Identification of Fluid Transient Temperature

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Abstract
Under steady-state conditions when fluid temperature is constant, there is no damping and time lag and temperature measurement can be accomplished with high degree of accuracy. However, when fluid temperature is varying rapidly as during start-up, quite appreciable differences occur between the actual fluid temperature and the measured temperature. This is due to the time required for the transfer of heat to the thermocouple placed inside a heavy thermometer pocket. In this paper, two different techniques for determining transient fluid temperature based on the first and second order thermometer model are presented. The fluid temperature was determined using the temperature indicated by the thermometer, which was suddenly immersed into boiling water. To demonstrate the applicability of the presented method to actual data, the time constants for the three sheathed thermocouples with different diameters, placed in the air stream, were estimated as a function of the air velocity.

Keywords: Temperature measurement, transient conditions, first and second order models, time constant, uncertainty analysis.

1. Introduction
Most of the books on temperature measurements concentrate on steady-state measurements of the fluid temperature [1-9]. Only a unit-step response of thermometers is considered to estimate the dynamic error of the temperature measurement. Little attention is paid to measurements of the transient fluid temperature, despite the great practical significance of the problem [10-12]. The measurement of the transient temperature of steam or flue gases in thermal power stations is very difficult. Massive housings and low heat transfer coefficient cause the actually measured temperature to differ significantly from the actual temperature of the fluid. Some particularly heavy thermometers may have time constants of 3 minutes or more, thus requiring about 15 minutes to settle for a single measurement. There are some thermometer designs where there is more than one time constant involved. In order to describe the transient response of a temperature sensor immersed in a thermowell the measuring of the medium temperature in a controlled process may have two or three time constants which characterise the transient thermometer response. The problem of a dynamic error during the measuring of the temperature of the superheated steam is particularly important for the superheated steam temperature control systems which use injection coolers (spray attemperators). Due to a large inertia of the thermometer, a measurement of the transient temperature of the fluid, and thus the automatic control of the superheated steam temperature can be inaccurate. A similar problem is encountered in flue gas temperature measurements, since the thermometer time constant and time delay are large. In this paper two methods of determining the transient temperature of the flowing fluid on the basis of the temperature time changes indicated by the thermometer are presented.

In the first method the thermometer is considered to be a first order inertia device and in the second method it is considered as a second order inertia device. A local polynomial approximation, based on 9 points was used for the approximation of the temperature changes. This assures that the first and the second derivative from the function representing the thermometer temperature changes in time will be determined with a great accuracy. An experimental analysis of the industrial thermometer at the step increase of the fluid temperature was conducted. The temperature time histories determined using the two proposed methods at the step increase of the fluid temperature were compared.

2. Mathematical models of the thermometers
Usually the thermometer is modeled as an element with lumped thermal capacity. In this way, it is assumed that the temperature of the thermometer is only the function of time, and temperature differences occurring within the thermometer are neglected. The temperature changes of the thermometer in time $T(t)$ have been described by an ordinary first order differential equation (first order thermometer model)
For thermometers with a complex structure used for measuring the temperature of the fluid under high pressure, the accuracy of the first order model (1) is inadequate.

3. Thermometer of a complex structure

To demonstrate that a dynamic response of a temperature sensor placed in a housing may be described by a second-order differential equation, a simple thermometer model shown in Fig. 1 will be considered.

![Cross section through the temperature sensor together with the housing](image)

An air gap can appear between the external housing and the temperature sensor. The thermal capacity of this air gap \( c \cdot \rho \) is neglected due to its small value (Fig. 1).

Introducing the overall heat transfer coefficient \( k_w \) referenced to the inner surface of the housing

\[
\frac{1}{k_w} = \frac{1}{\alpha_w} + \left( \frac{1 + D_o/d}{d} \right) \frac{D_w}{4 \lambda_p} + \frac{D_w}{d} \frac{1}{\alpha_t},
\]

(2)

and accounting for the radiation heat transfer from the housing to the inner sensor, the heat balance equation for the sensor located within the housing assumes the form:

\[
A \rho c \frac{dT}{dt} = P_w k_w (T_o - T) + C (T_o^4 - T^4),
\]

(3)

where the symbol \( C \) denotes:

\[
C = \frac{\pi d \sigma}{\frac{1}{e_t} + d \left( \frac{1}{e_t} - 1 \right)}.
\]

The convection and conduction heat transfer between the fluid and the thermometer housing is characterized by the overall heat transfer coefficient \( k_t \) referenced to the outer housing surface:

\[
\frac{1}{k_t} = \frac{1}{\alpha_t} + \frac{1 + D_c/D_o}{2} \frac{\delta_c}{\lambda_c}.
\]

(4)

The formulas (2) and (4) for the overall heat transfer coefficients were derived using the basic principles of heat transfer [2, 4, 8]. The heat transfer equation for the housing (thermowell) can be written in the following form:

\[
A_o \rho_c \frac{dT_o}{dt} = P_w k_w (T_o - T) - P_o k_o (T_o - T) - C (T_o^4 - T^4).
\]

(5)

In the analysis of the heat exchange between the housing and the thermocouple the radiation heat transfer will also be disregarded. Such a situation occurs, when a gap between the housing and temperature sensor is filled with non-transparent substance or one of the two emissivities \( e_t \) and \( e_o \) is close to zero.

After determining the temperature \( T_o \) from Eq.(3), we obtain:
Substituting Eq. (6) into Eq. (5) yields:

$$T_0 = \frac{A \rho c}{P_k} \frac{dT}{dt} + T.$$  \hfill (6)

Introducing the following coefficients:

$$a_2 = \frac{(A \rho c_0)(A \rho c)}{(P_k k_0)(P_k k_1)}, \quad a_1 = \frac{A \rho c_0}{P_k k_1} \left[ 1 + \frac{(P_k k_1)(A \rho c)}{(P_k k_0)(A \rho c_0)} + \frac{A \rho c}{A \rho c_0} \right],$$

the ordinary differential equation of the second order (7) can be written in the form:

$$a_2 \frac{d^2 T}{dt^2} + a_1 \frac{dT}{dt} + T = T_\infty.$$  \hfill (7)

The initial conditions are:

$$T(0) = T_0 = 0, \quad \frac{dT(t)}{dt}\bigg|_{t=0} = v_\tau = 0.$$  \hfill (8)

The initial problem (8-9) was solved using the Laplace transformation. The operator transmittance $G(s)$ assumes the following form:

$$G(s) = \frac{T(s)}{T_\infty(s)} = \frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)}.$$  \hfill (9)

The time constants $\tau_1$ and $\tau_2$ in Eq. (10) are:

$$\tau_{1,2} = \frac{2a_2}{a_1 \pm \sqrt{a_1^2 - 4a_2}}.$$  \hfill (10)

The differential Equation (8) can be written in the following form:

$$\tau_1 \tau_2 \frac{d^2 T}{dt^2} + (\tau_1 + \tau_2) \frac{dT}{dt} + T = T_\infty.$$  \hfill (11)

For the step increase of the fluid temperature from $T_0 = 0 ^\circ C$ to the constant value $T_\infty$ the Laplace transform of the fluid temperature assumes the form: $T_\infty(s) = \frac{T_\infty}{s}$ and the transmittance formula can be simplified to:

$$G(s) = \frac{T(s)}{T_\infty(s)} = \frac{1}{s(\tau_1 s + 1)(\tau_2 s + 1)}.$$  \hfill (12)

After writing Eq. (12) in the form:

$$\frac{T(s)}{T_\infty} = \frac{1}{s} + \frac{\tau_1}{\tau_2 - \tau_1} \left( \frac{1}{s + \frac{1}{\tau_1}} - \frac{\tau_2}{\tau_2 - \tau_1} \right) \left( \frac{1}{s + \frac{1}{\tau_2}} \right),$$

it is easy to find the inverse Laplace transformation and determine the thermometer temperature as a function of time:

$$u(t) = \frac{T(t)}{T_\infty} = 1 + \tau_1 \left( \frac{1}{\tau_2 - \tau_1} \right) \exp \left( -\frac{t}{\tau_1} \right) - \tau_2 \left( \frac{1}{\tau_2 - \tau_1} \right) \exp \left( -\frac{t}{\tau_2} \right).$$  \hfill (13)

For the first order model the thermometer response for a unit step fluid temperature change is determined by the simple expression:

$$u(t) = 1 - \exp \left( -\frac{t}{\tau_1} \right).$$  \hfill (14)

If we assume in Eq. (15) $\tau_2 = 0$ then we obtain Eq. (16) with $\tau = \tau_1$. In the time response of a first order system, given by Eq. (16), there is no a time delay (a dead time). Heavy thermometers for measuring fluid temperature at high pressure involve a time delay between the temperature sensor output and the fluid temperature changes. The second order thermometer model is appropriate to describe the response behaviour with a time delay.

The time constant $\tau$ in Eq. (16) or time constants $\tau_1$ and $\tau_2$ in Eq. (15) will be estimated from experimental data. The fluid temperature can be determined on the basis of measured histories of the thermometer temperature $T(t)$ and known time constants $\tau_1$ and $\tau_2$. 

$$u(t) = 1 - \exp \left( -\frac{t}{\tau_1} \right).$$  \hfill (15)
The changing in time fluid temperature $T_{cz}(t)$ can be determined from Eq. (1) or Eq. (12) after a priori determination of the time constant $\tau$ or time constants $\tau_1$ and $\tau_2$ respectively. The thermometer temperature changes $T(t)$, the first and second order time derivatives from the function $T(t)$ can be smoothed using the formulas [3]:

\[
T(t) = \frac{1}{693} \left[ -63 f(t - 4 \cdot \Delta t) + 42 f(t - 3 \cdot \Delta t) + 
+117 f(t - 2 \cdot \Delta t) + 162 f(t - \Delta t) + 177 f(t) + 162 f(t + \Delta t) + 
+117 f(t + 2 \cdot \Delta t) + 42 f(t + 3 \cdot \Delta t) - 63 f(t + 4 \cdot \Delta t) \right]
\]

\[
T'(t) = \frac{dT(t)}{dt} = \frac{1}{1188 \Delta t} \left[ 86 f(t - 4 \cdot \Delta t) - 142 f(t - 3 \cdot \Delta t) - 
-193 f(t - 2 \cdot \Delta t) - 126 f(t - \Delta t) + 126 f(t + \Delta t) + 
+193 f(t + 2 \cdot \Delta t) + 142 f(t + 3 \cdot \Delta t) - 86 f(t + 4 \cdot \Delta t) \right]
\]

\[
T''(t) = \frac{d^2 T(t)}{dt^2} = \frac{1}{462 \Delta t^2} \left[ 28 f(t - 4 \cdot \Delta t) + 7 f(t - 3 \cdot \Delta t) - 
-8 f(t - 2 \cdot \Delta t) - 17 f(t - \Delta t) - 20 f(t) - 17 f(t + \Delta t) - 
-8 f(t + 2 \cdot \Delta t) + 7 f(t + 3 \cdot \Delta t) + 28 f(t + 4 \cdot \Delta t) \right],
\]

in order to eliminate, at least partially, the influence of random errors in the thermometer temperature measurements $T(t)$ on the determined fluid temperature $T_{cz}(t)$. The symbol $f(t)$ in Eq. (17-19) denotes the temperature indicated by the thermometer and $\Delta t$ is a time step.

If measured temperature histories are not too noisy, the first and second order derivatives can be approximated by the central difference formulas

\[
T'(t) = \frac{f(t+\Delta t) - f(t-\Delta t)}{2\Delta t},
\]

\[
T''(t) = \frac{f(t+2\Delta t) - 2f(t+\Delta t) + 2f(t-\Delta t) - f(t-2\Delta t)}{(2\Delta t)^2}.
\]

Equation (1) and Equation (12) can also be used for determining fluid temperature $T_{cz}$ when the time constants of the thermocouple $\tau$ or $\tau_1$ and $\tau_2$ are a function of fluid velocity $w$. After substituting the time constant $\tau(w)$ into Eq. (1) we can determine fluid temperature $T_{cz}(t)$ for different fluid velocities using the proposed method.

4. Experimental determination of time constants

The method of least squares was used to determine the time constants $\tau_1$ and $\tau_2$ in Eq. (15) or the time constant $\tau$ in Eq. (16). The values for the time constants are found by minimising the function $S$:

\[
S = \sum_{i=1}^{N} \left[ u_i(t_i) - u(t_i) \right]^2 = \min,
\]

where $u(t)$ is the approximating function given by Eq. (15) or Eq. (16). The symbol $N$ denotes the number of measurements ($t_i, u_{i_d}(t_i)$). That is, the sum of the squares of the deviations of the measured values $u_{i_d}(t_i)$ from the fitted values $u(t_i)$ is minimized. Once the time constants $\tau_1$ and $\tau_2$ or $\tau$ have been determined they can be substituted into Eq. (22) to find the value for $S_{\text{min}}$.

The uncertainties in the calculated time constants $\tau_1$ and $\tau_2$ or in $\tau$ are estimated using the mean square error [13-15]:

\[
S_{\chi} = \frac{S_{\text{min}}}{\sqrt{N - m}},
\]

where $m$ is the number of time constants ($m = 2$ for Eq. (15) and $m = 1$ for Eq. (16)).

Based on the calculated mean square error $S_{\chi}$, which is an approximation of the standard deviation, the uncertainties in the determined time constants can be calculated using the formulas given in the TableCurve 2D software [15].

5. Determining the fluid temperature on the basis of time changes in the thermometer temperature

An industrial thermometer (Fig. 2) at the ambient temperature was suddenly immersed into hot water with saturation temperature. The thermometer temperature data was collected using the Hottinger-Baldwin Messtechnik data acquisition system. The measured temperature changes were approximated using functions (15)
and (16). The time constant $\tau$ in Eq. (16) and time constants $\tau_1$ and $\tau_2$ in function (15) were determined using the TableCurve 2D code [15]. The following values with the 95% confidence uncertainty were obtained: $\tau = 14.07 \pm 0.39 \, \text{s}$, $\tau_1 = 3.0 \pm 0.165 \, \text{s}$, and $\tau_2 = 10.90 \pm 0.2 \, \text{s}$.

Next, the fluid temperature changes were determined from Eq. (1) for the first order model and from Eq. (12) for the second order model (Fig. 3). The time step $\Delta t$ was 1.162 \, \text{s}.

The analysis of the results presented in Fig. 3 indicates that the second order model delivers more accurate results. The same tests were repeated for sheathed thermocouple with outer diameter 1.5 mm and the results are presented in Fig. 4. The estimated value of the time constant and the uncertainty at 95% confidence are: $\tau = 1.54 \pm 0.09 \, \text{s}$. As the thermocouple is thin, then Eq. (16) was used as the function approximating the transient response of the thermocouple. First, the transient fluid temperature $T_{cz} = u(t)$ was calculated using Eq. (1) together with Eqs. (17) and (18). Then, the raw temperature data was used. The first order time derivative $dT/dt$ in Eq. (1) was calculated using the central difference quotient (20). The inspection of the results displayed in Fig. 4 indicates, that the central difference approximation of the time derivative in Eq. (1) leads to less accurate results, since it is more sensitive to random errors in the experimental data.
The thermocouple time constant $\tau$ for various air velocities $w$, were determined in the open benchtop wind tunnel (Fig. 5). The WT4401-S benchtop wind tunnel is designed to give a uniform flow rate over 100 mm × 100 mm test cross section [16].

![Fig. 4. Fluid and thermometer temperature changes determined from the first order Eq. (1) for the sheathed thermocouple with outer diameter 1.5 mm](image1)

![Fig. 5. Benchtop wind tunnel used for determining thermocouple time constant](image2)

The experimental data points displayed in Fig. 6 were approximated by the least squares method for three different thermocouple diameters 0.5 mm, 1.0 mm and 1.5 mm. The following function was obtained:

$$\tau = \frac{1}{a + b\sqrt{w}},$$  \hspace{1cm} (24)

where $\tau$ is expressed in s, and $w$ in m/s.

The best estimates for the constants $a$ and $b$, with the 95 % confidence uncertainty in the results, are:

- thermocouple with outer diameter 0.5 mm
  
  $a = 0.004337 \pm 0.000622$ \, 1/s and $b = 0.022239 \pm 0.001103$ \, (m·s)$^{-1/2}$,  

- thermocouple with outer diameter 1.0 mm
  
  $a = 0.020974 \pm 0.006372$ \, 1/s and $b = 0.103870 \pm 0.011240$ \, (m·s)$^{-1/2}$

- thermocouple with outer diameter 1.5 mm
  
  $a = 0.040425 \pm 0.003301$ \, 1/s and $b = 0.056850 \pm 0.004479$ \, (m·s)$^{-1/2}$

The variations of the thermocouple time constants $\tau$ with the fluid velocity for the sheathed thermocouples with the outer diameter of 0.5 mm, 1.5 mm and 3.0 mm are shown in Fig. 6.
Fig. 6. Time constants $\tau$ of sheathed thermocouples with outer diameters of 0.5 mm, 1.5 mm and 3.0 mm as a function of air velocity $w$ with 95% confidence intervals.

Fig. 7. Temperature measurements: a) air velocity changes, b) temperature indicated by thermocouples and calculated air temperature.
The time constant of the thermocouple $\tau = m_T c_p (\alpha_T A_T)$ depends strongly on the heat transfer coefficient $\alpha_T$ on the outer thermometer surface, which in turn is a function of the air velocity [17].

When the velocity and temperature of the air stream change in time, the velocity dependent time constant (24) can be used in Eq. (1) to estimate the air temperature based on the temperature readings from the sheathed thermocouples.

In order to determine fluid temperature three thermocouples with different diameters of 0.5 mm, 1.0 mm and 1.5 mm were used simultaneously. Time constants $\tau$ as a function of air velocity for mentioned above thermocouples are presented in Fig. 6. Air velocity changes during temperature measurement depicts Fig. 7a. Temperature indicated by thermocouples and calculated fluid temperature are shown in Fig. 7b. A significant improvement in air temperature measurement accuracy can be observed when the proposed method was applied for determining air temperature. The agreement between the air temperature for the thermocouple with 0.5 mm and for the thermocouple with 1.0 mm outer diameter is very good. It seems that the thermocouple with 1.5 mm outer diameter is less adequate to follow fast changes fluid temperature, since its time constant is large.

6. Conclusions

Both methods of measuring the transient temperature of the fluid presented in this paper can be used for the on-line determining fluid temperature changes as a function of time.

The first method in which the thermometer is modelled using the ordinary, first order differential equation is appropriate for thermometers which have very small time constants. In such cases the delay of the thermometer indication is small in reference to the changes of the temperature of the fluid. For industrial thermometers, designed to measure temperature of fluids under a high pressure there is a significant time delay of the thermometer indication in reference to the actual changes of the fluid temperature. For such thermometers the second order thermometer model, allowing for modelling the signal delay, is more appropriate. Large stability and accuracy of the determination of the actual fluid temperature on the basis of the time temperature changes indicated by the thermometer can be achieved by using a 9 point digital filter.

Fluid temperature changes obtained using the two described methods were compared. It was established that the thermometer model of the second order gave better results for the industrial thermometer having the large thermowell. The techniques proposed in the paper can also be used, when time constants are functions of fluid velocity.

References