Application of a Hybrid Optimization Method for Identification of Steel Reinforcement in Concrete by Electrical Impedance Tomography

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Abstract

In this paper we consider the problem of detection steel bars used to reinforce concrete structures by the Electrical Impedance Tomography (EIT). Usually in this technique, electrical current is injected between electrodes placed on the external boundary of the body and electrical potentials are measured at the other electrodes. Such measures are used to achieve an image of conductivity distribution inside the transversal section analyzed, which is a non-linear inverse problem. The aim of the present work is to identify the position and size of the steel bars in a section of a reinforced concrete member. In the problem here considered a hybrid optimization strategy that combines Genetic Algorithms (GA) as a first step and the Levenberg-Marquardt method as the second step is proposed. The best solution found by the first step is used as initial guess to the second. In order to test this strategy, numerical experiments are presented. Since the measured data are not available, the boundary potential measurements are obtained computationally.

Keywords: Inverse Problems, Optimization, Genetic Algorithms, Levenberg-Marquardt.

1. Introduction

In this paper, we are considering the application of the Electrical Impedance Tomography (EIT) [1, 2] as a non-destructive technique to evaluate concrete structures [3]. In this technique, electrical current is injected between electrodes placed on the external boundary of the structure and electrical potentials are measured at the other electrodes. Such measures are used to iteratively achieve an image of the conductivity distribution inside the structure by minimizing the misfit between these and the ones obtained by the solution of a computational model of the physical problem, the forward problem. This minimization problem is a non-linear inverse problem as the measures depend nonlinearly on the distribution of conductivity inside the body.

One usual approach use the Finite Element Method to solve the direct problem and the conductivities of each finite element are used as the optimization parameters. In order to achieve acceptable results regularization procedures are necessary and the obtained solutions are smooth distribution of conductivities. In order to identify the boundary of inclusions a thresholding process must be applied introducing other approximations.

Here a different strategy is used, the the direct problem is solved by the Boundary Equation Method (BEM), the reinforcement bars are treated as inclusions with infinity conductivity and the optimization variables are geometrical shape and position parameters.

As the aim of the present work is to identify the position and size of an unknown number of steel bars in a section of the concrete structure, the number of optimization variables are a priori unknown. This motivates the adoption of an hybrid optimization strategy similar to the one proposed by Hsiao et al.[4]. First, Genetic Algorithms (GA) [5, 6, 7] are used to find some approximations for the number of inclusions, their position and size. Then, such approximations are provided as initial guess for the Levenberg-Marquardt (LM) [8, 9] method. In this step of the process, the number of inclusions and consequently, the number of optimization variables, are kept fixed. After that, the Levenberg-Marquardt method is used to improve the solution and increase the convergence speed. Although the GA require a large number of evaluations of the minimization function to reach the minimum, the convergence to a local extrema can be avoided. On the other hand, the Levenberg-Marquardt method is faster and converges to the solution if a good initial guess is provided. Therefore, the hybrid strategy proposed tries to take advantage of the best features of both methods.
2. Inverse Problem

The inverse problem consists on the identification of the size and position of an unknown number of steel bars in a section of concrete. Such identification is done using electrical potential measures on electrodes placed at the external boundary of the domain with respect to a previously defined current injection protocol. Since the inclusions are assumed to be homogeneous and their shape can be considered known, the interfaces between the concrete and the steel bars can be defined by a set of geometric parameters. Each inclusion is considered circular and its boundary is determined by its radius \( r \) and the position of the center \((x_c, y_c)\). So, the problem is to find the set of geometrical parameters \( \mathbf{X} \) that defines the conductivity distribution that minimizes the difference between the electrical potential measures \( \bar{V} \) and the computed ones \( \mathbf{V}(\mathbf{X}) \). Mathematically, it can be written as:

\[
\mathbf{X}^* = \arg \min_{\mathbf{X}} f(\mathbf{X}),
\]

with

\[
f(\mathbf{X}) = \frac{1}{2} \mathbf{R}(\mathbf{X})^T \mathbf{R}(\mathbf{X}), \quad \mathbf{R}(\mathbf{X}) = \mathbf{V}(\mathbf{X}) - \bar{\mathbf{V}}
\]

where \( f(\mathbf{X}) \) is called objective function and \( \mathbf{R}(\mathbf{X}) \) residual function. The number of measures \( m \) in \( \bar{\mathbf{V}} \) depends on the current injection and measures protocol. In this paper the opposite protocol is used. In this protocol, 8 cases of current injection, described in Table (1) below, generate 104 measures of electrical potential. The choice of this protocol is based on \([10, 11]\).

<table>
<thead>
<tr>
<th>Case</th>
<th>Injection Electrodes</th>
<th>Null Potential Electrode</th>
<th>Case</th>
<th>Injection Electrodes</th>
<th>Null Potential Electrode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 and 9</td>
<td>5</td>
<td>5</td>
<td>5 and 13</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>2 and 10</td>
<td>6</td>
<td>6</td>
<td>6 and 14</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>3 and 11</td>
<td>7</td>
<td>7</td>
<td>7 and 15</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>4 and 12</td>
<td>8</td>
<td>8</td>
<td>8 and 16</td>
<td>12</td>
</tr>
</tbody>
</table>

In previous works \([12, 10]\), the number of inclusions was considered known. So, the number of optimization variables \( n \) is determined and the minimization problem described in Eq. (1) could be solved by a gradient based classical method to solve non-linear least squares problems. However, in the problem treated here, the number of inclusions is considered unknown. So, an hybrid optimization process is proposed. Initially, a genetic algorithm is used to generate a set of solution candidates and in a second step, the Levenberg-Marquardt method is used to improve the approximation provided by the GA. For the GA, the optimization variables are the number of inclusions, their position and size. For Levenberg-Marquardt method, the number of inclusions remains unchanged and the variables are their size and position. The main objective of the hybrid strategy is to take advantage of the capability of GA in treating complex constrained problems without a predefined number of variables, improving the convergence speed in the neighborhood of the solution with Levenberg-Marquardt Method.

2.1. Genetic Algorithms

Genetic Algorithms \([5, 6, 7]\) are search procedures inspired in biological process of evolution. These optimization strategies have been used to solve successfully a large variety of complex problems. GA are based on genetics and natural selection of species and do not demand computation of the objective function derivatives with respect to optimization variables. In this strategy, the individual is a solution candidate and the population is a set of individuals. Each individual is coded by the chromosome, that represents the optimization variables. In this work, the chromosome is real coded. The fitness of the individual is the objective function value and according to this value the individuals are ranked.

The main operations that compose a GA are the selection, the mutation and the crossover. The selection mechanisms are responsible for the suitable choice of the individuals, based on their ranking.
The chosen individuals give origin to new ones by mutation or crossover. The basic idea is that an individual with good fitness will generate a new good one. In this way, one can expect that the population converges to the desired solution.

In this work the so called steady-state evolution scheme is used. In this scheme each new generated individual has its fitness evaluated, is included in the population and ranked. Then the individual with the worst fitness in this increased population is discarded, and the number of individuals is maintained unchanged during the process. In the GA used, the initial population is randomly generated. The stop criterion is the maximum number of evaluations of the objective function.

In the numerical experiments presented in this paper, two different kinds of chromosomes are tested. The first one can be represented as

\[ \{N|\{x_{c_i} | y_{c_i} | r_1 | x_{c_2} | y_{c_2} | \ldots | x_{c_N} | y_{c_N} | r_N \} \}, \]

where \(N\) is the number of inclusions, \((x_{c_i}, y_{c_i})\) are the coordinates of the center of inclusion \(i\) and \(r_i\) is its radius. A similar approach for the presented parametrization can be found in [13, 14].

The second kind of chromosome can be represented as

\[ \{N|\{r | x_{c_1} | y_{c_1} | x_{c_2} | y_{c_2} | \ldots | x_{c_N} | y_{c_N} \} \}. \]

Note that in this second chromosome, it is assumed that the size of all inclusions is the same. The correct adjust of their size should be done at the second step of the process by the Levenberg-Marquardt method.

Every individual has to satisfy the geometrical constraints of the problem: the inclusions must be inside the domain and them can not intercept themselves. If a new individual violates the geometrical constraints, another one is generated. Although the constraints exist, the problem is treated as unconstrained since such constraints are not active in the neighborhood of the solution.

In this paper, the operators used are: two-points crossover, discrete crossover, BLX-alpha crossover and random mutation.

2.2 Levenberg-Marquardt Method

Once the inverse problem was solved by the described GA, such solutions can be used as initial guesses for the Levenberg-Marquardt’s method. This method, described in [8, 9], can be considered as the Gauss-Newton method modified by the model trust region approach. At each iteration the new values of the optimization variables, \(X_+\), are obtained from the current ones, \(X_0\), through the solution of a modified minimization problem:

\[
\begin{align*}
\text{minimize} & \quad \|R(X_0) + J(X_0)(X_+ - X_0)\|_2 \\
\text{subject to} & \quad \|X_+ - X_0\|_2 \leq \delta_0.
\end{align*}
\]

With this method, the values of the variables of the next step \((X_+)\) are calculated as follows:

\[ X_+ = X_0 - (J(X_0)^TJ(X_0) + \mu_0 I)^{-1}J(X_0)^TR(X_0), \]

where \(I\) is the identity matrix and \(\mu\) is the parameter that modifies the Gauss-Newton method. If \(\delta_0 \geq \|J(X_0)^TJ(X_0)\|_2^{-1}\|J(X_0)^TR(X_0)\|_2\) then \(\mu_0 = 0\) and \(\mu_0 \neq 0\) otherwise. \(J \in \mathbb{R}^{n \times n}\) is the Jacobian matrix, the derivatives of the objective function with respect to the optimization variables \((J_{ij} = \partial f_i / \partial X_j)\).

In this work, the computation of the Jacobian matrix is based on finite differences. So, the element \(J_{ij}\) of the Jacobian matrix is computed as follows:

\[ J_{ij} = \frac{\partial f_i}{\partial X_j} \approx \frac{f_i(X + h_j e_j) - f_i(X)}{h_j} \]

where \(h_j\) is a small finite perturbation at the \(j\)-th element of the original vector of optimization variables \(X\) and \(e_j\) is the \(j\)-th column of the identity matrix.

The value of the parameter \(h_j\) is computed by the product \(\sqrt{\varepsilon} X_j\), where \(\varepsilon\) is a parameter provided by the user. If the machine precision is greater than the computed \(h_j\), this value is substituted by the machine precision.
The MINPACK-1, a standard package of subroutines implemented in Fortran to solve numerically non-linear equations and non-linear least square problems, was used in the present work and it can be found at Netlib repository (http://www.netlib.org/minpack). More details about this implementation, subroutine LMDIF, can be found in [15].

4. The Forward Problem
The forward problem consists on calculating the electrical potential on the electrodes of the external boundary of a body with known conductivity distribution, considering some current injection protocol. When the usual simplifying assumptions [16] are adopted, this problem is governed by Laplace’s equation. Thus, a domain composed by a main conductor material with inclusions of different conductivities, Fig. (1), can be divided in homogeneous sub-domains in which the potential value of the electrical potential $u$ at each point $x$ must satisfy Laplace’s equation,

$$\nabla^2 u_k(x) = 0, \quad x \in \Omega_k,$$

where $\Omega_k$ is the $k$-th sub-domain. Usually in this approach, the conductivity value ($\sigma_k$) of each sub-domain is considered known and constant. The main sub-region is represented by $\Omega_0$ and the inclusions are represented by $\Omega_k$, with $k > 0$.

The boundary conditions are:

$$u_0(x) = 0, \quad x \in \Gamma^U_0;$$

$$\sigma_0 \frac{\partial u_0}{\partial n} = J(x), \quad x \in \Gamma^J_0;$$

and the interface conditions are:

$$u_0(x) = u_k(x), \quad x \in \Gamma_{0k};$$

$$\sigma_0 \frac{\partial u_0}{\partial n}(x) = -\sigma_k \frac{\partial u_k}{\partial n}(x), \quad x \in \Gamma_{0k};$$

where $n$ is the outward normal to the boundary, $\frac{\partial u_k}{\partial n} = \nabla u_k \cdot n$ is the electrical flux, $\Gamma_0$ is the external boundary of the domain, $\Gamma_{0k}$ is the interface between the sub-domains $\Omega_0$ and $\Omega_k$ and $J(x)$ is the current density prescribed. For each loading case, two electrodes ($\Gamma^J_0$) are used for current injection and have unity current density prescribed. A third electrode ($\Gamma^U_0$) is taken as potential reference and it has null potential prescribed. The rest of the external boundary ($\Gamma^U_0$) has null current density prescribed.

In this paper, a particular case of this kind of problem is treated. The body is composed by concrete and inclusions of steel. So, the model of conductivity distribution assumes that the section of the body is composed by a main material with homogeneous electrical properties and inclusions of a material with infinity conductivity. In this case, the interface conditions must be treated in a specialized way:

$$u_0(x) = u_k(x) = U_k, \quad x \in \Gamma_{0k};$$

$$\int_{\Gamma_{0k}} \frac{\partial u_k}{\partial n} d\Gamma = 0, \quad x \in \Gamma_{0k};$$

Figure 1: An heterogeneous domain divided in three homogeneous sub-domains and the boundary conditions.
In order to solve Eq. (9) for each subregion the Boundary Elements Method (BEM) is used. Further details about this technique can be found in [17]. The integral equation of BEM for this problem is

\[ c(\xi)u(\xi) + \int_{\Gamma} p^*(\xi; x)u(\xi; x)d\Gamma(x) = \int_{\Gamma} u^*(\xi; x)p(\xi; x)d\Gamma(x), \]  

where \( \xi \) is the collocation point, \( \Gamma \) is the boundary of the sub-domain, \( u \) is the electrical potential, \( p \) is the current density, \( u^* \) and \( p^* \) are the fundamental solutions for potential and current density, respectively and \( c(\xi) \) is a function of the boundary shape, whose value is 0 if \( \xi \) is outside of the domain, 1 if \( \xi \in \Omega \) and \( \beta/2\pi \) if \( \xi \in \Gamma \). The parameter \( \beta \) is the angle between the left and right tangents at the collocation point \( \xi \).

In order to obtain a numerical solution for Eq. (16), the boundary of the body is discretized. The external boundary is divided in \( N_0 \) elements and each inclusion boundary in \( N_k \) elements. In this paper, the element adopted approximates the geometry linearly and the value of the electrical potential is considered constant in each element. In this case, the parameter \( \beta = \pi \) and then \( c(\xi) = 0.5 \) if \( \xi \in \Gamma \).

Each boundary element has two nodes for the geometrical definition and a centered node, called function node, for the potential and flux definition. Thus, the discretized form of Eq. (16) for each sub-domain \( k \) allows evaluating the potential at each functional node as follows

\[ c(\xi_j)u(\xi_i) + \sum_{j=1}^{N_k} u_{j} \int_{\Gamma_j} p^* d\Gamma_j = \sum_{j=1}^{N_k} p_{j} \int_{\Gamma_j} u^* d\Gamma_j, \]  

where \( u_j \) and \( p_j \) represent the potential and the current density at the \( j \)-th functional node, the regular integrals are computed numerically by the usual Gauss Quadrature scheme and the singular ones are computed analytically.

In problems with inclusions, both potential and current densities are unknowns at interfaces. BEM solutions to this kind of problem are usually obtained by the application of Eq. (17) for each sub-domain \( \Omega_k \), in addition to the compatibility conditions, Eq. (12) and (13), for the potential and current density at the functional nodes of the interface elements at \( \Gamma_k \). In the present case, however, besides the application of compatibility equations Eqs. (14) and (15) it suffices to apply Eq. (17) to the main sub-region boundary.

The number of unknowns in the resulting system of equations is the number of boundary elements used in discretization of external and inclusion boundaries plus one potential value per inclusion.

After determining the unknowns at the boundary, the values of the electrical potential at the nodes in the center of the electrodes without prescribed values are collected in the vector \( \mathbf{V}(\mathbf{X}) \) in order to compute the objective function, Eq. (1). The fact that no computation of internal values is needed also enhances BEM performance for this kind of problem.

5. Numerical Experiments

Two identification problems were proposed in order to test the hybrid strategy. The first problem consists on the identification of 3 inclusions with infinity conductivity. One can observe that the middle inclusion has its size greater than the other 2 inclusions. In the second problem, 4 inclusions with the same limitations: 1 \( \leq N \leq 5 \), \(-15 \leq x_c \leq 15\), \(-30 \leq y_c \leq 30\), \(0.3 \leq r \leq 2.0\).

In both cases, the variables have the same limitations: 1 \( \leq N \leq 5 \), \(-15 \leq x_c \leq 15\), \(-30 \leq y_c \leq 30\), \(0.3 \leq r \leq 2.0\).

In all the experiments with the GA, the probabilities of the operators are 30% for the two-points crossover, 30% for the discrete crossover, 20% for the BLX-alpha crossover and 20% for the random mutation. Four runs of the GA were used to solve the problem with each kind of chromosome.

The best solution candidate obtained in each run of GA were used as initial guess for the Levenberg-Marquardt method. In this method, the following parameters were used: size of the trust region \( \delta = 0.1 \),
parameter for the jacobian approximation $\varepsilon = 10^{-4}$ and stop criteria for the optimization variables $xtol = 10^{-3}$, objective function $ftol = 10^{-6}$ and gradient $gtol = 10^{-6}$.

The measurements of electrical potential for both problems were obtained computationally by the solution of the forward problem for the considered actual conductivity distribution. In this case, the BEM was used to solve the forward problem, but the size of the elements was different of the size used during the solution of the optimization problem in order to avoid the called “inverse crime” [18].

Figure (2) shows the discretization used and the distribution of the electrical potential in a section with 4 inclusions of infinity conductivity for one current injection case.

Figure 2: Electrodes positioning and discretization of external boundary (left) and potential distribution for current injection between electrodes 1 and 9 (right).

5.1. First Problem

In order to solve the first problem, the population with 150 individuals was initialized randomly. Each run stops with 300000 evaluations of the objective function.

Figures (3) and (4) show the geometrical results obtained with the hybrid optimization strategy. Table (2) presents the values of the objective function reached with the GA and the Levenberg-Marquard Method for each run.
Figure 3: Results obtained for the first problem in the four runs. In this results, the first kind of chromosome was used.

Figure 4: Results obtained for the first problem. Here, the second kind of chromosome was used.

Table 2: First problem: values of the objective function.

<table>
<thead>
<tr>
<th>Run</th>
<th>N</th>
<th>$f_{GA}$</th>
<th>$f_{LM}$</th>
<th>Run</th>
<th>N</th>
<th>$f_{GA}$</th>
<th>$f_{LM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>0.0065</td>
<td>0.0018</td>
<td>1</td>
<td>1</td>
<td>0.0726</td>
<td>0.0723</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.0313</td>
<td>0.0018</td>
<td>2</td>
<td>2</td>
<td>0.0417</td>
<td>0.0343</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.0398</td>
<td>0.0365</td>
<td>3</td>
<td>3</td>
<td>0.0240</td>
<td>0.0018</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>0.0084</td>
<td>0.0018</td>
<td>4</td>
<td>2</td>
<td>0.0403</td>
<td>0.0343</td>
</tr>
</tbody>
</table>

It is important to note that, using the first kind of chromosome, the number of inclusions was correctly identified in 3 runs. Using the second kind of chromosome, this feature was correctly identified in only one run. Using the Levenberg-Marquardt method, in all cases, the value of objective function was decreased. However, if the number of inclusions was correctly identified by GA, the value of the objective function reaches a lowest level. This can indicate that the correct solution for the inverse problem was found. In each run, while the GA spends 300000 evaluations of the objective function, the LM finishes the optimization process with 70 evaluations, including the evaluations used to approximate the jacobian matrix.
5.2. Second Problem

This problem was solved by GA with 150 individuals in the population initialized randomly. Each run stops with 200000 evaluations of the objective function.

Figures (5) and (6) show the geometrical results obtained with the hybrid optimization strategy. Table (3) presents the values of the objective function reached with the GA and the Levenberg-Marquard Method for each run.

Figure 5: Results obtained for the second problem in each run. In this results, the first kind of chromosome was used.

Figure 6: Results obtained for the second problem. In this results the second kind of chromosome was used.
Table 3: Second problem: values of the objective function.

<table>
<thead>
<tr>
<th>Run</th>
<th>N</th>
<th>$f_{GA}$</th>
<th>$f_{LM}$</th>
<th>Run</th>
<th>N</th>
<th>$f_{GA}$</th>
<th>$f_{LM}$</th>
</tr>
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<tbody>
<tr>
<td>1</td>
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<td>0.0129</td>
<td>0.0120</td>
<td>1</td>
<td>4</td>
<td>0.0181</td>
<td>0.0017</td>
</tr>
<tr>
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<td>2</td>
<td>0.0463</td>
<td>0.0463</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.0349</td>
<td>0.0092</td>
<td>3</td>
<td>3</td>
<td>0.0200</td>
<td>0.0092</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.0094</td>
<td>0.0080</td>
<td>4</td>
<td>2</td>
<td>0.0463</td>
<td>0.0463</td>
</tr>
</tbody>
</table>

In this problem, the correct number of inclusions was found by the GA using the first kind of chromosome in 2 runs. Despite of that, the solutions obtained with Levenberg-Marquardt method have not converged to the correct solution, as indicated by the relatively high value of the objective function. By the other hand, using the second kind of chromosome, the correct number of inclusions was identified in a single run. With this initial guess, the correct solution for the inverse problem was reached by the Levenberg-Marquard method. For this success case (first run using the second chromosome), in which the values of the objective function is lowest, the GA demand 200000 evaluations and the LM method evaluates this function 94 times.

6. Conclusions

Although the exact position and size of the inclusions were found only in some of the results, the experiments showed that it is possible to solve the described inverse problem with the proposed hybrid strategy. Some results for the second problem presented small values of objective function without a good geometry adjust. This fact indicates that it would be necessary to include more information in this objective function, increasing the number of cases of current injection.

The two problems solved here show that the GA can deal with the difficulties of the proposed problem providing useful results. On the other hand, the Levenberg-Marquardt method can be considered satisfactory, since it provides good solutions when the GA provides suitable approximations as initial guess. About the kinds of chromosomes tested, it is necessary more tests to conclude which one is more suitable, furthermore, other kinds of chromosomes should also be experimented [13, 14].

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References


