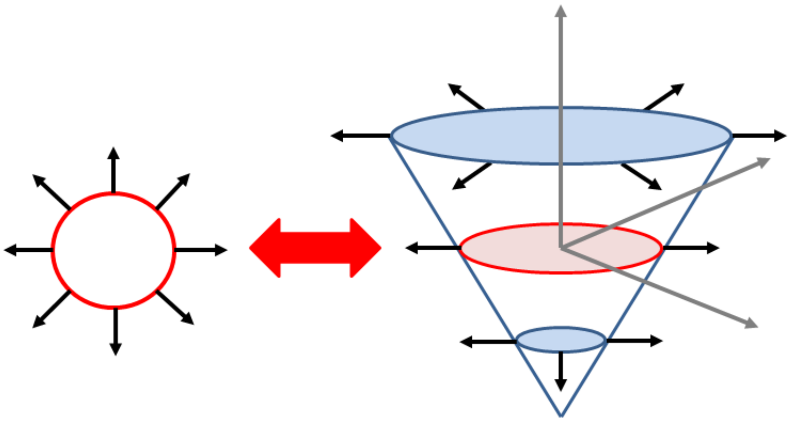

Figure 1: Level Set Function. Circle evolution due to cylinder surface.

4 STRUCTURAL RELIABILITY THEORY AND METHODS FOR ANALYSIS

4.1 General Concepts

The reliability analysis aims at calculating the probability of failure regarding a specific failure scenario, known as limit state. It is worth to mention that reliability R and probability of failure P_f are complementary concepts, in which $R = 1 - P_f$.

The first step in the reliability assessment is to identify the basic set of random variables $X = [x_1, x_2, \cdots, x_n]^T$ for which uncertainties have to be considered. For all these variables, probability distributions are attributed to model its randomness. These probability distributions have to be defined by physical observations, statistical studies, laboratory analysis or expert opinion. The number of random variables is an important parameter to determine the computational time consumed during the reliability analysis. To reduce the size of the random variable space, it is strongly recommended to consider as deterministic all variables whose uncertainties lead to minor effects on the value of the probability of failure.

The second step consists in defining a number of potentially critical failure modes. For each of them, a limit state function $G(X)$ separates the space into two regions as described in Figure 2.

The entire domain is divided into safe, where $G(X) > 0$, and failure where $G(X) < 0$. The boundary between these two domains is defined by $G(X) = 0$, known as the limit state itself. It is worth to mention that an explicit expression for the limit state function is not often possible.

The probability of failure is evaluated by integrating, over the failure domain, the joint density function, [24]:

$$P_f = \int_{G \leq 0} f_X \left(x_1, x_2, \ldots, x_n \right) \; dx_1, dx_2, \cdots dx_n \tag{5}$$

where $f_x = (x_1, x_2, \cdots, x_n)$ is the joint density function of the variables X. As the evaluation of the above integration is impossible in practice, as the joint density function has not an explicit form, alternative procedures have been developed on the basis of the concept of reliability index, β [25]. This parameter is defined by the distance between the mean point and the failure point placed at the limit state function $G(X) = 0$ in the standard space of random variables. The reliability index allows calculating the probability of failure as follow:

$$P_f = \Phi(-\beta) \tag{6}$$

in which $\Phi(.)$ is the standard Gaussian cumulated distribution function.

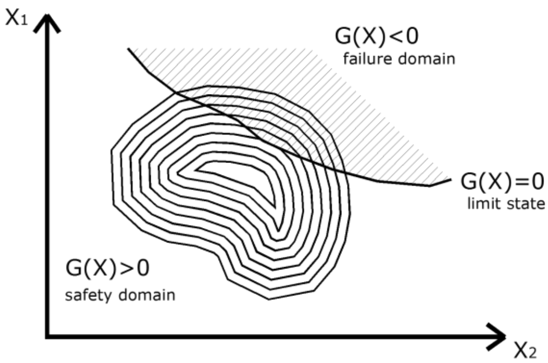

Figure 2: Failure and safety domains considering two random variables. [26]

There are alternative procedures available in order to calculate probabilities of failures, which are based on numerical simulation techniques. Among them, it is worth to cite Monte Carlo simulation. This approach is adopted in this study to determine the probability of structural failure. Another possible approach to achieve the probability of failure is the direct coupling between a mechanical model and the First Order Reliability Method (FORM). The mechanical model allows assessing the limit state function values and FORM the probability of failure. These approaches will be discussed in the following sections.

4.2 Monte Carlo Simulation

Monte Carlo method is a numerical simulation procedure widely used in reliability problems. In this method, a sampling of random variables is used to construct a set of values aiming to describe the failure and safe spaces in order to calculate Eq. (5). The sampling is constructed based on the statistical distribution assigned for each random variable considered in the problem. As this method deals the simulation of the limit state function, as bigger be the sampling adopted accurater will be the space description and accurater will be the probability of failure achieved.
The kernel of this method consists on the construction of a sampling for the random variables involved in the problem, as described in Figure 3.
The probability of failure is calculated, for Monte Carlo simulation, using the following equation:

$$P_f = \int_{G \leq 0} f_X(x_i)dx_i = \int_{G \leq 0} I(x_i) f_X(x_i)dx_i = E[I(x_i)] \tag{7}$$

The function $I(x_i)$ can be estimated as follows:

$$I(x_i) = \begin{cases} 1 \rightarrow G \leq 0 \\ 0 \rightarrow G > 0 \end{cases} \tag{8}$$

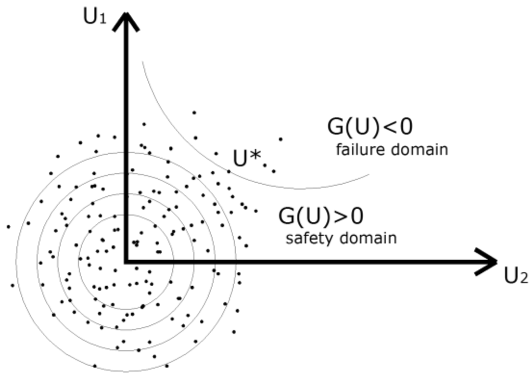

Figure 3: Monte Carlo simulation considering two random variables. [26]

By simulating the limit state function for a convenient range of sampling, the mean value of $I(x_i)$ will be an estimator for the probability of failure as follows:

$$\overline{P}_f = E[I(x_i)] = \frac{1}{N}\sum_{i=1}^{N} I(x_i) \tag{9}$$

The disadvantage of this method is related to the high number of simulations required to compute accurately the probability of failure. Normally, to estimate accurately the probability of failure of 10^{-n}, the number of simulations must be higher than 10^{n+2}or 10^{n+3}. It means, in engineering structures, where the probability of failure is in between $10^{-3}$to 10^{-6}, it is required 10^5 to 10^9 realizations of the limit state function. When complex numerical mechanical models are involved, which lead to high computational time consuming, this method may be not reliable. However, theoretically, when the number of simulations tends to infinity, the probability of failure calculated tends to its real value.

4.3 Direct Method. Coupling BEM-FORM

The basic procedure consists in direct coupling the reliability model First Order Reliability Method (FORM) to the mechanical BEM model. As previously presented, the limit state function defines the safe and failure domains. In general form, the limit state function can be written as follows:

$$G(X) = \text{Resistance}(X) - \text{Solicitation}(X) \tag{10}$$

in which $\text{Resistance}(X)$indicates the structural resistance considering a particular failure scenario and $\text{Solicitation}(X)$represents the external actions. In order to give invariance measure of safety, the random variables, defined in the physical space, are transformed into independent standard Gaussian variables [27], by using appropriate probabilistic transformation. Figure 4 illustrates this transformation, showing that the function $G(X)$ in the physical space is transformed to $H(U)$ in the standard space, where $U = [u_1, u_2, \ldots, u_n]^T$ denote the standard Gaussian variables.
In the standard space, the reliability index β is given by the minimum distance between the failure domain and the origin of the standard space. Therefore such parameter is evaluated by solving the following constrained optimisation problem:

$$\begin{gathered}\text{find: } U^* \\ \text{which minimizes: } \beta = \sqrt{U^T \cdot U} \\ \text{subject to: } \mathrm{H}(U) = 0\end{gathered} \tag{11}$$

The solution of this problem converges to the failure point nearest to the space origin, known as the design point or the most probable failure point X*. In the standard space, the distance between this point and the origin is the reliability index. The reliability index β can be achieved by applying the Rackwitz-Fiessler algorithm [25], directly to the mechanical model.

As the structural resistance is known only point-by-point, the limit state function is implicit. Then, the optimisation problem Eq.(11) is solved using numerical derivatives from the mechanical responses provided by BEM.

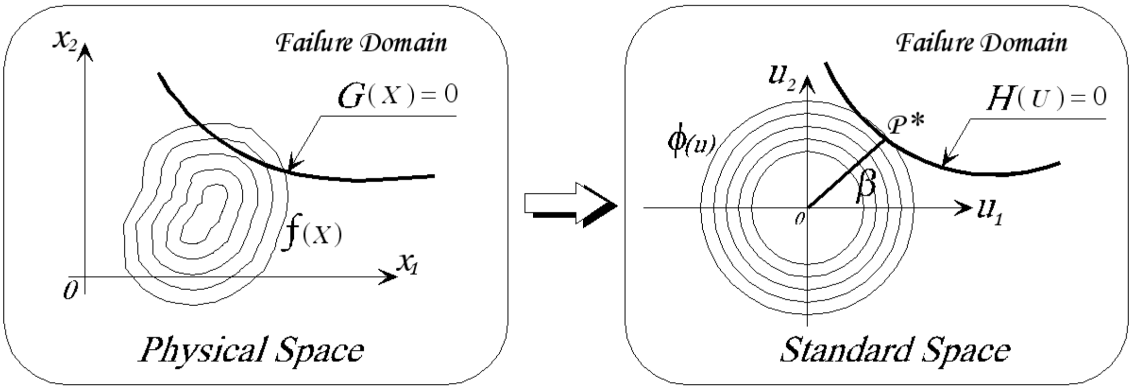

Figure 4: Probabilistic transformation from physical to standard space.

5 TOPOLOGY OPTIMISATION BY COUPLING BEM-LSM

As previously mentioned, in the present study the topology optimisation analysis is developed by the coupling BEM-LSM. The proposed algorithm consists of three parts: mechanical model, topology optimisation procedure and structure reconstruction.

Considering the mechanical model, the BEM requires the structure discretization by nodes and elements to determine stresses and displacements at the structural boundaries. The structural boundaries are described by the level set function evaluated at its zero level which is discretized by linear discontinuous boundary elements.

To solve the topology optimisation problem, an Eulerian mesh defined by a rectangular grid is required. The structure is immersed into this grid and its geometrical coordinates are part of Eulerian mesh [28]. Thus, for each grid point, the Level Set function is determined. This function is defined as the orientated distance between the grid points and the structural boundary. The grid points positioned internally to the boundary are represented by negative values of $\phi(x)$. Positive values of $\phi(x)$ indicate points positioned outside the boundary. Null value of $\phi(x)$ defines the zero level set and, consequently, the structural boundary. Thus, the function $\phi(x)$ is written as presented in Eq. (12), where $\partial\Omega = \Gamma$.

$$\begin{cases} 0 < \phi(x) & to\ \forall \boldsymbol{x} \in \Omega \backslash \partial\Omega \\ \phi(x) = 0 & to\ \forall \boldsymbol{x} \in \partial\Omega \\ \phi(x) < 0 & to\ \forall \boldsymbol{x} \in D \backslash \Omega \end{cases} \tag{12}$$

In this study, Eq.(4) is solved using upwind differences method. As ϕ_t and ϕ are known, a velocity function $\vec{V}$ must be defined. The velocity function depends on the Von Mises stress, Eq.(13), which is evaluated for each grid node.

$$\sigma_{vm} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} \tag{13}$$

in which σ_1, σ_2 e σ_3 are the principal stresses

Based on the Von Mises stress evaluated by BEM, inefficient materials are progressively removed as well as materials are added where necessary. The regions that satisfy the Eq. (14) have removal material condition.

$$\sigma_{vm} < RR\sigma_{max} \tag{14}$$

where RR is the material reduction module defined in the beginning of analysis and σ_{max} is the maximum von Mises stress calculated considering the initial structure. Similarly, the regions where material must be added are identified considering Eq. (15). If Eq.(15) is true material is added.

$$\sigma_{vm} > min(\sigma_{max}, \sigma_y) \tag{15}$$

in which σ_y represents the yield stress of the material in question.

The circular cavities are included into the structural domain in points where the Von Mises stress reaches minimum values. Just one cavity per iteration is inserted.

The Level Set equation is discretized through upwind difference method and the velocities for each grid point are calculated according to the intensity of Von Mises stress. The upwind difference method calculates the partial derivatives of the Level Set function in relation to x and y directions. The time interval used by the method is considered as fictitious. Another input parameter is the number of evolutions carried out at each iteration.

To define the velocity range according to the intensity of Von Mises stress, stress intervals are defined, [18]. The intervals are characterized as a function of σ_{vm}, RR, σ_y e σ_{max}. The velocity criteria adopted in this study, [18], is presented in Eq. (16).

$$\begin{gathered}
\sigma_{vm} \in [0, \sigma_{t1}]: \sigma_{t1} = 0{,}5RR\sigma_{max}, \qquad VN = -1 \\
\sigma_{vm} \in [\sigma_{t1}, \sigma_{t2}]: \sigma_{t2} = 0{,}9RR\sigma_{max}, \qquad VN \in [-1; 0] \\
\sigma_{vm} \in [\sigma_{t2}, \sigma_{t3}]: \sigma_{t3} = 0{,}95\, min\left(\sigma_{max}, \sigma_y\right), \qquad VN = 0 \\
\sigma_{vm} \in [\sigma_{t3}, \sigma_{t4}]: \sigma_{t4} = min\left(\sigma_{max}, \sigma_y\right), \qquad VN \in [0; 1] \\
\sigma_{vm} \in [\sigma_{t4}, \infty]: VN = 1
\end{gathered} \tag{16}$$

The negative velocity indicates that the grid node movement is orientated for inside the boundary. Therefore, negative velocities eliminate inefficient material. Similarly, the movement of grid nodes orientated to the outside boundary direction occurs when the velocity calculated is positive. In this condition, material is added in the structure. Null velocity indicates non-movement of grid/boundary nodes. The criteria presented in Eq.(16) is graphically represented in Figure 5.

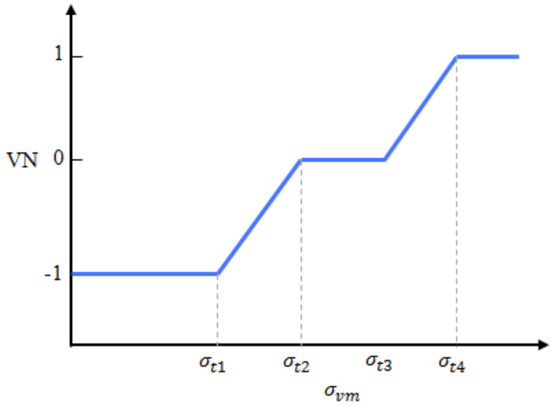

Figure 5: Von Mises stress × Velocity presented by [18].

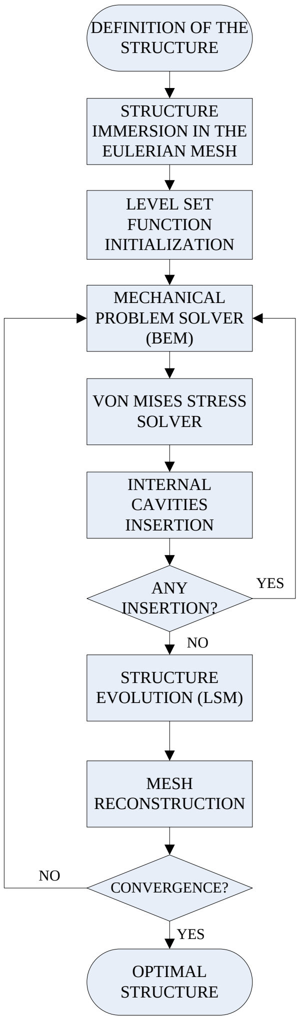

Figure 6: BEM-LSM coupling for two-dimensional structures topology optimisation.

The LSM requires an update mesh procedure, i.e. the mesh considered in the previous iteration must be eliminated and a new mesh is constructed at the new zero level set. Thus, a remeshing procedure must be performed. The developed algorithm is capable to recognize the amount of curves present in the structure (outer and internal boundaries) and achieve the zero level as final boundary. Then, the zero level set is discretized in nodes and linear boundary elements for further analysis of the BEM. This procedure is performed by linear interpolation of the ϕ function along the entire grid. This process is repeated until the desired optimal area be reached. To assist computational implementation by the reader, a flowchart is introduced in Figure 6.

6 UNCERTAINTIES ANALYSIS BY COUPLING BEM-RELIABILY

The pure deterministic optimisation process considers as convergence criterion a target structural volume. Therefore, the numerical amount of material prescribed into design. In this classical optimisation procedure, any information is provided considering the structural safety. In addition to that, any information is taken into account during the optimisation analysis to evaluate the structural safety as the volume reduces.

In this study, the structural safety is evaluated during the optimisation process. Then, the proposed model is capable to associate to the geometry obtained at each optimisation iteration the probability of structural failure. The reliability index and the probability of structural failure are determined using the Monte Carlo simulation and the direct coupling between BEM and FORM. In both cases, the limit state is achieved using the algebraic BEM equations. The limit state function considered in this study concerns the maximum displacement observed into the structural domain.

In this article, one example is presented in order to illustrate the potential of application of the proposed model. It introduces, certainly, an important contribution in the domain of topology optimisation, despite the fact that many improvements be possible.

7 RESULTS

The developed topology optimisation scheme was applied to a cantilever beam subjected to a vertical load. Figure 7 illustrates the initial structure including dimensions (in meters) and load position and intensity.

The following material properties were adopted: Young's modulus (E) $210\ GPa$, Poisson ratio (ν) 0.3 and yield stress σ_y $280\ MPa$. The reduction factor adopted is $RR = 0.25$ and 5 evolutions for each step were performed by the upwind difference method. The fictitious time used is $\Delta t = 0.003$. The grid is composed by a regular mesh of 67×45 points, i.e. 3015 points. The values of Δx and Δy are equal to $0.1\ m$. The goal of this application is the determination of the structural geometry that contains 40% of the initial area.

shows the structural geometry evolution from the initial step until de convergence. The creation of internal cavities must be observed which accelerated the convergence. It is worth to mention that the obtained results agree with classical references, [16,18].

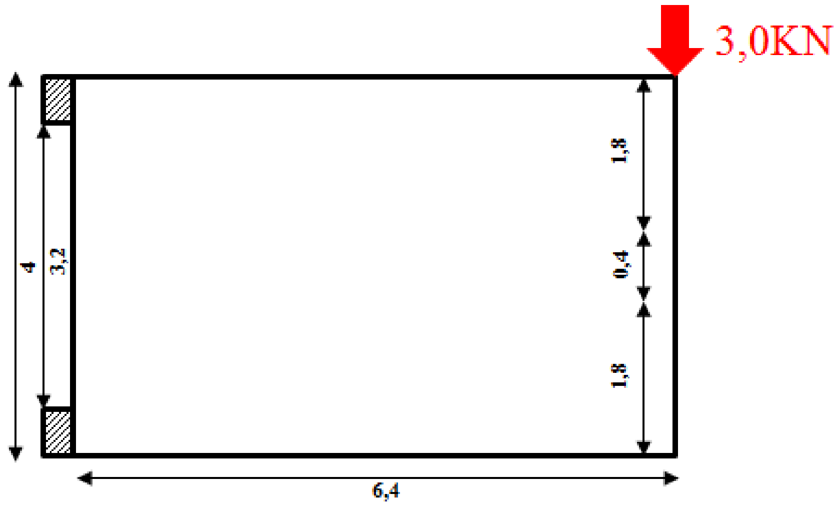

Figure 7: Cantilever beam.

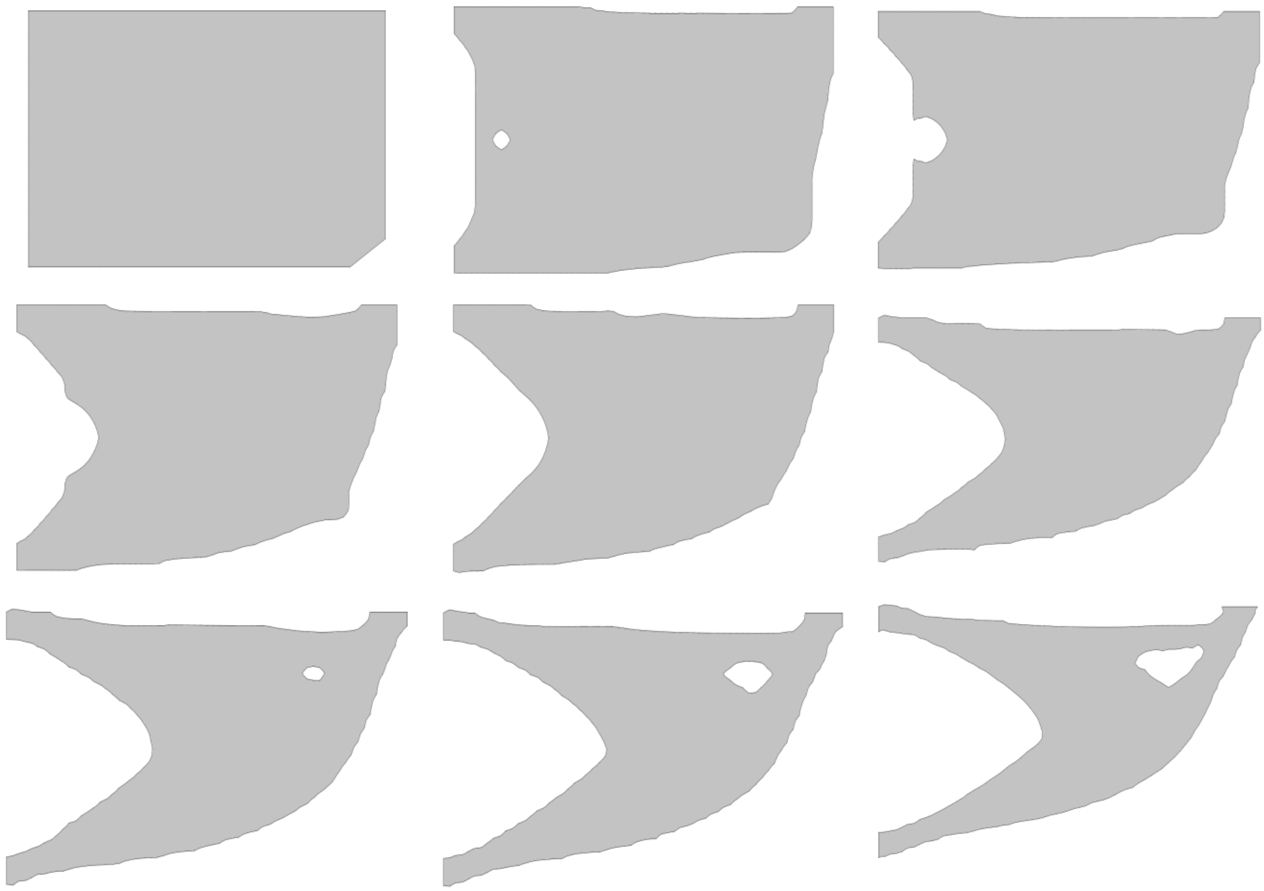

Figure 8: Evolution process.

Figure 9 illustrates the reduction of area during the iterative process. According to this figure a smooth convergence is observed which indicates the numerical stability of the developed procedure. Figure 8 and Figure 10 presents the evolution of the level set function until the convergence. According to these figure, the geometry complexity of the level set function must be mentioned.

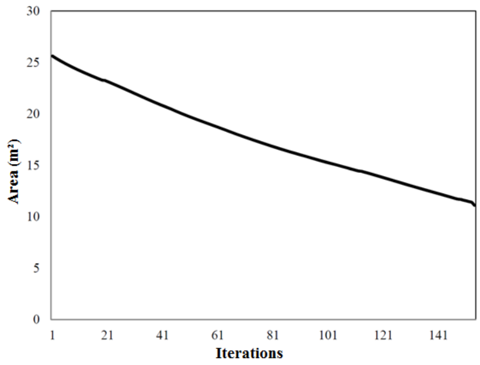

Figure 9: Reduction of area during the numerical procedure.

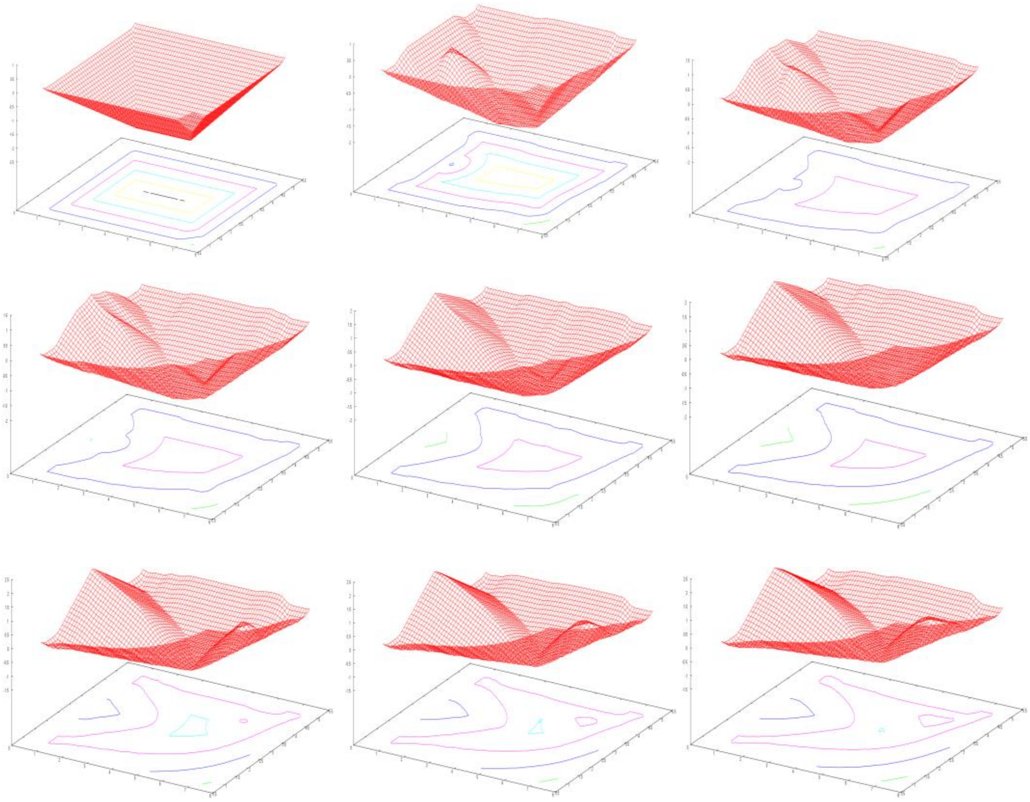

Figure 10: Optimisation process – level set surfaces.

Figure 11 contains the structures chosen for the reliability analysis.

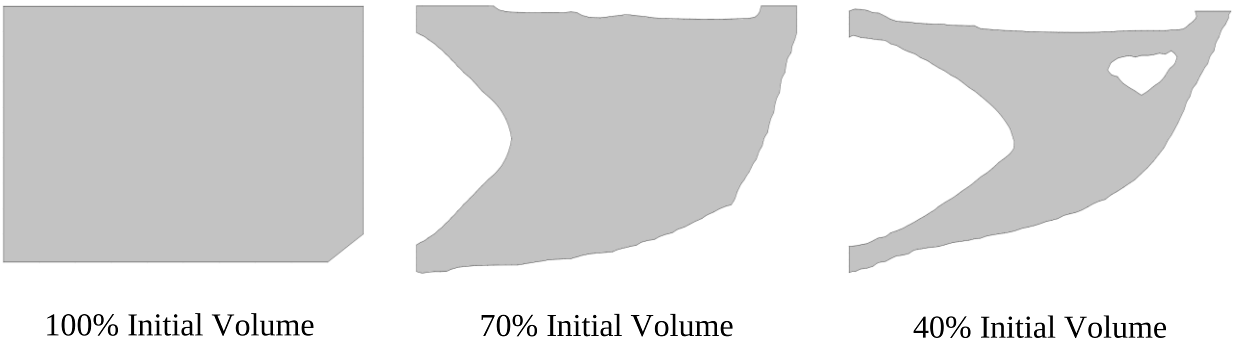

Figure 11: Structures for reliability analysis.

As previously mentioned, the deterministic topology optimisation convergence is based on the target structural volume and any randomness is considered. In order to improve the classical model, reliability analyses were carried out considering the random variables presented in Table 1.

Table 1: Random variables of the problem.

Random Variable	**Probability distribution**	**Mean (μ)**	**Standard deviation (σ)**
Young's modulus [GPa]	Lognormal	210	10
P (applied load) [kN]	Normal	30	3

The limit state function, $g(X)$, is defined in terms of maximum displacement. The vertical displacement must be lower than a specified threshold. Equation (17) presents the limit state function considered in this study.

$$g(X) = R - S \qquad g(X) = \frac{L}{250} - u_{end\_node} \tag{17}$$

in which L is the geometry length and u_{end_node} is the node of the upper right end, where the maximum displacement is observed.
The reliability results obtained are presented in Table 2 and Table 3, where the reliability index and the probability of structural failure are presented.

Table 2: Reliability index for the volume reduction.

Method	β (100% Initial Volume)	β (70% Initial Volume)	β (40% Initial Volume)
FORM	5.1509	4.6835	2.8826
MONTE CARLO SIMULATION	5.1653	4.6877	2.8880

Table 3: Failure probability for the volume reduction.

Method	PF (100% initial Volume)	PF (70% initial Volume)	PF (40% initial Volume)
FORM	1.30E-07	1.41E-06	1.97E-03
MONTE CARLO SIMULATION	1.20E-07	1.38E-06	1.94E-03

As presented in Table 2 and Table 3 good agreement is observed between the reliability index values and failure probability achieved from FORM and Monte Carlo simulation. This behaviour illustrates the numerical stability observed during the analysis.

As expected, the reliability index reduces according to the material elimination. As smaller be the amount of material lesser is the value of reliability index.

[29] introduces the reliability index target according to the expected consequences of failure. These values are shown in Table 4. According to the values presented in Table 4 and based on the probabilistic results achieved, it is observed that the final structure is recommended if consequences of failure local effects and irrelevant be expected. Therefore, such geometry cannot be applied in major structural applications.

Table 4: Reliability index target according to the consequences of failure [29].

Consequences of Failure	Reliability Index Target	Probability of Failure
Very Serious	4.2	1.4×10^{-5}
Serious	3.7	1.1×10^{-4}
Not Serious	3.14	9.7×10^{-4}
Local Effects	2.3	1.0×10^{-2}
Irrelevant	1.0	1.0×10^{-1}

8 CONCLUSIONS

In this study the analysis of topology optimisation was performed by the coupling BEM-LSM. In such type of coupling the advantages of each method are used to obtain optimal structural geometries in an efficient and accurate manner.

According to the results obtained, good agreement was observed among the results achieved by the developed BEM-LSM coupled model and the responses available in literature. Smooth and stable numerical convergence was observed indicating that the numerical model is stable.

The reliability approach was adopted to evaluate the probability of structural failure during the optimisation procedure. As expected, the probability of failure grows as the material is eliminated. In addition to that, such probability is not linearly dependent on the amount of material removed.

In this study, a simple structure was analysed aiming to illustrate the potential of application of the proposed model. Some improvements are due in course such as the consideration of stresses as limit state, the parallelization of the developed code and the continuous evaluation of the probability of failure during the optimisation analysis.

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