Numerical simulation of contact problems under large 3d elastoplastic deformation

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Abstract. In this work new simulations of 3D contact problems with friction in solid mechanics are presented. The bodies in contact undergo finite deformation within an elastoplastic range. A brick element based on a compressible elastoplastic material is applied in the numerical examples. The augmented Lagrangian method is used to solve the contact problems. For the contact formulation within the finite element method, the matrix formulation for a node-to-surface element consisting of a master surface with four nodes and a contacting node is derived. Here, the discretised contact surfaces are not smooth, i.e. there is no continuity of the normal vector between the adjacent surfaces. At the edge between the surfaces the normal is not uniquely defined. That needs a special algorithmic treatment. When one slave node is sliding from one surface to the adjacent one, the node-to-surface contact formulation is not sufficient to solve the contact problems. Then, for the special cases the node-to-edge and node-to-node contact formulations are employed, see Bandeira et al. (2005). The purpose of this paper is to present numerical examples using the formulation of contact mechanics and the elastoplasticity algorithm for large 3D deformation, considering the possible sliding of slave node from one surface to the adjacent one. The formulation is derived based on exact linearization. Several numerical examples for contact problems in elastoplastic range are presented.

Keywords: Finite element, plasticity, contact mechanics, computational mechanics, non-linear analysis, numerical simulation.

1. INTRODUCTION

Numerous contact formulations have been presented in the literature. Three-dimensional node-to-surface contact formulation can be found in e.g. Laursen et al. (1993a), Laursen (2002), Hallquist et al. (1985), Heegaard et al. (1993), Wriggers (2002), Alart (1991) or Bandeira et al. (2005, 2006). The non-penetration condition as a purely geometrical constraint is used to impose contact in normal direction. For the frictional response, the constitutive equation of the classical Coulomb’s law, like in Curnier (1984) and Laursen et al. (1993a and 1993b) is used. Frictional phenomena have been considered within the framework of plasticity theory. The response in tangential direction can be divided into two different actions. First, no tangential relative displacement of the two bodies occurs which is so-called stick condition. The second action is associated with a relative tangential movement in the contact interface, so-called slip condition.

In this work a three-dimensional brick element with eight nodes is used for the treatment of finite elastoplastic deformation of the contacting surfaces. The finite element formulation for the brick element can be found in Zienkiewicz et al. (2005), Holzapfel (2001) and Truesdell and Noll (2004). An augmented Lagrangian method is applied to solve the frictional contact problems. The mathematical theories concerning augmented Lagrangian method in the context of mathematical programming problems subjected to equality and inequality constraints are well established by Fletcher (1980), Luenberger (1984) and Bertsekas (1995). In the context of finite element methods, augmented Lagrangian techniques have been successfully applied to frictional contact problems in solid mechanics by Simo et al. (1992) and Laursen & Maker (1995).

When large deformations occur in the contact mechanics, a large amount of sliding can happen within the contact interface. Two methods can be followed to discretise the contact interfaces. One, which applied here, is to discretise the contact interface by the isoparametric interpolation, where there is no continuity in the normal vector between adjacent surfaces. Thus, in the edge between two adjacent surfaces the normal is not uniquely defined and a special algorithmic treatment is needed. The other one is the smooth contact discretization, which allows for a smooth sliding of contacting nodes on the master surface. Within this approach, a Hermitian, Spline
or Bézier interpolation is used to discretise the master surface. This leads to a C1 or even C2 interpolation of the surface. In three-dimensions it is difficult to develop one general formulation for smooth contact.

The main goal of this work is to present simulations of contact problems using the node-to-surface, node-to-edge and node-to-node contact formulations, which together, can solve contact problems with large sliding. The complete formulation presented is characterized by algorithmic stability, short evaluation time, high performance and quadratic rate of convergence within a Newton equation-solving strategy, owing to the exact linearization employed. It is important to mention, that the complete contact formulation was presented in Bandeira et al. (2005).

The objective of this paper is to show some numerical results using the complete contact formulation that allow the sliding of the slave node from one surface to the adjacent one. Here, all numerical examples undergoing large three-dimensional elastoplastic deformation.

The plasticity algorithmic is based on the Von Mises theory on the principal directions. For details see Pimenta (1992). Theory of plasticity can be founded also in Kachanov (2004) and Hearn (1997).

It is important to mention that the contact formulations and the plasticity theory and its algorithmic are not presented in this paper because these fields are very complicated to describe here. For details refer Bandeira et al. (2005, 2006).

2. FRICtIONAL CONTACT MECHANICS FORMULATION

This section summarises the continuum formulations applied to solve multibody frictional contact problems undergoing large 3D deformation. The node-to-surface, node-to-edge and node-to-node contact formulations used in this paper are developed in Bandeira et al. (2005) and Laursen et al. (1993a). Within this reason, it will be not present in this paper.

2.1. Contact Kinematics

The normal and tangential gaps define contact contribution. The impenetrability condition can be formulated as

\[ g_N = (x^s - \bar{x}^m) \cdot \bar{n}_c \leq 0 \quad (1) \]

and furthermore, the penetration function can be formulated in following manner

\[ g_N = \begin{cases} 
(x^s - \bar{x}^m) \cdot \bar{n}_c , & \text{if } (x^s - \bar{x}^m) \cdot \bar{n}_c \geq 0 \\
0 , & \text{if } (x^s - \bar{x}^m) \cdot \bar{n}_c < 0 
\end{cases} \quad (2) \]

where the bar denotes the closest distance of a point \( x^s \) to the surfaces of \( B^m \).

The sliding path of the slave node \( x^s \) on the contact surface \( \Gamma^m \) is described by the total tangential gap as

\[ g_T = \int_{t_0}^1 \| \hat{\xi}^a \| \bar{a}_a \| \, dt, \quad (3) \]

where \( \alpha = 1, 2 \). The parameters \( \xi^1 \) or \( \xi^2 \) are introduced to describe the surface of the solids and can be interpreted as convective co-ordinates. \( \bar{a}_a \) are the tangents at the solution points \( \xi^1 \) and \( \xi^2 \) of the distance function minimization

\[ \hat{d}(\xi^1, \xi^2, t) = \text{minimum} \| x^s - \bar{x}^m(\xi^1, \xi^2, t) \|. \quad (4) \]

These tangential vectors are obtained from

\[ \bar{a}_a = \frac{\partial \bar{x}^m(\xi^a, t)}{\partial \xi^a} = \bar{x}^m, \quad (5) \]
The tangential relative slip between two bodies is related to the change of the solution point \( (\xi_1, \xi_2) \) of the minimum distance problem (4). The time derivatives of the parameters \( \xi^\alpha \) from (3) are obtained using the orthogonality condition of both tangent and normal vector. This yields

\[
\dot{\xi}^\beta = \frac{1}{\alpha_{\beta\alpha} - g_{\alpha\beta}} \left\{ [v^s - \bar{v}^m] \cdot \alpha_a + g_{\alpha\beta} \bar{n}_c \cdot \bar{v}^m \right\},
\]

(6)

where \( \alpha_{\beta\alpha} = \alpha_\beta \cdot \alpha_a \) is the metric and \( \bar{b}_{\alpha\beta} = \dot{k}_m^{\alpha\beta} \cdot \bar{n}_c \) is the curvature of the contact surface. When the frictional contact problem is solved in an exact way, the slave node moves on the master surface. With this condition \( g_{\alpha\beta} \) can be neglected and the equation (6) can be simplified as \( v^s - \bar{v}^m = \dot{\xi}_2^\alpha \bar{n}_a \). The relative tangential velocity at the contact point is defined by

\[
T_g \dot{\xi}_2 = v^s - \bar{v}^m.
\]

2.2. Contact Contribution to the Weak Form

The contact contribution to the weak form can be formulated in following manner

\[
W_c(u,v) = \int_{\partial \Omega_{oc}} \left( t_n \bar{n}_c + t_T \right) \cdot (v^s - \bar{v}^m) \, dA
\]

(7)

where \( t_n \) is the contact pressure, \( t_T \) is the tangential traction vector, \( \bar{n}_c \) is the internal normal at the slave node \( x^s \), \( v^s \) the velocity at the slave node and \( \bar{v}^m \) the velocity at the master nodes. The variation of normal gap is defined by

\[
\delta g_N = (v^s - \bar{v}^m - \bar{n}_a \dot{\xi}_2^\alpha) \cdot \bar{n}_c + (x^s - \bar{x}^m) \cdot \delta \bar{n}_c = (v^s - \bar{v}^m) \cdot \bar{n}_c.
\]

(8)

In equation (8), the orthogonality condition \( \bar{n}_a \cdot \bar{n}_c = 0 \) and the simple relationship \( \bar{n}_c \cdot \bar{n}_c = 1 \Rightarrow \delta \bar{n}_c \cdot \bar{n}_c = 0 \) are used. Following the same idea, \( \Delta g_N \) can be written as \( \Delta g_N = (\Delta u^s - \Delta \bar{u}^m) \cdot \bar{n}_c \). With these results, equation (7) can be rewritten as

\[
W_c(u,v) = \int_{\partial \Omega_{oc}} t_n \bar{n}_c \cdot (v^s - \bar{v}^m) \, dA + \int_{\partial \Omega_{oc}} t_T \cdot (v^s - \bar{v}^m) \, dA
\]

\[
= \int_{\partial \Omega_{oc}} t_n \delta g_N \, dA + \int_{\partial \Omega_{oc}} t_T \cdot \delta g_T \, dA = \int_{\partial \Omega_{oc}} t_n \delta g_N \, dA + \int_{\partial \Omega_{oc}} t_{fa} \delta \xi^\alpha \, dA
\]

(9)

with \( t_{fa} = t_T \cdot \bar{n}_a \). The linearization of the virtual work (9) is defined by

\[
\frac{\partial W_c}{\partial u} \Delta u = \int_{\partial \Omega_{oc}} [\Delta t_n \delta g_N + t_n \Delta (\bar{g}_N)] \, dA + \int_{\partial \Omega_{oc}} [\Delta t_{fa} \delta \xi^\alpha + t_{fa} \Delta (\bar{g}_N)] \, dA.
\]

(10)

The terms in equation (10) is defined in details in Laursen (1993a, 1993b). The non-symmetry has been encountered in the linearization of \( W_c \) for the stick case stemming only from the term \( \Delta t_{fa} \delta \xi^\alpha \) in (10). For the slip case the tangential stiffness matrix is clearly non-symmetrical. The algorithm used to integrate the frictional equations is completely developed in Laursen et al. (1993a). In this work, a simple non-associated Coulomb friction law is used.

3. NUMERICAL EXAMPLES
In this chapter some examples are analysed using the formulation presented in this paper. A finite element code was written by Bandeira using C++. All cases presented here are characterised by quadratic rate of convergence within a Newton equation-solving strategy, owing to employed exact linearization. The numerical examples are based on three-dimensional calculations with finite deformation. The 8-noded brick elements are used to discretise the bodies.

3.1. Example 1: Block and Pipe

The following example corresponds to a contact between a block and a pipe. The geometrical properties and the material features are presented in Figure 1.

![Figure 1: Initial configuration](image1)

In the initial configuration the pipe rests on the block without generating contact forces. The contact area in the initial configuration is exactly a line. The base of the block is restrained completely in all directions. To avoid the sliding of the cylinder along x-axis the displacements of the nodes on the contact line are restrained in the x-direction. A vertical displacement of 0.8 is applied on the top of the cylinder in 40 increments. The block is modeled with a mesh of 40x20x4 elements and the pipe with a mesh of 82x4x20 elements. The problem involves 36099 degrees of freedom. The penalty parameters used are $\varepsilon_N = 100$ and $\varepsilon_T = 1.0$. The friction coefficient between the contact surfaces is set to $\mu = 0.4$. The deformed configuration obtained after the computational analysis is shown in Figure 2. The plasticity zone developed in the body is plotted in Figure 2. This zone is defined by the equivalent plasticity strain, parameter $\alpha$, of the plasticity algorithmic developed by Pimenta (1992). The superior structure lost convergence in the 53th increment of load.

3.2. Example 2

The numerical example shown in Figure 3 corresponds to a contact between two cylindrical shells. The geometrical properties and the characteristics of the materials are presented in Figure 4.

![Figure 2: Equilibrium configuration (step 53)](image2)

In the initial configuration cylindrical shells lie one on another without generating contact forces. The base of the inferior shell is restrained in all directions. On the top faces of the superior shell vertical displacements of 0.35 are imposed in 70 load increments. To avoid the sliding in the x-direction, all nodes on the contact line are restrained. Both bodies are modeled by mesh of 80x4x20 elements. The problem involves 49728 degrees of freedom. The penalty parameters used in the analysis are $\varepsilon_N = 100$ and $\varepsilon_T = 1.0$. The coefficient of friction between the contact surfaces is $\mu = 0.4$. The final deformed configuration is shown in Figure 4. In this picture we can see the plasticity zone developed at the superior body. The superior structure lost convergence in the 43th increment of load.
3.3. Example 3

In this example, Figure 5, the number of master and slave surfaces is around 2403 elements and the complete mesh is around 21627 bricks elements. Each block has the same geometry of 90mm×45mm×15mm and material properties defined by elasticity module of 70 GPa, Poisson coefficient of 0.3 and adopted initial yield stress of 200 MPa. The base of the master block is fixed and lateral displacements of both blocks are released. The contact surfaces of the bodies are irregular. The hardness is modeled in the Finite element mesh. The micromechanical formulations are presented in detail in Zavarise et al. (1992). The hardening parameters used is defined by $h = E / 100$. The contact surfaces are modified according to the theory presented Bandeira et al. (2005, 2006), such that the maximum initial asperities height $\xi$ is 0.180394 mm. A uniform displacement of 8 mm is prescribed at the top of the slave block in several increments. Each analysis ends when the current mean plane distance $d$ approaches zero. The mean plane distance goes to zero in the 35th increment of load.

After all generated surfaces were analyzed, the mean value curve of the normal pressures is depicted in Figure 6, which represents the constitutive interface law for different hardness.

The plastic zone developed at the master surface can be analyzed in each increment of loads by the equivalent plastic strain, see Figure 7, Figure 8, Figure 9 and Figure 10.
4. CONCLUSIONS

The analysis of contact problems in three-dimensions involving large deformations is a topic under intensive development. The contact formulations have been usually developed in two dimensions. For the three-dimensional analysis the formulation of contact is much more complicated. One of the reasons, obviously, is the complexity involving space geometry when the projection of the slave node on the master surface is calculated. This requires a sophisticated algorithm. Laursen et al. (1993a) developed the node-to-surface contact element considering the contact between a slave node and a surface element. In that formulation the sliding of the slave node from a master surface to the adjacent one is not allowed. The contact formulation presented in Bandeira et al. (2005) allows for the general sliding of a slave node on the master surfaces. Therefore, this formulation constitutes a general and consistent contribution to the contact mechanics. To achieve this, the node-to-edge and the node-to-node contact formulations are employed. An outstanding feature is the quadratic rate of convergence within a Newton equation-solving strategy, owing to the exact linearization employed. It is important to mention that the linearization of the node-to-edge contact formulation was performed exactly and it can also be applied in the two-dimensional contact problems.

All numerical examples presented in this article addresses three-dimensional problems with large deformations. In the considered numerical examples it was numerically proven that the theories developed in Bandeira et al. (2005) work perfectly. The overall performance of the developed algorithms was remarkably efficient yielding good results and quadratic rate of convergence.

This work can be regarded as a complementary study of Bandeira et al. (2005). In example 3, the plasticity of the asperities is taken into account by assuming a constitutive equation based on associated von Mises yield function formulated in principal axes. The authors can concluded that is possible to modeling the micromechanical phenomena developed at the contact surfaces and obtained results for constitutive equations derived by numerical simulations with good agreements with the theoretical laws. In the numerical examples the normal and tangential contact pressures are plotted to show the contact surfaces behaviour. The plasticity evolution developed at the contact surfaces are also presented in the numerical results. It can be seen that the contact surfaces developed high plasticity effects during the contact mechanics.

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6. REFERENCES


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