Abstract

This work presents an element addition strategy for topology optimization of thermally actuated compliant mechanisms under uniform temperature fields. The proposed procedure is based in the evolutionary structural optimization (ESO) method, which has been successfully applied to several optimum material distribution problems, but not for thermal compliant mechanisms. The present paper aims to progress on this work line and an extension of this procedure, based on an additive version of the method, is developed to approach the more complicated case of thermal actuators. As an initial simple approach, this research considers only uniform heating of the system. The adopted method has been tested in several numerical applications and benchmark examples to illustrate and validate the approach.

Keywords: Compliant, topology, optimization, evolutionary, thermal.

1. Introduction

A compliant mechanism is a mechanism that obtains force and motion transmission capabilities through elastic deformation and from the flexibility of its components. Traditional compliant mechanisms function under the application of a force at an input port and generate the desired force or deflection at the output port. Thermal transducers, which can be described as thermally actuated compliant mechanisms, are those mechanisms where the input load is a thermal load instead of a force. These systems function based on the thermal expansion of the compliant mechanism material when heated, converting thermal auction into deflection.

An important area where the use of compliant thermal actuators is advantageous is in Micro Electro Mechanical Systems (MEMS) design, where friction and wear problems prohibit use of conventional rigid-body mechanisms. Early thermal actuators were mostly bimorph actuators made of two materials with different thermal expansion coefficients and were widely used in MEMS. More recently, the idea of using a single material to obtain the same effect based on the geometry of the mechanism was introduced. In these actuators, the structure is not heated uniformly as in classical bimorph systems and deformation is achieved by virtue of electro-thermal non-uniform Joule heating. Changing widths and lengths it is possible to obtain the deformation needed to achieve the displacement of a point in any direction. Therefore, it is clear that the topology of the compliant mechanism is a critical aspect of electro-thermal actuators, which lead to the development of several topology optimization techniques for automatic thermally actuated compliant mechanism design.

Recently topology optimization has been successfully applied to optimize compliant mechanisms in many practical engineering design problems with the use of finite element analysis. The basic idea is to distribute material in a previously defined design domain to obtain a compliant mechanism that deforms in a desirable manner when heated. The original idea for continuum topology optimization was first introduced by Kikuchi and Bendsoe [1], treating the reference domain as if it was made of composite material consisting of a solid and void periodic microstructure. The idea of using a penalized variable density approach (SIMP) for numerically approximating a material-void design problem was first tested in Bendsoe [2] and Rozvany et al.[3]. Here a density variable is associated to each finite element and exponential law is applied to compute effective properties. The recently developed level-set method has been successfully used in the field of optimization [4] and seems to be tremendously promising. Complimenting these methods a number of heuristic or intuition based methods have been proposed as well, like the Evolutionary Structural Optimization (ESO) [5].

In the field of compliant mechanisms topology optimization for continuum synthesis approach, the first applications appeared in Ananthasuresh et al. [6]. A later approach by Sigmund [7] modelled the output load by a spring which captures the nature of the work piece held at the output port of the compliant mechanism. Frecr et al. [8] presented also the synthesis of compliant topologies with multiple input and output ports. Concerning the parameterization methods employed for the solution of the topology optimization problem for compliant mechanisms, we can cite the microstructure based homogeneization method [9], the SIMP interpolation [10], the level-set method [11] or a simple version of the ESO method, successfully applied by this research group for planar compliant mechanisms design with directly applied forces [12].

This paper presents a simple initial approach for thermally actuated compliant mechanism design where the loads arise due to a uniform change in the temperature and applies an extension of the evolutionary structural optimization method in order to understand the viability of this technique to solve these kind of problems. This
procedure has been successfully applied to several structural optimization problems so far, but has not been tested for thermally actuated actuators design yet. This paper shows that it can be used for thermal compliant mechanisms topology design subjected to uniform temperature fields by means of an additive version of the method [13].

2. Optimization problem formulation

The overall objective of thermal compliant mechanisms optimization problem is to maximize the displacement $u_{out}$ of the output port performed on a workpiece, usually modelled by a spring with stiffness $K$, when a uniform rise in temperature ($\Delta T$) is applied. Assuming a linear elastic body occupying a two dimensional domain $\Omega$ where the mechanism is supposed to lie, the topology optimization problem is posed as a material distribution problem where a limited amount of material is to be distributed in the specified design domain, when a uniform heating is applied to the system (see Figure 1a). This objective function can be formulated by using the concept of mutual mean compliance. A unit dummy load is applied at the output port in the direction of the desired displacement, as shown in Figure 1b.

![Figure 1: Design domain subjected to real and dummy load cases](image)

The output displacement $u_{out}$ can be expressed in terms of the so called mutual mean compliance using finite element notation as

$$u_{out} = \{u_2\}^T[K]\{u_1\}$$  \hspace{1cm} (1)

where $[K]$ is the global stiffness matrix of the structure, and $\{u_1\}$ and $\{u_2\}$ denote the nodal displacement vectors due to the thermal load and the unit dummy load, respectively. These displacement vectors can be obtained by solving equilibrium equations for two load cases. The first load case consists of the uniform thermal actuation and the second load case is a pseudo force of unit magnitude:

$$[K]\{u_1\} = \{f_1\} \hspace{1cm} [K]\{u_2\} = \{f_2\}$$  \hspace{1cm} (2)

where $\{f_1\}$ and $\{f_2\}$ are the nodal force vectors containing the equivalent thermal force and the dummy force, respectively. As previously stated, the goal of the optimization problem is to maximize the displacement $u_{out}$ at the output port with a limit of the material resource at our disposal. An optimization problem incorporating these ideas can be written as

$$\text{Minimize} \hspace{0.5cm} -u_{out}$$  \hspace{1cm} (3)

$$\text{Subject to} \hspace{0.5cm} \sum_{i=1}^{n} v_i \leq V^*$$

$$v_i \in [0, v_i^e]$$

where $v_i$ is the actual element volume, $v_i^e$ represent the total volume of the element and $V^*$ refers to the prescribed
total volume in the design. Here we will take $v_i$ as discrete design variables that state the absence ($0$) or presence ($v_i^e$) of an element. The overall objective of the formulated problem is to gradually add elements of volume $v_i^e$ which results in the maximum increase of output displacement until the constrained total volume reaches its given limit.

3. Evolutionary optimization method

The proposed element addition strategy is based on the ESO (Evolutionary Structural Optimization) method. The principle of ESO is that the structure evolves towards an optimum by systematically removing inefficient elements inside the discretized design domain. The first step to begin the evolutionary optimization process is the definition of a design domain with a given boundary and load conditions. Then finite element analysis is performed to determine displacements for the load cases considered, and a sensitivity number $\alpha_i$ is calculated for all elements. Finally volume can be reduced gradually by eliminating under-utilized portions from the structure removing elements of the smallest or highest sensitivity number, depending on the optimization problem. In this case we will adopt the additive version of the method, where elements with largest $\alpha_i$ will be added to the design domain. This process is repeated until the structure reaches the prescribed volume. Here an additive version of the ESO method is adopted, where the sensitivity numbers are used to optimize the topology with a heuristic update scheme. The optimal design of the mechanism is obtained by repeating the cycle of finite element analysis and element additions until the volume reaches the prescribed value, producing the largest increase of $u_{out}$ for the given volume.

The process starts with a discretized design domain full of elements with a low elastic modulus and when an element is added, it is assigned the real isotropic elastic modulus. Flowchart in figure 2 shows steps involved in the evolutionary optimization process and the iteration loop the program follows.

![Flowchart](image)

Figure 2: Flowchart
4. Sensitivity number calculation

The effect of element addition on the output displacement can be assessed using the finite element analysis and computing the sensitivity number $\alpha_i$ for each element. The addition of an element will be traduced in a change of volume $\Delta V_i$, equal to the total volume of the selected element, $V_i$. The change in the output displacement can be determined by considering equilibrium conditions before and after the change. From equation (2) this gives

$\begin{bmatrix} K + \Delta K \end{bmatrix} \{ u_i + \Delta u_i \} = \{ f_i + \Delta f_i \}
\begin{bmatrix} K + \Delta K \end{bmatrix} \{ u_j + \Delta u_j \} = \{ f_j \}
(4)$

By subtracting (4) from (2) and neglecting higher order elements, we get:

$\begin{bmatrix} \Delta K \end{bmatrix} \{ u_i \} + \begin{bmatrix} K \end{bmatrix} \{ \Delta u_i \} = \{ \Delta f_i \}
\begin{bmatrix} \Delta K \end{bmatrix} \{ u_j \} + \begin{bmatrix} K \end{bmatrix} \{ \Delta u_j \} = 0
(5)$

It must be noted that for the first load case of thermal analysis, the applied load depends on the design variables. The main difference in this design problem as compared to directly applied forces in the input port is that sensitivities have to take the nodal load vector change into account. No change in the nodal force vector of the second load case is assumed. Similarly, the variation in the mutual mean compliance can be written from equation (1) in the following way:

$\Delta u_{out} = \{ u_j \}^T \begin{bmatrix} K \end{bmatrix} \{ u_i \} + \{ u_j \}^T \begin{bmatrix} \Delta K \end{bmatrix} \{ u_i \} + \{ u_j \}^T \begin{bmatrix} [K] \end{bmatrix} \{ \Delta u_i \}
(6)$

Substituting the expressions in (5)

$\Delta u_{out} = -\{ u_j \}^T \begin{bmatrix} \Delta K \end{bmatrix} \{ u_i \} + \{ u_j \}^T \begin{bmatrix} \Delta K \end{bmatrix} \{ u_i \} + \{ u_j \}^T \{ \Delta f_i \} - \begin{bmatrix} \Delta K \end{bmatrix} \{ u_i \}
(7)$

and simplifying

$\Delta u_{out} = \{ u_j \}^T \{ \Delta f_i \} - \{ u_j \}^T \begin{bmatrix} \Delta K \end{bmatrix} \{ u_i \}
(8)$

When $i$-th element is introduced to the domain, only the stiffness corresponding to the added element is affected:

$[\Delta K] = [K] - [K] = [K']$

$\{ \Delta f_i \} = \{ f_i \} - \{ f_i \} = \{ f_i \}
(9)$

where $[K']$ is the stiffness matrix of the added element and $\{ f_i \}$ the equivalent thermal load vector on the added element. Therefore, substituting in (8), the final sensitivity number for addition of element $i$ can be defined as:

$\alpha_i = \{ u_j \}^T \{ f_i \} - \{ u_j \}^T \begin{bmatrix} K \end{bmatrix} \{ u_i \}
(10)$

This sensitivity number can be easily calculated using the results available from the finite element analysis of the thermoelastic and static problems for both the thermal load and the virtual unit load, respectively, defined in the two load cases of equation (1).

5. Checkerboard elimination

The sensitivity smoothing is necessary to prevent checkerboard patterns in topology optimization of structures when the finite element method is used. This phenomenon of alternating presence of solid and void elements ordered in a checkerboard like fashion makes the interpretation of optimal material distribution and geometric extraction for manufacturing difficult. Here we will use an enhanced version of the smoothing technique proposed by Li [14]. This approach is based on a weighted average of sensitivity numbers and smoothes the sensitivity value of each element in terms of the surrounding elements’ sensitivity values. It consists of two basic steps: first a reference value at each node of the considered element is calculated by averaging the sensitivities of the elements connecting to this node. Then the smothered sensitivity number for the candidate element is computed by averaging the values of all nodal ones of this element. This approach is known as first-order smoothing technique. Nevertheless, the new sensitivity numbers can be smoothed further, applying a second-order, third-order, or higher
order smoothing. We have often seen that using a low order smoothing technique may be effective at the initial steps of the process but becomes ineffective at the final steps when the volume fraction raises. To overcome this difficulty a continuation method is proposed in this work because it is often advisable to use a variable order smoothing method. This means that the optimization process should start with a low order smoothing until the structure is at least roughly defined, and increase gradually because the optimization algorithm is prone to form checkerboard patterns at the last step of the process. This way the smoothing order can be made adjustable for efficiency of the method.

6. Example

A numerical application example is presented in this section, similar to the problem solved by Jonsmann et al. [15]. The optimum topology obtained using the procedure presented in this paper agree well with solutions obtained by these authors, showing the capability of the method to solve these type of problems. The dimensions of the maximum design domain and position of supports are given in figure 6. The design domain with the dimension of 20 × 20 cm is meshed by 40 × 40 four node elements, totalling 1600 elements. Due to the symmetry of the problem only a half of the full model was optimized. The compliant mechanism is optimized to maximize the displacement of the output port and actuate a 100 KN/m spring, attached to the central right part of the domain, while the rest of the edges are fixed. The thickness of the plate is 1 mm. The material properties adopted for the example are chosen to those of a common plastic material, Young’s modulus E = 1.0 GPa, Poisson’s ratio 0.4 an thermal expansion coefficient $\alpha = 10.10^{-5}$ ºC$^{-1}$. Uniform heating applied is a constant temperature of 100 ºC. The amount of material is by choice limited to 25% of the total area of the design domain, imposed to obtain relatively narrow beams.

![image](image)

Figure 3: Design domain and boundary conditions

The optimum topology for this example is shown in Figure 7a, which is a reasonable solution compared to the optimum topology obtained by Jonsmann et al. Figure 7b shows the deformed geometry of the compliant mechanism. As seen in the figure deformation primarily takes place in the narrow hinges. The evolution history of this example is given in figure 8, where output displacement is plotted versus the number of added elements for the symmetric problem, and thick line represents the gradual improvement of the output displacement until the process is finished.

![image](image)

Figure 4: Optimized topology and deformed geometry
It is apparent from this topological evolution that the method works successfully. It can be observed that the output displacement may jump in the optimization process and suddenly drop, because a wrong choice of ESO method in adding an element may happen, since sometimes differences between sensitivity values and real changes in objective function may happen when a discrete element is added to the design domain. This indicates also that probably the element addition ratio is still high, and a smaller ratio should be used, but computational time would be higher.

![Figure 5: Evolution history](image)

7. Conclusions
This work analyzes the viability of the evolutionary structural optimization (ESO) method for thermal compliant actuators topology design under uniform temperature fields. The addition of elements in regions with highest sensitivity number values leads to an increase in the displacement of the output port under the applied temperature load. Based on the this sensitivity number, the most efficient discrete element addition is achieved during the optimization process, which gives the largest increase of the output displacement for the prescribed volume. The proposed method has been illustrated with a numerical example, showing that solutions converge to the topologies expected if compared to approximated analytical calculations and are very similar to the designs obtained by other authors. Despite the well known shortcomings of the method, results obtained in this paper prove that the proposed additive strategy is capable of producing correct optimal topologies and may be of considerable interest. Furthermore, ESO method could be extended to more complicated thermally actuated compliant mechanisms specifications, like electro-thermal actuators subjected to non uniform temperature fields actuated by Joule heating.

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References
References should be listed at the end of the paper and consecutively numbered. Refer to references in the text with reference number in brackets as [1]. Style the reference list according to the following examples.


