Shape Optimization of a Feet Support Using Finite Element Analysis

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Abstract
In the framework of Eco Veículo by DEM-FCTUC project, there is the need for each part to obtain the desired level of performance with minimum mass. Because of this we need to use a method to optimize each part of the vehicle. Moreover we have a limited time available for development which imposes that the optimization method must produce results in a fast and efficient way, using available means. The use of CAD tools that include stress analysis by finite element method, allow the stress and deformation distributions to be evaluated in a short time for a given load scenario. With CAD tools the mass of a part can be calculated too, and then performance parameters can be defined for stress and deformation using the mass of the part and the maximum stress and deformation. A global performance parameter can be defined based on the performance parameters for stress and deformation, and so the geometric optimization of the part can be done based on the value of the global parameter. Using a spreadsheet a mathematical model can be applied to optimize the shape of the part by changing a specific geometric variable and calculating the global performance parameter. The value of the specific variable that gives the maximum of the global performance parameter is the optimum value for this geometric variable. Then we skip to another geometric variable and use the same method to find its optimum value and continue the same procedure with the other geometric variables of the part until we reach the optimum geometrical shape. Moreover, we used restrictions for the maximum values of the stress and deformation levels, in each iteration of the process. This method was applied in a real part that had already been built and it allowed to reduce by 3 times the mass of the part and maintain stress and deformation levels within acceptable levels for the material used in the manufacture of the part and functional requirements of the component, so the part performs its function properly and is much lighter. This mass reduction method is easy to use, is pretty fast and can be applied to obtain the optimal shape of many complex parts found in machine construction.

Keywords: Mass reduction, Finite element analysis

1. Introduction
The optimization of a feet support fits the project Eco Veículo by DEM FCTUC who participates in international competitions for energy efficiency. For the vehicles involved in those events the mass is an important factor in the overall performance of the vehicle where the lower is its overall mass, the better the performance achieved by the vehicle. To minimize the mass of a vehicle component, tools are needed to evaluate the levels of stress and deformation of the component and set performance parameters of the same in terms of stress and deformation. These performance parameters allows us to compare different configurations of the part geometry being the best those with higher value of the global performance parameter and also have levels of maximum stress and deformation below the values considered acceptable. In this work the tool used to evaluate levels of stress and deformation of the feet support was the software for structural analysis by finite element method of the program Autodesk Inventor 2009 Professional. With the help of a spreadsheet, all values of stress, deformation and volume were listed to evaluate and compare the performance of each geometry and advance to the next geometry change. The optimization was performed starting with a part whose design was already set and the material was carbon fiber with epoxy resin matrix. In the structural analysis an important simplifying assumption was to consider the part as an isotropic solid, because the software does not allow us to analyze the support differently.
2. Optimization Procedure

During the optimization performance parameters were defined, as referred they will allow geometry evaluation. The stress parameter ($S$), deformation parameter ($D$), and the global parameter ($G$) are calculated using the following equations.

\[ S \left[ \frac{m.s^2}{kg^2} \right] \cdot 10^{-6} = \frac{1}{\sigma[MPa] \cdot m[kg]} \] (1)

\[ D \left[ \frac{1}{kg.m} \right] \cdot 10^3 = \frac{1}{\varepsilon[mm] \cdot m[kg]} \] (2)

\[ G \left[ \frac{s^2}{kg^2} \right] \cdot 10^{-3} = S \left[ \frac{m.s^2}{kg^2} \right] \cdot 10^{-6} \cdot D \left[ \frac{1}{kg.m} \right] \cdot 10^3 \cdot m[kg] \] (3)

With these equations we can evaluate the performance of a particular geometry of the part. When we change to the next geometry of the part, we can evaluate its performance and see if it is improving or not, and then choose the best geometry. The geometric changes are made in areas where the stress is lower in the part that are typically identified with the color blue in the software used. The geometric figure chosen was the circle because it allows low stress concentration. The ellipse was also a figure to consider, but due to the introduction of more variables would make the process more complicated and time consuming, not going to result in clearly better results. Defined using the circle as the geometrical figure, just now starting to change the parameters that define the geometry of which is the same geometric position of the center and diameter of it. For the analysis of the part is necessary to define the state of charge that it is subject and all boundary conditions. For the case were considered pin constraint on all the screws and frictionless constraint in the areas of contact with the structure. The load considered was 500 N per foot, in a direction perpendicular to the surface where the support is applied, and the same applied to an area with an elliptical shape that represents the contact of the heel with the support. Also was considered pre-tension on the screws equivalent to 1000 N per screw.
The only restriction was that the geometric width of the foot contact area should not be less than 180 mm, which equates to an area of 11176 mm$^2$. After these considerations, we performed the analysis with the initial design to see which the areas with the lowest stress level were.

3. Results of the Optimization Procedure

Made after the iterative processes and tried various geometric solutions, we now present the final solutions and the path to the final shape and geometry of the feet support. The optimization was done following the order of the indices of the diameters of the circles (d). The final description of the piece is shown in Fig.3.

![Final geometry and diameter identification](image)

3.1. Optimization of d1 and d2

d1 and d2 centers were defined as the half distance between the feet contact area and the upper part of the feet support. As established in the procedure, diameter values were changed, maximum stresses and deformation levels in the part were evaluated using the FEA, and then the performance parameters calculated. This procedure was applied to evaluate the effect of the geometry changes in order to get the best global performance parameter. Each parameter was changed in a wide range first with a coarse variation and then with a fine variation in the zones where the performance parameter was higher.

![Evolution of the global parameter with d1 and d2](image)
Both diameters were changed so the method was first to pick up one and fix it and then change the other. As an example d2_d1(140mm) means that d1 was fixed and d2 was changed while d1 stayed at the 140 mm. In Fig.4 it can be seen that best global performance parameter value was obtained for d2 equal to 140 mm and d1 equal to 125 mm. The shape of the part after optimization of d1 and d2 is shown in Fig.5.

![Figure 5 - Shape after d1, d2 optimization](image)

3.2. Optimization of d3
In section 3.1 two diameters were chosen to be changed in the same process, but here just one was optimized. It is a much easier and fast task to optimize just one parameter each time, and most often the final solution can be quickly achieved simply by evaluating three points and adjusting a parabola to the evolution of the global performance parameter with the geometric variable.

![Figure 6 - Evolution of the global parameter with d3](image)

In Fig.6 it can be seen that the best performance is obtained for a 40 mm diameter and also can be seen that the global parameter decreased compared to its value at the end of the d1, d2 optimization. So we can say that this result was the best for this geometry change but the global parameter slightly decreased. It was a controlled variation of the parameter because weight reduced and the stress, deformation levels didn’t increased much. We decided to make this change despite this small reduction in the global performance.
3.2. Optimization of d4 and d5
These two diameters, d4 and d5, don’t appear in Fig.3 because they were at each side of the foot contact area in a zone that as disappeared after some material in each side in the foot contact area was removed. The foot contact area was reduced to a minimum width of 180 mm and d4 and d5 disappeared after this change. This change was made with extra care because it was a big change in the design. Despite the large material removal involved in this change the stress and deformation were still at good levels. Mass reduced from 0.084 kg to 0.0647 kg and stress and deformation increased from 440 MPa to 513 MPa and from 2.77 mm to 3.43 mm respectively. After this large change in design the shape of the part is shown below in Fig.8.

3.3. Optimization of d6
In this geometry optimization two parameters were involved, d6 and I2. I2 doesn’t appear in Fig.3 but it is the distance between the center of the circle and the back part of the support.
Fig. 9 shows that each $d_6$ have an optimum $I_2$ that can maximize the global performance parameter value and all of them get more or less the same value so we needed to get a way to select one of the optimum points. As the main goal for this study is mass reduction we have decided to select it for the mass, the lightest is the best.

Fig. 10 shows that the optimum value of $d_6$ is 650 mm because it results in the largest mass reduction.
3.4. Optimization of d7
The center of d7 was defined as the half distance between the two central holes for the screws in the upper part and was concurrent with the straight line that passes in the holes centers. After the optimization of the diameter d7 a fillet radius, r1, was created to minimize the stress.

![Figure 12 - Evolution of the global parameter with d7, r1](image)

The global performance parameter has a strange evolution with d7 and the final diameter chosen was 85 because more than that would make the holes disappear and the holes must be preserved. As expected the fillet radius increased the global performance parameter. For a 35 mm radius value the maximum of the parameter was achieved as shown in Fig.11.

![Figure 13 – Shape after d7 and r1 optimization](image)

3.5. Optimization of d8
Before proceeding to the optimization of d8 another change was made to the geometry without involving variables or performance parameters calculations. The change was basically to remove the material in the upper side of the feet support between the holes, leaving a width equal to the washer that was going to be used. So the final result is shown in Fig.14.
For d8 optimization another variable was considered, I3. It is the distance between the two centers. I3 was ultimately not a variable because the position where the two circles were fitted didn’t have room for much movement so they have just been centered, and the value for I3 was 120 mm.

The d8 value that allows the best global performance parameter is 35 mm. The global parameter again slightly decreased but the part stress and deformation values stayed at acceptable levels.
3.6. Optimization of d9
For d9 the same policy as for d8 was taken so here the two circles were just positioned in the middle of the space that it was available. The optimum value for d9 is 10 mm as shown in Fig. 17.

![Figure 17 - Evolution of the global performance parameter with d9](image)

3.7. Optimization of d10, d11 and d12
Fig. 3 describes the geometry of circles with diameter d10, d11 and d12. Fig. 19 shows that the optimum value for d12 is 12 mm. This value was fixed for optimization of d10 and d11.

![Figure 18 - Shape after d9 optimization](image)

![Figure 19 – Evolution of the global performance parameter with d12](image)
Fig. 20 shows that the optimum value for $d_{10}$ is 10 mm. This value was also fixed for the optimization of $d_{11}$.

![Figure 20 - Evolution of the global performance parameter with $d_{10}$](image)

Fig. 21 shows that the optimum value for $d_{11}$ was 18 mm.

![Figure 21 - Evolution of the global performance parameter with $d_{11}$](image)

Fig. 22 shows the shape after $d_{10}$, $d_{11}$, $d_{12}$ optimization.

![Figure 22 - Shape after $d_{10}$, $d_{11}$, $d_{12}$ optimization](image)

4. **Conclusion**

This optimization method allowed us to reduce the mass of the part from 0.127 kg to 0.0454 kg, stress increased from 420.9 MPa to 547.3 MPa and deformation from 2.58 mm to 4.86 mm. This was a good improvement, the part was made and it is operating since two and a half years from now. Since then the part performed as expected so the objective of the optimization method was achieved.