Piezoelectric Energy Harvesting System Optimization

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Abstract
The search for alternative sources of energy has been becoming each time bigger. It has occurred for many reasons, among them the most important are the necessity to develop new sources of clean energy due environment problems and the problem of exhaustive sources of energy in front of a growing demand. In this context, a sector that has attracted much interest are the devices which are able convert other types of energy into electrical energy. This area is named by Power or Energy Harvesting and is based on transducers that provide changes in the energy type. This work is based in these devices able to convert mechanical vibration energy in electrical energy. For these cases, the most used are magnetic, electrostatic and piezoelectric transducers. However, when an electrical circuit is coupled to the transducer the mechanical system is strongly influenced it. This paper proposes a model for Piezoelectric energy harvesting that describes all this interaction between the mechanical and electrical system. The present model is based on Impedance Methodology and was used for find the optimal conditions for the device using Genetic Algorithms. The structure modeled was a piezobeam with free-sliding boundary conditions. The results illustrated that the generated power can be maximized if some optimal conditions are set simultaneously in the mechanical and electrical system of the Piezoelectric Energy Harvesting.

Keywords: Energy Harvesting, Piezoelectric Transducer, Genetic Algorithms

1. Introduction
Energy Harvesting, Power Harvesting or Energy Scavenging is about the act of converting ambient energy in electrical energy (electrical power). In every cases this energy was been wasted or lost before. Normally, the electrical energy converted is stored in a kind of battery to be used later but that don’t prevent the energy be used in the same time that is converted too.

In this form, the Energy Harvesting may be a solution for source energy in many cases, mainly in remote, inaccessible or hostile environments applications where the connection with the electrical energy network is difficult. Frequently, these devices are small, wireless autonomous, like those used in wearable electronics and wireless sensor networks.

The external source can be solar, wind, thermal, salinity gradients and kinetic. In this paper the source is kinetic; specifically, vibration sources that can be anything that have periodic motion. For example the small vibrations of a machine, the motion of walking, even the motion of blood circulation. However, for this conversion to be possible the transducer should transform mechanical energy in electrical energy. The transducers mostly used for this are electromagnetic, electrostatic and piezoelectric. In this work, the harvesting of energy is through piezoelectric transducers.

Piezoelectric transducers are constituted by piezoelectric materials. This material has the ability to directly convert applied strain into electrical charge. According to Cook-Chennault et al (2008) it happens because when a load is applied in the material it causes the molecular structure deformation that in turn causes a separation of the positive and negative gravity centers, resulting in the macroscopic polarization of the material. Piezoelectric transducers are been applied in many types of applications among then we can quote; military sector, implantable devices; shoe inserts, Eels (to get energy from wave energy) and others.

In this context, many studies had been conducted they: Sodano et al. (2002) performed a study to investigate the amount of power generated through the vibration of a piezoelectric plate, as well as methods of power storage; Lesieutre et al. (2002) investigated the damping added to a structure due to the removal of electrical energy from the system during power harvesting; Leffeuvre et al. (2005) constructed an electromechanical structure, trying to optimize the power flow of vibration-based piezoelectric energy-harvesters. The biggest problem when an electric circuit is connected in the transducer is that occurs a interaction between the electrical and mechanical systems. This paper works with a model proposed by Nakano et al (2007) for describes
this interaction. This model is based a two-port network model of a transducer. The methodology is applied for a piezoelectric transducers connected a resistive load and the behaviour of these systems is studied. The Genetic Algorithm is implemented to finding the optimum parameters of the structure and load for maximum power harvested.

2. Two-Port Network Model

To model the harvesting system a two-port network model of a transducer is used connected to the Thevenin equivalent for the vibrating structure and an electric load. This model was proposed by Nakano et al (2007) and can be seen in figure (1).

![Two-port network model](image)

In figure (1) \( f_b \) is the blocked force, \( Z_{ms} \) is the Mechanical Impedance of the system, \( u \) is the velocity, \( f \) is the force on transducer, \( Z_{mt} \) is the Mechanical Impedance of the transducer, \( Z_{et} \) is the Electrical Impedance of the transducer, \( Z_{et} \) is the Impedance of the external load, \( i \) is the current, \( v \) is the voltage on load, \( T_{em} \) and \( T_{me} \) are the transduction coefficients. \( T_{me} \) describes the force produced per unit electric current and similarly \( T_{em} \) represents the voltage generated per unit velocity.

For the transducer the relationship between the mechanical and electrical variables is expressed by:

\[
\begin{bmatrix} f \\ v \end{bmatrix} = \begin{bmatrix} Z_{mt} & T_{me} \\ T_{em} & Z_{et} \end{bmatrix} \begin{bmatrix} u \\ i \end{bmatrix}
\]

where

\[|T_{me}| = |T_{em}|\]

The voltage across the external load is given by:

\[v = -Z_{et}i\]

Substituting equation (3) into the equation for voltage for the transducer given in equation (1) gives the current as a function of velocity:

\[i = -\frac{T_{em}}{Z_{et} + Z_{et}}u\]

Substituting equation (4) into the equation for the force in equation (1) gives the force as a function of velocity:

\[f = \left(Z_{mt} - \frac{T_{em}T_{me}}{Z_{et} + Z_{et}}\right)u\]

The force in the transducer can be expressed as:

\[f = f_b - Z_{ms}u\]

Now, substituting equation (6) in (5) gives the expression for the velocity:

\[u = \frac{f_b}{Z_m - \frac{T_{em}T_{me}}{Z_{et} + Z_{et}}}\]

where

\[Z_m = Z_{mt} + Z_{ms}\]
Power harvested is considered as the power dissipated in the electric load. Under the harmonic excitation this power is:

\[ P_h = \frac{1}{2} \text{Re}[-iv^*] \]  

(9)

where \((-^*)\) means the conjugate complex number.

From equation (3) and (4):

\[ P_h = \frac{1}{2} \text{Re}[Z_{el}] \left| \frac{-Z_{em}}{Z_{el} + Z_{et}} u \right|^2 \]

(10)

3. Piezoelectric Transducer

The piezoelectric transducer model showed in this work was based on previous study of Preumont (2006) and Nakano et al (2007). This model is used to find the mechanical and electrical impedances and transduction coefficients for this transducer.

Figure (2) shows the piezoelectric transducer used in this work. In this picture \(E_f\) is Electric Field, \(T\) is Stress, \(l_t\), \(b_t\) and \(t_t\) are length, width and thickness of transducer.

![Piezoelectric transducer](image)

Figure 2. Piezoelectric transducer

This transducer can be modelled as a uniaxial and element the constitutive equations can be written as:

\[
\begin{bmatrix}
Q \\
S
\end{bmatrix} =
\begin{bmatrix}
\varepsilon^T & d_{31} \\
d_{31} & s^E
\end{bmatrix}
\begin{bmatrix}
E_f \\
T
\end{bmatrix}
\]

(11)

where \(Q\) is Electric displacement, \(S\) is Strain, \(s^E\) is compliance of material under constant electric field, \(d_{31}\) is piezoelectric constant, \(\varepsilon^T\) is permittivity when the stress is constant.

Assuming a harmonic force the constitutive equation can be transformed to:

\[
\begin{bmatrix}
f \\
v
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{C_{mt}} & -D_{31} & 0 \\
-D_{31} & \frac{1}{C_{et}} & 0 \\
0 & 0 & \frac{1}{C_{et}}
\end{bmatrix}
\begin{bmatrix}
q_{mt} \\
q_{et}
\end{bmatrix}
\]

(12)

where \(f\) is force, \(v\) is voltage, \(q_{mt}\) is mechanical deflection and \(q_{et}\) is electrical charge, \(D_{31}\) is the piezoelectric transducer constant, \(C_{mt}\) is mechanical compliance with open electrodes \((q_{et} = 0)\) and \(C_{et}\) is electric capacitance of the transducer for a fixed geometry \((q_{mt} = 0)\). Defining a coupling coefficient, \(\kappa\), by:

\[ \kappa = \frac{|d_{31}|}{\sqrt{s^E \varepsilon^T}} \]

(13)

The others parameters of equation (12) are given by:

\[ \frac{1}{C_{mt}} = \frac{K_a}{1 - \kappa^2} \]

(14)

\[ \frac{1}{C_{et}} = \frac{1}{C (1 - \kappa^2)} \]

(15)

\[ D_{31} = \frac{d_{31} K_a}{C (1 - \kappa^2)} \]

(16)
where
\[ C = \frac{\varepsilon T_A}{l} \]  
(17)

and
\[ K_a = \frac{A}{\varepsilon E_l} \]  
(18)

In these equations C is the capacitance of the transducer with no external load \((f_t = 0)\), \(K_a\) is the stiffness of the transducer with short-circuited electrodes \((v_i = 0)\) and \(A\) is the cross section area.

Finally, the mechanical and electrical impedances and the transduction coefficients are given, respectively, by:

\[ Z_{mt} = \frac{1}{j\omega C_{me}} (1 + j\eta_{mt}) \]  
(19)

\[ Z_{et} = \frac{1}{j\omega C_{et}} (1 + j\eta_{et}) \]  
(20)

\[ T_{me} = T_{em} = \frac{b_{31}}{j\omega} \]  
(21)

where \(\eta_{mt}\) and \(\eta_{et}\) are the loss factors in the mechanical and electrical compliances.

4. Model of Finite Beam
The system investigated in this work was a Euler-Bernoulli beam with a piezoelectric patch bonded on a surface. For a finite beam with a piezoelectric element bounded is necessary to find the uniform equivalent beam for applying this theory. Fig. (3) shows the finite beam element and the cross-section before and after determines the equivalent beam.

![Figure 3. Piezobeam; (a) Finite element; (b) cross-section of beam and equivalent beam](image)

The equation of motion for a uniform Euler-Bernoulli beam is derived by Linear Theory of Elasticity and for flexural vibration motion due to a transverse distributed force per unit length, \(f_x(x,t)\), is given by Bishop and Johnson (1960).

\[ EI \frac{\partial^4 w(x)}{\partial x^4} + \rho S \frac{\partial^2 w(x,t)}{\partial t^2} = f_x(x,t) \]  
(22)

where \(E\) is the Young’s modulus, \(I\) is the second moment of area, \(\rho\) is the density of the material, \(S\) is cross-sectional area and \(w\) is displacement. The solution for this equation in terms of trigonometric and hyperbolic functions is given by:

\[ w(x) = A \sin kx + B \cos kx + C \sinh kx + D \cosh kx \]  
(23)

where \(A, B, C\) and \(D\) are constants and \(k\) is the flexural wave number given by:

\[ k = \left( \frac{\rho S}{EI} \right)^{\frac{1}{4}} \omega^{\frac{1}{2}} \]  
(24)

The mechanical Impedance of the beam depends of the boundary conditions and was determined using the methodology shown by Gardonio and Brennan (2004) using the 200th first modes. The transfer impedance due a force excitation at \(x_i\) and a velocity response at \(x_j\) is given by the inverse of the mobility:

\[ Z_{ij} = \frac{1}{Y_{ij}} \]  
(25)
where \( \psi_n(x) \) is the \( n \)th natural mode, \( \omega_n \) is the natural frequency for the \( n \)th natural mode, \( l \) is the length of the finite beam and \( \eta \) is the loss factor for the material of beam. The natural modes \( \psi_n(x) \) can be obtained in many text books. Here we work with the approach developed by Gonçalves et al (2007).

5. Optimization Procedure

Optimization problems can be defined as the determination of variables so that an objective function reaches an extreme value (maximum or minimum) subjected to some constraints of the problem. Algorithms to solve this kind of problems are often classified in two groups: the classic methods based on the computation of gradient values (derivatives) that provide the search direction of the algorithm \[9\], and the heuristic methods that changes the optimization parameters based on random decisions \[10\]. Despite the popularity of the classical methods, often is not possible to ensure that the final solution found by these strategies is actually the global optimum. That characteristic is dependent on the level of complexity of the optimization problem \[11\]. In this case is usual to apply heuristic algorithms as: genetic algorithms, simulated annealing, particle swarm, etc.

Genetic algorithms are methods of search and optimization that simulates the natural process of evolution, by means of the natural selection of the species described by Charles Darwin \[12\], \[13\]. These algorithms are robust methods and applicable in the solution of various problems. In the general procedure of a basic genetic algorithm, in the first step, an initial random population of chromosomes or individuals is created which represents possible solutions to the problem codified usually in binary code. In the next step, each chromosome is evaluated by a quality measure called fitness that is related to the objective function of the problem. After this, crossover and mutation operators are applied generating a new population of chromosomes. This process is iteratively repeated until a pre-established stop criterion is reached or until a maximum number of generations. The flowchart of the basic genetic algorithm is showed in Fig. (4).

![Figure 4: Flowchart of a basic genetic algorithm](image)

6. Numerical Results

In this work genetic algorithms were implemented to find the optimum parameters that give a maximum power harvested. For this analysis four parameters were chosen to the optimization process: thickness of transducer \( t_t \), thickness of the beam \( t \), mechanical loss factor of the beam \( \eta \) and electrical impedance \( Z_e \). These values were codifed in binary vectors which represent the genetic code of each chromosome. It was used an elitist model which saves the best chromosomes of each generation (elite chromosome) \[13\]. This ensures that the best fitness of each generation only remain the same or evolves over generations. The objective function in this problem, which is also the fitness function of the genetic algorithm, can be described as:

\[
\max J(p) = Ph(p)
\]

where \( J \) is the objective function, \( p \) is a vector with the optimization parameters and \( Ph \) is the power harvested. Important to note that the parameter vector is restricted in a range of values with physical meaning. Figure (5) shows the beam of interest and the Tab. (1) the properties of the system and the piezoelectric material even as the intervals of parameters optimized that are showed in bold. The input of the system was represented in the figure above too and it was a blocked force with unitary amplitude. The load connected was just a resistance representing a battery. The genetic algorithm used to solve this problem has the configurations showed in Tab.(2).
Figure 5. Piezobeam free-sliding with harmonic excitation

<table>
<thead>
<tr>
<th>Descriptions</th>
<th>Symbols</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the beam and transducer</td>
<td>( l )</td>
<td>0.1 ([\text{m}])</td>
</tr>
<tr>
<td>Width of the beam and transducer</td>
<td>( b )</td>
<td>0.02 ([\text{m}])</td>
</tr>
<tr>
<td>Thickness of the transducer</td>
<td>( t_t )</td>
<td>0.00127 - 0.00026 ([\text{m}])</td>
</tr>
<tr>
<td>Thickness of the beam</td>
<td>( t )</td>
<td>0.001 - 0.005 ([\text{m}])</td>
</tr>
<tr>
<td>Piezoelectric constant of material</td>
<td>( d_{33} )</td>
<td>-320 x 10^{-12} ([\text{C/N}])</td>
</tr>
<tr>
<td>Young’s modulus of the transducer</td>
<td>( 1/s_E )</td>
<td>62 ([\text{GPa}])</td>
</tr>
<tr>
<td>Dielectric constant of the transducer</td>
<td>( \varepsilon_T )</td>
<td>3.36452 x 10^{-8} ([\text{F/m}])</td>
</tr>
<tr>
<td>Electrical loss factor of the transducer</td>
<td>( \eta_{et} )</td>
<td>0.003</td>
</tr>
<tr>
<td>Mechanical loss factor of the transducer</td>
<td>( \eta_{mt} )</td>
<td>0.000056</td>
</tr>
<tr>
<td>Density of the transducer</td>
<td>( \rho_t )</td>
<td>7600 ([\text{m}^3/\text{kg}])</td>
</tr>
<tr>
<td>Density of the beam</td>
<td>( \rho )</td>
<td>2700 ([\text{Ns/m}])</td>
</tr>
<tr>
<td>Young’s modulus of the beam</td>
<td>( E )</td>
<td>70 ([\text{GPa}])</td>
</tr>
<tr>
<td>Mechanical loss factor of the beam</td>
<td>( \eta )</td>
<td>0.001 - 0.005</td>
</tr>
<tr>
<td>Electrical Impedance</td>
<td>( Z_{el} )</td>
<td>0 - 5000 ([\Omega])</td>
</tr>
</tbody>
</table>

Table 2: Genetic algorithm configuration

<table>
<thead>
<tr>
<th>Type of selection</th>
<th>Type of crossover</th>
<th>Population size</th>
<th>Participants of tournament</th>
<th>Number of iterations (generations)</th>
<th>Crossover tax</th>
<th>Mutation tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tournament</td>
<td>One point</td>
<td>100</td>
<td>5</td>
<td>50</td>
<td>0.8</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The best chromosome found in five simulations using genetic algorithms with the configuration of Tab. (2) is showed in Tab. (3).

Table 3: Results of the simulation

<table>
<thead>
<tr>
<th>( t_t ) ([\text{m}])</th>
<th>( t ) ([\text{m}])</th>
<th>( \eta )</th>
<th>( Z_{el} ) ([\Omega])</th>
<th>Fitness ([\text{W}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000267</td>
<td>0.001</td>
<td>0.001</td>
<td>713</td>
<td>0.0891</td>
</tr>
</tbody>
</table>

How expected the thickness optimum for the beam is the smaller of the interval because, in this case, the beam is more flexibly and consequently more stress is generated in the PZT. Figure (6) shows a comparison between the first and the last generations of chromosomes of one of the simulations.

Figure 6. Comparison between the first and the last generation of chromosomes

It shows that despite the small fitness of most of the chromosomes of the first generation (red points), the genetic algorithm could improve the fitness of chromosomes to higher values.
Figure (7) shows the evolution of the fitness of the populations over the generations in one of the simulations.

The blue line shows the fitness of the elite chromosome of each generation, showing the characteristic of the elitist model of sustaining growth of the fitness of the best chromosome. Looking at Fig. (7), it can be noted that the best chromosome of the simulation was found by the algorithm before ten generations. A similar behavior can be observed in the other simulations with the genetic algorithms.

7. Concluding Remarks
This work used Genetic Algorithm to optimization the power harvested from a piezobeam using a two-port network model of the transducer and a resistive load for the electric circuit. The optimum parameters for the mechanical structure and for the load impedance were determined in five different simulations with genetic algorithms.

The Genetic Algorithm has shown efficient for this application and has big importance once the interaction between the electric and mechanical systems is strongly depend of the parameters of the both system.

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