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A NUMERICAL STUDY FOR THE SIMPLIFICATION OF LARGE SCENARIOS OF SEEPAGE UNDER DAMS

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Abstract *The seepage flow under concrete dams, besides its dependence on soil permeability and the different total head upstream and downstream, is strongly determined by the geometry of the scenario. However, there are limits in the length and depth of the ground for which the quantity of seepage converges to a maximum value. These limits values, which are not independent, separate two types of scenarios: those that determine an infiltration flow that depends on the geometrical parameters (finite scenarios), and those that determine a discharge flow independent of these parameters (infinite scenarios). In order to reduce the computing times required for the simulation, the last scenarios could be shortened accordingly to their limit values. In this work, the curve that separates the finite and infinite scenarios is determined as a function of the dimensionless groups that characterize the anisotropic medium problem. The points of this curve have been chosen for a quantity of seepage that is a high percentage of its convergence value. For a given data of a current scenario, the obtained curve allows the user to set these groups, to check if the scenario under study is finite or infinite, and hence to reduce it in the second case to the limit values of the domain, so optimizing the computing time. Numerical simulation is carried out by network method.*

1. INTRODUCTION

Steady state groundwater flow under concrete dams, weirs founded and cofferdams in non-homogeneous, anisotropic and permeable soils, is governed by Laplace equation in terms of the total head (or piezometric level) variable. The relation between the soil and the percolated water (seepage) is important in the design of foundations and failures due to piping caused by the excess pressure of water. Therefore, understanding the hydraulic conditions is important to design structures correctly [1,2]. In these scenarios, water flows and pore pressure changes adjust very rapidly reaching steady state conditions nearly instantaneously.

In most cases, the geometry of the typical scenarios may be approximated by a 2-D domain, whose upper layer is separated in two lateral sides by the dam, with impermeable vertical wall at the foot of the dam. Although the boundary conditions are generally of the first (Dirichlet) or second (homogeneous Neumann) type, the complex semianalytical solution is formed by mathematical series of slow convergence. Flow of water through earth masses is in most of cases three dimensional, but it is too complicated and flow problems are usually solved on acceptance that the seepage flow is two-dimensional. Thus, flow lines are parallel to the plane of the structure.

Only a small number of seepage problems have been solved analytically. They have a lot of difficulties due to the boundary conditions of the flow equation that cannot be satisfied in all cases. Harr [3] has solved some simple hydraulic structures and Mandel [4] developed a conformal mapping technique to solve analytically seepage related to excavations and cofferdams.

As an alternative, civil engineers make used of graphical solutions based on the so called flow net construction. The flow net represents two orthogonal families of curves, the flow lines (a line along which particle travel from upstream to downstream) and the equipotential lines (a line along that join points have the same potential head). The equipotential lines intersect flow lines at right angles. Calculation of seepage from a flow net can be tedious and require a lot of time. The main unknowns of interest reached are the steady state seepage loss, the upthrust on the base of the dam and the maximum exit hydraulic gradient. Standard commercial codes can also be used for the numerical solution.

The present work investigates a model, based on network method [5] and seepage theory, capable of solving these problems with sufficient accuracy and negligible computing time, using a standard circuit simulation code such as Pspice [6]. Network simulation method is a numerical tool widely used for the solution of non-lineal, coupled or uncoupled problems, in many engineering fields such as heat transfer, tribology, corrosion, elastostatic and vibrations [7-9]. The method goes beyond the scope of classical electric analogy that is currently used in many text books of different engineering fields, particularly in heat transfer, since it is capable of working with non-lineal and coupled problems type. For the first time, it is applied in the field of geosciences in this work; particularly to groundwater.

The proposed model uses as dependent variable the piezometric head (h), related to the saturated water flow through the Darcy´s equation. This lineal relation allows of deriving the value of the four resistors that form the network model of the elementary cell or volume element. Flow conservation is directly assumed by the Kirchhoff´s theorem referred to the conservation of electric current in the network. Once solved the state steady field h(x,y), programming routines of MATLAB [10] provides the flow lines or the field of values of the streamfunction, ψ(x,y). The representation of suitable flow nets from the solution, with an arbitrary number of iso-lines for each variable, provides a clear vision of the flow and piezometric head distribution through the domain. Applications to isotropic and anisotropic soils are presented.

2. EQUATIONS AND BOUNDARY CONDITIONS

The basic problem-scheme presents in Figure 1, while physical representation shows in Figure 2. A concrete structure is confined between two finite regions with of different piezometric head that causes water flows underground from the larger water level (left) to the lower level (right).

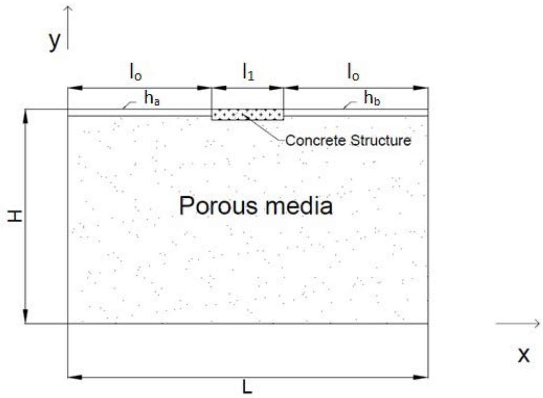

Figure 1: Scheme representation of the problem

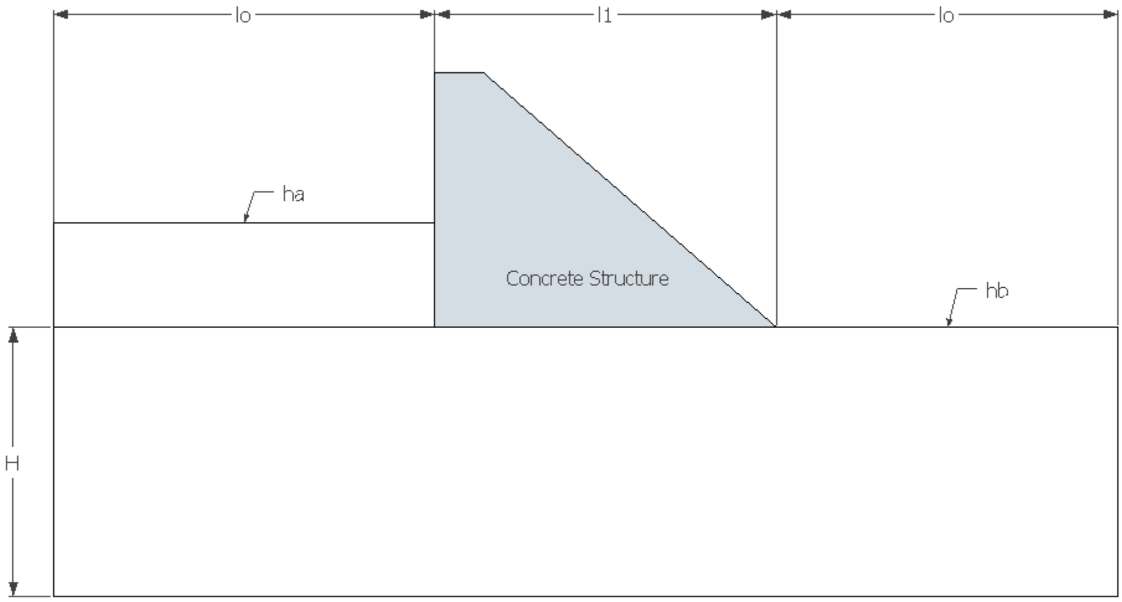

Figure 2: Physical representation of the problem

Under the hypothesis of incompressible fluid and no volume changes in the soil, the governing equation is given by that of Laplace; assuming an anisotropic hydraulic conductivity (permeability), this is written as

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0 \qquad (1)$$

l_1 (m_x):	dam length
l_0 (m_x):	upstream and downstream length
l^*_y (m_y):	length that define
K_x (m_x^2):	Horizontal permeability
K_y (m_y^2):	Vertical permeability
h_a:	Total head upstream
h_b:	Total head downstream
Δh: h_a- h_b	Difference of total head upstream and downstream

while the boundary conditions are described by the equations:

$$\begin{array}{ll} (y=H,\ 0<x<l_0) & h=h_a \\ (y=H,\ l_0\le x\le l_1) & \frac{\partial h}{\partial y}=0 \\ (y=H,\ l_0+l_1\le x\le L) & h=h_b \\ (x=0,y)\ \text{ and }\ (x=L,y) & \frac{\partial h}{\partial x}=0 \\ (y=0,x) & \frac{\partial h}{\partial y}=0 \end{array} \qquad (2)$$

where k_x and k_y are the horizontal and vertical permeability, respectively, and h the piezometric level; h_a and h_b denote the values of Dirichlet conditions applied to the left and right sides of the dam.

3. DIMENSIONLESS NUMBERS

In dimensional analysis theory [11] –or π theorem– it is shown that every complete equation in physics, i.e. every equation, which subsists if an arbitrary change is made in the fundamental units, can be written in the form

$F(\pi_1, \pi_2, \ldots) = 0$

where the π_I are all the dimensionless monomials, independent of one another, which occur in the problem [12]. According to the physical and geometrical characteristics and making use of the discrimination –an extension of the classical dimensional analysis [13]–, it is immediate to define the two dimensionless numbers that rule the seepage problem under dams. These are:

$$\pi_1 = \frac{l_0}{l_1}, \qquad \pi_2 = \left(\frac{K_x}{K_y}\right)\left(\frac{l_y^{*2}}{l_1^2}\right) \qquad (3)$$

where l_y^* denote the characteristic vertical length of the scenario in which seepage is not negligible (the unknown of the problem), and K_x and K_y the intrinsic anisotropic permeability of the soil. Hence, we can get through the relation between the two dimensionless group π_2 = f (π_1), the order of magnitude of the unknown:

$$l_y^* \sim l_1\sqrt{\left(\frac{K_y}{K_x}\right)}\ f\left(\frac{l_0}{l_1}\right) \tag{4}$$

The dependence π_2 = f (π_1) is obtained numerically by simulating the mathematical model of the problem, equations (1) and (2), by network method [5]. To ensure that the numerical results are reliable a grid of 50×50 has been used (this provides errors less than 1% in this kind of problems. Network model runs in the standard code of circuit simulation Ppice [6]. The dependence that separates the region finite from the region infinite is given by the curve shown in Figure 3. The use of this curve starts from the actual data of a given scenario from which we determine the monomials

$$\pi_1 = \frac{l_0}{l_1}, \qquad \pi_2 = \left(\frac{K_x}{K_y}\right)\left(\frac{H}{l_1^2}\right)$$

Locating these values in Figure 3 a point is marked. If this point is above the curve, l_o may be diminished until the point of the curve located at the same value of π_2. If the point is below the curve, the depth of the scenario, H, may be diminish until the value pointes out by the curve for the same value of π_1. So that, the curve marks the lowest limit geometrical values of the scenario that provide the same solution of the actual scenario. The simplification of the scenario to other of small dimension short the total computing times and provides a more depth physical understanding of this kind of problem.

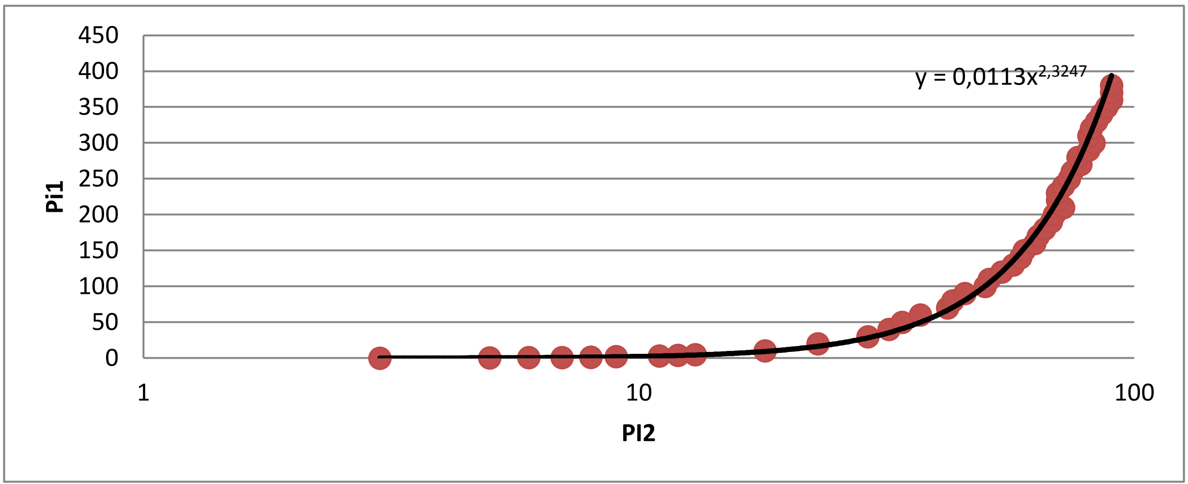

Figure 3. Relation between the two dimensionless groups

CONCLUSIONS

Dimensional analysis has been used to define the dependence between monomials that rule the problem of seepage under dams, Based on this dependence the actual scenarios can be simplified to other with small geometrical dimensions, so reducing the computing time. A standard (universal) curve is obtained base on the former dependence.

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