Global Optimal Design of Electricity and Fresh Water Plants

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Abstract
This paper presents a NLP mathematical model of a Dual Purpose Plant (cogeneration of electricity and fresh water). Precisely, the main goal is to determine a set of thermodynamic non-ideal optimal solutions by applying a deterministic global optimization algorithm proposed by the authors. General Algebraic Modeling System GAMS is used to implement the model and the resolution strategy as well.

The considered dual purpose process involves a gas turbine discharging exhaust gas to a backpressure turbine cycle which is coupled to a thermal desalination process and to a condensation turbine cycle. This system has been previously studied by the authors but using local optimization algorithm [1] and consequently it was not possible to ensure the globality of solutions. The obtained global optimal solutions provide preliminary designs that satisfy optimum thermodynamic and process functional criteria.

Keywords: Dual purpose plant; optimal design, deterministic global optimization

1. Introduction
Cogeneration desalination plants are large-scale facilities that produce both electric power and fresh water. Desalination thermal methods, in particular, are suitable for cogeneration. The high-pressure steam that runs electric generators can be efficiently used in the brine heater of the desalination unit. This implementation significantly reduces fuel consumption compared with what would be required if both processes operate in stand-alone mode. Cogeneration desalination plants using gas turbines are very common around the world and they have been in operation for the past 20 years.

Certainly, a gas turbine is an internal combustion engine capable of producing electric power with an efficiency of about 30%. This production is accompanied by a great amount of exhaust gases discharge at a temperature of about 800 K which can be used in a Heat Recovery Steam Generator to produce steam.

Figure 1 shows the cogeneration desalination plant which is studied in this paper.

2. Problem statement. Optimal criterion
The optimization problem can be stated as follows. Given the total heat transfer area, the objective is to determine the optimal heat exchange area distribution between power generation cycles (PGCs) and the desalator in order to obtain the maximum power generated per unit of both fuel consumed (unit of fuel) and per unit of distillate produced. The heat exchange area of the PGCs must be also distributed among all the cycles.

3. Mathematical model
In this section, the assumed hypothesis and the mathematical model for the entire process are presented.

3.1 Assumptions for the multi effect desalination model
- Heat losses are negligible
- Mean values are adopted for heat capacity coefficients and boiling point elevation of the brine; that is, the effects of the brine concentration, temperature and pressure are neglected.
- The heat transfer coefficients and the latent heat of vaporization are assumed as constant values, neglecting the effect of chamber geometry, temperature, pressure and fluid parameters; also, the non-condensable effects are neglected.
- Number of effects (N) is assumed as a continuous variable.
3.2 Assumptions for power cycles

Basically, the temperatures of working fluid on the power cycles \((\tau_0, \tau_1, \tau'_0, \tau'_1)\) are assumed as constants but with unknown values. The same assumptions are also applied for the temperatures of cold and hot sources. The values corresponding to the total heat transfer coefficients in coolers and heaters are assumed as known constants.

Based on these assumptions, the following mathematical model for the dual purpose plant illustrated in Fig. 1 is derived.

![Figure 1. Schematic layout of a dual purpose desalination](image)

3.3. Mathematical model

*Gas Turbine model*

\[
\frac{T_{t2}}{T_{t1}} = 1 + \frac{\eta \gamma}{\eta_c - 1} \left( \frac{1}{\eta_c} - 1 \right) \tag{1}
\]

where \(T_{t2}, \eta_c\), and \(\eta\) refer to the compressor outlet air temperature, compressor efficiency and compressor ratio, respectively.

The turbine outlet gas temperature \([T_{t4}]\) is computed by the following constraint:

\[
\frac{T_{t4}}{T_{t3}} = 1 + \eta \left( \frac{1}{\eta_c} - 1 \right) \tag{2}
\]

The electric power \([P_{GASC}]\) demanded by the air compressor and the electric power produced by the expansor of the gas turbine \([P_{GAST}]\) are determined by:

\[
P_{GASC} = M_a C_p (T_{t1} - T_{t2}) \tag{3}
\]
\[
P_{\text{GAST}} = (M_a + M_c) C_p(a+c) (T_t^3 - T_t^4) \quad (4)
\]

**Back-pressure and condensation turbine cycles:**
The heat area transfer for the back-pressure cycle is computed by eq. (5)

\[
A_b = \frac{Q_v}{U_0 (T_0 - (T_0 + \Delta T))} + \frac{W}{U_1} \ln \left( \frac{T_0}{T_{t4}} \right) \quad (5)
\]

where \( Q_v, W, T_g^1 \) refer, respectively, to the discharged heat to the desalting unit, the heat capacity flow-rate of the hot source and the outlet temperature. \( U_0 \) and \( U_1 \) denote the total heat transfer coefficients while \( \tau_0 \) and \( \tau_1 \) are the the operating fluid of the back-pressure turbine cycle for cold and hot sources, respectively.

The efficiency of the back-pressure cycle is determined by:

\[
\eta = \frac{Q_b^1 - Q_v}{Q_b^1 - Q_v} = \frac{Q_b^1 - Q_v}{Q_b^1 (1 - \frac{\tau_0}{\tau_1})} \quad (6)
\]

The power production for this cycle is computed as follows:

\[
P_{\text{BPT}} = Q_b^1 - Q_v = W(T_t^4 - T_g^1) - Q_v \quad (7)
\]

where the heat capacity flow-rate of the hot source \([W]\) is given by:

\[
W = (M_a + M_c) C_p(a+c) \quad (8)
\]

here, \( C_p \) is the heat capacity of air/fuel mixture.

On the other hand, the following constraints correspond to the condensation turbine cycle:

\[
P_{\text{CT}} = W \eta_{se} (T_g^1 - T_g^0) \quad (9)
\]

\[
Q_1 = W (T_g^1 - T_g^0) \quad (10)
\]

\[
\eta = \frac{Q_1 - Q_0}{Q_1 - Q_0} = \frac{Q_1 - Q_0}{Q_1 (1 - \frac{\tau_0}{\tau_1})} \quad (11)
\]

\[
Q_0 = U_0 A_0 (\tau_0 - 298) \quad (12)
\]

The total electric power generation is given as follows:

\[
P_T = P_{\text{GAST}} - P_{\text{GASC}} + P_{\text{BPT}} + P_{\text{CT}} \quad (13)
\]

**Multi Effect Desalination Process**
The specific energy consumption \([Q_v]\) in the desalting unit is determined as follows:

\[
Q_v = \frac{\lambda}{N} + \frac{f}{f - 1} C_p \Delta t \quad (14)
\]

The specific heat transfer of the desalting unit is computed by the following constraint:
\[
A_d = \frac{f}{f-1} \frac{C_p N}{U} \left( 1 - \frac{\Delta t}{\Delta T} \right) \ln \left( \frac{\Delta T}{N \Delta t} + 1 \right) + \frac{\lambda_v (N-1)}{U} \left( \frac{\Delta T}{N \Delta t} - N \text{BPE} \right)
\]

(15)

where \( \lambda_v \), \( f \), \( N \) and BPE are the heat of condensation, the maximum ratio of admissible salinity, number of effect and boiling point elevation, respectively.

Formally, the optimization problem can be formulated as follows:

\[
\text{Max } p_T(x)/M_c(x)
\]

\[
\text{Subject to : } A_T(x) \leq A_T^{(0)}
\]

\[
x \in X
\]

Where the set \( X \) is the feasible region described by eq. (1) to (15). The proposed model involves 32 variables and 33 constraints.

3.4. Model implementation and strategy solution

The model was implemented in General Algebraic Modeling System GAMS [2]. The generalized reduced gradient algorithm CONOPT 2.041 [3] is used as NLP local solver. A deterministic global optimization algorithm ME-D (Minimization of the Error and Discard) which finds the \( \varepsilon \)-global optimum of non linear programming problems is applied to solve the resulting model [4]. In order to apply the ME-D algorithm, the problem must be reformulated in an appropriated way. This reformulation admits two kind of non convex functions: univariate and bilinear functions. The convex hulls associated to the bilinear terms are well known and are used to formulate the main problem: MP. Linear over and sub estimation of the univariate functions are also included. Therefore, the feasible region of the MP problem is defined only by linear constraints. Meanwhile the objective function minimizes the error caused by the linear approximation. Furthermore, a constraint forcing the original objective function to improve is included in MP.

The main problem is solved in each iteration of the ME-D algorithm and the next step is decided according to the obtained solution. If MP is infeasible, the analyzed region can be eliminated for future considerations, since the original problem would be infeasible too. On the other hand, if the obtained local optimum of MP has objective value 0, then the point is a better solution of the original problem. Finally, if the obtained local optimum of MP has positive objective value, a region that can be eliminated is constructed. Furthermore, the complement of the eliminated region can be described through appropriated bounds for the variables. Therefore, it is not necessary to add any new constraint in order to define the region where the search must continue.

This is the goal of the ME-D algorithm: the total feasible region is reduced in each iteration by discarding points infeasible for the original problem and even being some of them feasible for the convex overestimation.

4. Model application.

This section presents the optimal values obtained by solving the optimization problem defined in section 2 using the global optimization algorithm described above.

The optimal preliminary solutions for a dual-purpose desalination process with a gas turbine and extraction-condensation turbines with a maximum potentiality to produce electric power are shown through Figures 1, 2 and 3.

\[\text{Figure 2. Optimal values of } \frac{p_t}{M_c} \text{ and } M_c \text{ vs. total heat transfer area}\]
These solutions were obtained by fixing the total heat transfer area of the whole plant. The reformulation of the dual purpose plant model required 6 univariate concave functions and 27 bilinear terms. The absolute tolerance for the global optimality was set at 0.001 which represents about 0.002% of the global optimal objective value. 14 case studies were solved. The algorithm required between 1203 and 3589 iterations and between 474.0 and 1354.3 CPUs for determining the global optimal solutions. Even though, the model presents an important number of non convexities, no local solutions appear during the resolutions. Then, it is presume that the model has only one optimal solution for each case study.

The family of thermodynamic solutions presented in the figures is very useful because they can be used as initial values for an economic optimization. Moreover, the obtained solutions can be also used to predict the range for the optimal values of the main model variables [5, 6, 7].

**4. Conclusions and future works.**

This paper presented a simplified mathematical model of a dual purpose plant. Precisely, a NLP optimization model and a deterministic global optimization algorithm have been implemented in GAMS. In spite of the presence of non-convexities, local and multiple solutions have not been found by the optimization algorithm. The obtained global optimal solutions provide preliminary designs that satisfy optimum thermodynamic and process
As future work, a more detailed model for all process-equipments as well as an economic objective function will be considered.

Acknowledgements
This work was financially supported by Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), the Agencia Nacional para la Promoción de la Ciencia y la Tecnología (ANPCyT) and the Universidad Tecnológica Nacional Facultad Regional Rosario (UTN-FRRo) from Argentina and the Laboratório Nacional de Energia e Geologia (LNEG) from Portugal.

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