September 6 - 9, 2010, Lisbon, Portugal

Multi-Objective Optimization of Centrifugal Pumps Using Particle Swarm Optimization Method

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Abstract

In the present study, multi-objective optimization of centrifugal pumps is performed at three steps. At the first step, η and *NPSHr* in a set of centrifugal pump are numerically investigated using commercial software. Two meta-models based on the evolved group method of data handling (GMDH) type neural networks are obtained, at the second step, for modeling of η and *NPSHr* with respect to geometrical design variables. Finally, using obtained polynomial neural networks, Multi-Objective Particle Swarm Optimization method (MOPSO) are used for Pareto based optimization of centrifugal pumps considering two conflicting objectives, η and *NPSHr*. The Pareto results of PSO method are also compared with that of multi-objective genetic algorithm (NSGA II). It is shown that some interesting and important relationships as useful optimal design principles involved in the performance of centrifugal pumps can be discovered by Pareto based multi-objective optimization of the obtained polynomial meta-models representing their η and *NPSHr* characteristics.

Keywords: Particle Swarm Optimization; Centrifugal Pumps; Multi-Objective Optimization; NPSHr; CFD; Pareto.

1. Introduction

Centrifugal pumps are the group of turbo machines which are used industrially in large scales. Optimization of centrifugal pumps is indeed a multi-objective optimization problem rather than a single objective optimization problem that has been considered so far in the literature. Demeulenaere et al. [1] investigated an optimization process on centrifugal pumps using Fine/ Design 3D environment of NumecaTM software and genetic algorithms. They tried to increase efficiency and head and decrease the *NPSHr* at two different flow rates and finally showed that the new blade geometry should have more curvature in the camber line definition. Nariman-zadeh et al. [2] investigated a multi-objective optimization process on centrifugal pumps and suggested four optimal point that designer can select each of them. They tried to increase the hydraulic efficiency and head and decrease the input power. They did not use CFD in their simulation and just used the analytical equations for hydraulic efficiency, head and the input power.

Both the efficiency and *NPSHr* in centrifugal pumps are important objective functions to be optimized simultaneously in such a real world complex multi-objective optimization problem. These objective functions are either obtained from experiments or computed using very timely and high-cost CFD approaches, which cannot be used in an iterative optimization task unless a simple but effective meta-model is constructed over the response surface from the numerical or experimental data. Therefore, modeling and optimization of the parameters is investigated in the present study, by using GMDH-type neural networks and multi-objective Particle Swarm Optimization method (PSO) in order to maximize the efficiency and minimize the *NPSHr*.

Particle Swarm Optimization (PSO), first introduced by Kennedy and Eberhart [3], is one of the modern heuristic algorithms. It was developed through simulation of a simplified social system, and has been found to be robust in solving continuous nonlinear optimization problems [3, 4]. The PSO technique can generate a high-quality solution within short calculation time and stable convergence characteristic than other stochastic methods [5, 6].

In this paper efficiency and the required *NPSH* in a set of centrifugal pumps are numerically investigated using NumecaTM software. Next, genetically optimized GMDH type neural networks are used to obtained polynomial

September 6 - 9, 2010, Lisbon, Portugal

models for the effects of geometrical parameters of the pumps on both efficiency and *NPSHr*. Such an approach of meta-modeling of those CFD results allows for iterative optimization techniques to design optimally the centrifugal pumps computationally affordably. The obtained simple polynomial models are then used in a Pareto based multi-objective PSO optimization approach to find the best possible combinations of efficiency and *NPSHr*, known as the Pareto front. The Pareto results of PSO method are also compared with that of multi-objective genetic algorithm (NSGA II). The corresponding variations of design variables, namely, geometrical parameters, known as the Pareto set, constitute some important and informative design principles.

2. CFD simulation of centrifugal pumps

2.1. Definition of objective functions

Both the efficiency and *NPSHr* in centrifugal pumps are important objective functions to be optimized simultaneously. The efficiency of a centrifugal pump is defined by

$$\eta = \frac{P_{out}}{P_{in}} \tag{1}$$

Where Pout is the useful power transferred by the pump to the liquid and given by

$$P_{out} = \rho.g.H.Q \tag{2}$$

And P_{in} is input power or shaft power.

NPSHr defines the cavitation characteristic of a pump. It is a characteristic of the pump and is indicated on the pump's curve. It varies by design, size, and the operating conditions [7, 8] and can be determined with the following formula:

$$NPSHr = ATM + Pgs + Hv - Hvp \tag{3}$$

Where *ATM* is the atmospheric pressure at the elevation of the installation, *Pgs* is the suction pressure gauge reading taken at the pump centerline and converted into head, *.Hv* is the velocity head $(V^2/2g)$, *Hvp* is the vapor pressure of the fluid . As mentioned above the *NPSHr* is a function of pump design and should be minimized in centrifugal pumps.

2.2. Definition of case study and the design variables

The case study in present paper is an ETANORM 65-160 centrifugal pump. The simulations are performed using NumecaTM software. Firstly one blade is modeled in Auto blade 3.6 and then the Design 3D environment of NumecaTM can automatically generate the database with different design variables.



September 6 - 9, 2010, Lisbon, Portugal



To parameterize the camber line curve, the simple Bezier method is used [9]. Schematically definition of simple Bezier method is shown in Figure 1. The design variables in this method are leading edge angle (β_1), trailing edge angle (β_2) and the stagger angle (γ). In the present paper three sections are defined in the blades, one on hub, one on shroud and the third one on the middle plane of hub and shroud, as shown in Figure 2.



Figure 2. Defining three sections on centrifugal pumps blade

In data base generation we suppose that β_2 is the same at the three defined sections of blade [7]. This problem is mathematically given by

$$\beta_{2Hub} = \beta_{2Shroud} = \beta_{2MidSpan} = DesignVariable$$
(4)

Moreover β_1 at mid span is equal to the average of β_1 at hub and shroud sections

$$\beta_{1MidSpan} = \frac{\beta_{1Hub} + \beta_{1Shroud}}{2}$$
(5)

So there are four design variables namely: $\gamma_{mid span}$, $\beta_{1 Hub}$, $\beta_{1 Shroud}$, and β_{2} . In fact $\gamma_{mid span}$ is the average γ of three sections. By changing the geometrical independent parameters various designs will be generated and evaluated by CFD. Consequently, some meta-models can be optimally constructed using the GMDH-type neural networks, which will be further used for multi-objective Pareto based design of such centrifugal pumps using PSO method. In this way, 135 various CFD analyses have been performed due to those different design geometrics.

2.3. Numerical scheme

For an incompressible fluid flow, the equations of continuity and balance of momentum are given as

$$\frac{\partial V_i}{\partial x_i} = 0 \tag{6}$$

September 6 - 9, 2010, Lisbon, Portugal

$$\frac{DV_i}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + v \frac{\partial^2 V_i}{\partial x_i \partial x_j} - \frac{\partial}{\partial x_i} \overline{u_i u_j}$$
(7)

The physical model used in the solver is the Reynolds-Averaged Navier–Stokes equations and the k- ϵ turbulence model is used [10]. The k- ϵ equations are given as

$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_j} \left[\left(C_k \frac{k^2}{\varepsilon} + \nu \right) \frac{\partial k}{\partial x_i} \right] - \overline{u_i u_j} \frac{\partial V_i}{\partial x_j}$$

$$\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial x_j} \left[\left(C_k \frac{k^2}{\varepsilon} + \nu \right) \frac{\partial \varepsilon}{\partial x_j} \right] - C_{\varepsilon 1} \frac{\varepsilon}{k} \overline{u_i u_j} \frac{\partial V_i}{\partial x_j} - C_{\varepsilon 2} \frac{\varepsilon^2}{k}$$
(8)

A mass flow inlet boundary condition is used at the pumps inlet, a pressure outlet boundary condition is used at the outlet and finally periodic boundary condition is applied between two blades. The computation is continued until the solution converged with a total residual of less than (-5).

Num		Inpu	Outpu	Output Data		
	ү _{Міd} (deg)	β _{1Hub} (deg)	β _{1Shroud} (deg)	β_2 (deg)	η (%)	NPSHr (m)
1	30	0	60	40	63.067	3.541
2	30	15	60	50	64.110	3.543
3	40	15	89	40	68.535	3.671
4	40	30	75	60	78.901	4.232
5	70	30	75	60	92.121	5.361
6	50	0	75	50	81.901	4.172
7	60	30	60	60	85.952	5.761
8	60	30	89	60	88.106	4.901
9	70	30	89	50	86.351	4.555
10	30	0	89	40	65.380	5.216
134	70	15	60	50	80.710	5.104
135	50	15	75	40	78.108	3.819

Table 1: Samples of numerical results using CFD

Samples of numerical results, using CFD are shown in Table 1. The results obtained in such CFD analysis can now be used to build the response surface of both the efficiency and the *NPSHr* for those different 135 geometries using GMDH-type polynomial neural networks. Such meta-models will, in turn, be used for the Pareto-based multi-objective optimization of the centrifugal pumps using PSO method. A post analysis using CFD is also performed to verify the optimum results using the meta-modeling approach. Finally, the solutions obtained by the approach of this paper exhibit some important trade-offs among those objective functions which can be simply used by a designer to optimally compromise among the obtained solutions.

3. Modeling of efficiency and NPSHr using GMDH-type neural network

By means of GMDH algorithm a model can be represented as set of neurons in which different pairs of them in each layer are connected through a quadratic polynomial and thus produce new neurons in the next layer. Such representation can be used in modeling to map inputs to outputs. The formal definition of the identification problem is to find a function \hat{f} so that can be approximately used instead of actual one, f in order to predict output \hat{y} for a given input vector $X = (x_1, x_2, x_3, ..., x_n)$ as close as possible to its actual output y. Therefore, given M observation of multi-input-single-output data pairs so that

September 6 - 9, 2010, Lisbon, Portugal

$$y_i = f(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) \quad (i=1, 2 \dots M),$$
(9)

It is now possible to train a GMDH-type neural network to predict the output values \hat{y}_{i} for any given input vector

$$X = (x_{i1}, x_{i2}, x_{i3}, ..., x_{in})$$
, that is

$$\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3}, ..., x_{in})$$
 (*i*=1, 2... *M*), (10)

The problem is now to determine a GMDH-type neural network so that the square of difference between the actual output and the predicted one is minimized, that is

$$\sum_{i=1}^{M} \left[\hat{f}(x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) - y_i \right]^2 \to \min$$
(11)

General connection between inputs and output variables can be expressed by a complicated discrete form of the Volterra functional series in the form of

$$y = a_0 + \sum_{i=1}^n a_i x_i + \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i x_j + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n a_{ijk} x_i x_j x_k + \dots,$$
(12)

Where is known as the Kolmogorov-Gabor polynomial [11]. This full form of mathematical description can be represented by a system of partial quadratic polynomials consisting of only two variables (neurons) in the form of

$$\hat{y} = G(x_i, x_j) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i x_j + a_4 x_i^2 + a_5 x_j^2.$$
(13)

There are two main concepts involved within GMDH-type neural networks design, namely, the parametric and the structural identification problems. In this way, some authors presented a hybrid GA and singular value decomposition (SVD) method to optimally design such polynomial neural networks [12]. The methodology in these references has been successfully used in this paper to obtain the polynomial models of efficiency and *NPSHr*. The obtained GMDH-type polynomial models have shown very good prediction ability of unforeseen data pairs during the training process which will be presented in the following sections.

The input-output data pairs used in such modeling involve two different data tables obtained from CFD simulation discussed in Section 2. Both of the tables consists of three variables as inputs, namely, the geometrical parameters of the pumps, $\gamma_{\text{Mid span}}$, β_{1Hub} , $\beta_{1Shroud}$ and β_2 (Figures 1 and 2), and outputs, which are efficiency and *NPSHr*. The tables consist of a total of 135 patterns, which have been obtained from the numerical solutions to train and test such GMDH type neural networks.

However, in order to demonstrate the prediction ability of the evolved GMDH type neural networks, the data in both input–output data tables have been divided into two different sets, namely, training and testing sets. The training set, which consists of 115 out of the 135 input–output data pairs for efficiency and *NPSHr*, is used for training the neural network models. The testing set, which consists of 20 unforeseen input–output data samples for η and *NPSHr* during the training process, is merely used for testing to show the prediction ability of such evolved GMDH type neural network models. The GMDH type neural networks are now used for such input–output data to find the polynomial models of efficiency and *NPSHr* with respect to their effective input parameters. In order to design genetically such GMDH type neural networks described in the previous section, a population of 10 individuals with a crossover probability (P_c) of 0.7 and mutation probability (P_m) 0.07 has been used in 500 generations for η and *NPSHr*. The corresponding polynomial representation for efficiency is as follows:

$$Y_{1} = .476 - .33\beta_{1\text{Hub}} + 2.027\beta_{1\text{Shroud}} + 0.014\beta_{1\text{Hub}}^{2} - .0130\beta_{1\text{Shroud}}^{2} + .0001\beta_{1\text{Hub}}\beta_{1\text{Shroud}}$$
(14a)

$$Y_2 = 20.3595 + 1.1797 \gamma_{mid} + .5391 \beta_2 - .00787 \gamma_{mid}^2 - .00397 \beta_2^2 + .00115 \gamma_{mid} \beta_2$$
(14b)

September 6 - 9, 2010, Lisbon, Portugal

$$Y_{3} = -17.93 + 2.01\beta_{1\text{Shroud}} + .5805\beta_{2} - .0130\beta_{1\text{Shroud}}^{2} - .00397\beta_{2}^{2} + .0002\beta_{1\text{Shroud}}\beta_{2}$$
(14c)

$$Y_4 = 37.03 + 1.228\gamma_{mid} - .352\beta_{1\text{Hub}} - .0078\gamma_{mid}^2 + 0.0142\beta_{1\text{Hub}}^2 + 0.00062\gamma_{mid}\beta_{1\text{Hub}}$$
(14d)

$$Y_5 = 60.5535 - .66962Y_1 - 0.91212Y_2 + 0.004299Y_1^2 + 0.005978Y_2^2 + 0.012869Y_1Y_2$$
(14e)

$$Y_6 = 57.0403 - .52137Y_3 - 0.97334Y_4 + .003361Y_3^2 + 0.0063741Y_4^2 + 0.01280Y_4Y_3$$
(14f)

$$\eta = .68350 - 3.42012Y_5 + 4.39817Y_6 - 2.330201Y_5^2 - 2.37611Y_6^2 + 4.706481Y_5Y_6$$
(14g)

Similarly, the corresponding polynomial representation of the model for NPSHr is in the form

$$Y_{1}' = -1.62 - .014\beta_{1Hub} + .16\beta_{1Shroud} + .0005\beta_{1Hub}^{2} - .0010\beta_{1Shroud}^{2} + 2.2e - 6\beta_{1Hub}\beta_{1Shroud}$$
(15a)

$$Y'_{2} = 3.426 - .0152\beta_{1\text{Hub}} + .0177 \ \beta_{2} + .0005\beta_{1\text{Hub}}^{2} + 3.109e - 12\beta_{2}^{2} + 1.22e - 5\beta_{2} \ \beta_{1\text{Hub}}$$
(15b)

$$Y'_{3} = 6.175 - .1292 \gamma_{mid} - .01609 \beta_{1Hub} + .001 \gamma_{mid}^{2} + .0005 \beta_{1Hub}^{2} + 2.83e - 5\gamma_{mid} \beta_{1Hub}$$
(15c)

$$Y'_{4} = -2.47 + .159 \beta_{1Shroud} + .017 \beta_{2} - .0010 \beta_{1Shroud}^{2} + 1.27e - 11\beta_{2}^{2} + 1.35e - 5\beta_{1Shroud}\beta_{2}$$
(15d)

$$Y'_{5} = 7.5491 - 3.5231Y'_{1} + 0.42880Y'_{2} + 0.46290Y'^{2}_{1} + 0.019501Y'^{2}_{2} + 0.060303Y'_{1}Y'_{2}$$
(15e)

$$Y_{6}' = 5.9118 - 1.963Y_{3}' - .6936Y_{4}' + .21291308Y_{3}'^{2} + 0.0801906Y_{4}'^{2} + 0.2316076Y_{3}'Y_{4}'$$
(15f)

$$NPSHr = -.3809 - .0652 Y'_{5} + 1.217 Y'_{6} + .0704 {Y'_{5}}^{2} + .030280 {Y'_{6}}^{2} - .1160804 Y'_{5} Y'_{6}$$
(15g)



Figure 3: CFD vs. Network

The very good behavior of such GMDH type neural network model for objective functions are also depicted in Figure 3, both for the training and testing data. It is evident that the evolved GMDH type neural network in terms of simple polynomial equations successfully model and predict the outputs of the testing data that have not been used during the training process. The models obtained in this section can now be utilized in a Pareto multi-objective optimization of the centrifugal pumps considering both efficiency and *NPSHr* as conflicting objectives. Such study may unveil some interesting and important optimal design principles that would not have been obtained without the use of a multi-objective optimization approach.

September 6 - 9, 2010, Lisbon, Portugal

4. Multi-objective optimization of centrifugal pumps using PSO method

James Kennedy and Russell Eberhart [3] originally proposed the PSO algorithm for optimization. PSO is a population-based search algorithm based on the simulation of the social behavior of birds within a flock. Although originally adopted for balancing weights in neural networks [13], PSO soon became a very popular global optimizer, mainly in problems in which the decision variables are real numbers [14].

In order to establish a common terminology, in the following we provide some definitions of several technical terms commonly used:

• **Swarm:** Population of the algorithm.

• **Particle:** Member (individual) of the swarm. Each particle represents a potential solution to the problem being solved. The position of a particle is determined by the solution it currently represents.

• **pbest** (personal best): Personal best position of a given particle, so far. That is, the position of the particle that has provided the greatest success (measured in terms of a scalar value analogous to the fitness adopted in evolutionary algorithms).

- **Ibest** (local best): Position of the best particle member of the neighborhood of a given particle.
- gbest (global best): Position of the best particle of the entire swarm.
- Leader: Particle that is used to guide another particle towards better regions of the search space.

In PSO, particles are "flown" through hyper-dimensional search space. Changes to the position of the particles within the search space are based on the social-psychological tendency of individuals to emulate the success of other individuals. The position of each particle is changed according to its own experience and that of its neighbors. Let \rightarrow

 $\vec{x_i(t)}$ denote the position of particle p_i , at time step t. The position of p_i is then changed by adding a velocity $\vec{v_i(t)}$ to the current position, i.e.:

$$\vec{x}_i(t+1) = \vec{x}_i(t) + \vec{v}_i(t+1)$$
(16a)

The velocity vector reflects the socially exchanged information and, in general, is defined in the following way:

$$\vec{v}_i(t+1) = W \vec{v}_i(t) + C_1 r_1 (\vec{x}_{pbest_i} - \vec{x}_i(t)) + C_2 r_2 (\vec{x}_{leader} - \vec{x}_i(t))$$
(16b)

where $r_1, r_2 \in [0,1]$ are random values.

In order to investigate the optimal performance of the centrifugal pumps, the polynomial neural network models obtained in section 3 are now employed in a multi-objective optimization procedure using PSO method. The two conflicting objectives in this study are efficiency and *NPSHr* that are to be simultaneously optimized with respect to the design variables $\gamma_{mid span}$, $\beta_{1 Hub}$, $\beta_{1 Shroud}$, and β_2 (Figures 1 and 2). The multi-objective optimization problem can be formulated in the following form:

September 6 - 9, 2010, Lisbon, Portugal

Figure 4 depicts the obtained non-dominated optimum design points as a Pareto front of those two objective functions. There are five optimum design points, namely, *A*, *B*, *C*, *D* and *E* whose corresponding design variables and objective functions are shown in Table 2. These points clearly demonstrate tradeoffs in objective functions efficiency and *NPSHr* from which an appropriate design can be compromisingly chosen. It is clear from Figure 4 that all the optimum design points in the Pareto front are non-dominated and could be chosen by a designer as optimum pump. Evidently, choosing a better value for any objective function in the Pareto front would cause a worse value for another objective. The corresponding decision variables of the Pareto front shown in Figure 4 are the best possible design points so that if any other set of decision variables is chosen, the corresponding values of the pair of objectives will locate a point inferior to this Pareto front. Such inferior area in the space of the two objectives is in fact bottom/right side of Figure 4.



Figure 4. Pareto front of efficiency and NPSHr using PSO method

In Figure 4, the design points A and E stand for the best NPSHr and the best efficiency. Moreover, the other optimum design points, B and D can be simply recognized from Figure 4. The design point, B exhibit important optimal design concepts. In fact, optimum design point B obtained in this paper exhibits an increase in NPSHr (about 1.17%) in comparison with that of point A whilst its efficiency improves about 30.56% in comparison with that of A, similarly optimum design point D exhibits a decrease in efficiency (about 11.29%) in comparison with that of point E whilst its NPSHr improves about 51.76% in comparison with that of E.

It is now desired to find a trade-off optimum design points compromising both objective functions. This can be achieved by the method employed in this paper, namely, the mapping method. In this method, the values of objective functions of all non-dominated points are mapped into interval 0 and 1. Using the sum of these values for each non-dominated point, the trade-off point simply is one having the minimum sum of those values. Consequently, optimum design point C is the trade-off points which have been obtained from the mapping method.

The Pareto front obtained from PSO method (Figure 4) has been superimposed with the Pareto front of NSGA II and the corresponding CFD simulation results, in Figure 5.

September 6 - 9, 2010, Lisbon, Portugal

Point	γ Mid span (deg)	β _{1Hub} (deg)	β _{1Shroud} (deg)	β ₂ (deg)	η (%)	NPSHr (m)
A	31.269	13.333	60.000	41.269	62.908	3.4863
В	44.603	23.809	60.000	40.000	72.107	3.5186
С	52.857	30.000	86.698	51.746	84.862	4.1400
D	59.206	30.000	79.333	57.142	89.633	4.7117
E	70.000	30.000	76.111	60.000	93.057	6.0344

 Table 2: The values of objective functions and their associated design

Variables of the optimum points

It can be clearly seen from this figure that PSO Pareto front lies on the best possible combination of the objective values of CFD data, which demonstrate the effectiveness of this paper in obtaining the Pareto front. As seen from this figure and region (A), 6 data samples have been located between PSO and NSGA II boundary. It means that PSO method detect the boundary of CFD data better than NSGA II for present case study.



Figure 5. Overlay graph of the obtained optimal Pareto front of PSO and NSGA II with the CFD simulation data

5. Conclusion

Two different polynomial relations for efficiency and *NPSHr* have been found by evolved GS-GMDH type neural networks using some experimentally validated CFD simulations for input–output data of the centrifugal pumps. The derived polynomial models have been then used in an multi-objective PSO optimization process so that some interesting and informative optimum design aspects have been revealed for pumps with respect to the control variables such as geometrical parameters of $\gamma_{Mid Span}$, β_{1Hub} , $\beta_{1Shroud}$ and β_2 (Figures 1 and 2) and Consequently, some very important tradeoffs in the optimum design of centrifugal pumps have been obtained and proposed based on the Pareto front of two conflicting objective functions. The Pareto front of PSO method and NSGA II have been compared and showed that PSO detect the boundary of CFD data better than NSGA II for present case study.

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September 6 - 9, 2010, Lisbon, Portugal

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