Fin shape optimization in tube heat exchangers by means of CFD program

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Abstract
The optimization of finned tube heat exchanger is presented focusing on the thermal efficiency improvement and weight reduction by the fin/tube group shape modification. In the cross-flow heat exchanger, the air streams are not heated and cooled evenly. The heat flux depends on the temperature difference between the local plate/tube and local air temperatures. This means that the temperatures vary along the cross section of the air stream and along the fluid flow direction. The fin and tube geometry affects the flow direction and has the effect on the temperature changes. Numerical analyses are carried out to examine finned tube heat exchanger. Three-dimensional models are performed to find heat transfer characteristics between a finned tube and the air. The heat is transferred from the tube/fins to the air that flows over the fin. Area weighted fin surface average temperature and mass flow weighted average temperature of air volume flow rate are calculated in the outlet section and compared for different fin/tube shapes in order to optimize the heat transfer between the material of the fin and air during the air flow in the cross flow heat exchanger.

Keywords: fin tube, optimization, flow, CFD, heat transfer

Nomenclature

\(A_{\text{fin}}\) – fin surface area, \(\text{m}^2\)
\(a_{\text{fin}}\) – the mesh element size on the fin surface that surrounds the local node, \(\text{m}^2\)
\(c_f\) – fluid (air) specific heat capacity, \(\text{J/(kg K)}\)
\(f(r)\) – fin profile function
\(m_f\) – fluid (air) mass flow rate, kg/s
\(m_{f_n}\) – local fluid (air) mass flow at a node, kg/s
\(m_s\) – tube and fin mass (solid), kg
\(r\) – radial coordinate, m
\(r_n\) – radial coordinate of fin tip, m
\(r_b\) – radial coordinate of fin base, m
\(r_{ch}\) – radial coordinate of chamfer, m
\(p_f\) – fin pitch, m
\(p_t\) – tube pitch, m
\(Q\) – heat flow removed from fin and tube to the fluid (air), \(\text{J/s}\)
\(T(r)\) – temperature on the fin surface (depends on \(r\), \(^{\circ}\text{C}\))
\(T_{f_n}\) – local fluid (air) temperature at a node, \(^{\circ}\text{C}\)
\(T_{\text{fin}_n}\) – local fin temperature at a surface node, \(^{\circ}\text{C}\)
\(T_{\text{fin}}\) – average temperature at a fin surface, \(^{\circ}\text{C}\)
\(T_{IN}\) – fluid (air) temperature in the inlet section, \(^{\circ}\text{C}\)
\(T_S\) – surrounding temperature, \(^{\circ}\text{C}\)
\(T_T\) – internal tube surface temperature, \(^{\circ}\text{C}\)
\(T_{OUT}\) – average fluid (air) temperature in the outlet section, \(^{\circ}\text{C}\)
\(V_s\) – volume of tube and fin material (solid), \(\text{m}^3\)
IN

– fluid (air) velocity in the inlet section, m/s

Greek symbols

$\alpha$ – heat transfer coefficient, W/(m$^2$ K)
$\beta_1$ – fin profile angle between tube and fin surface, deg
$\beta_2$ – fin profile angle between two fin surfaces after chamfer introduction, deg

$\Delta T = T_{OUT} - T_{IN}$ – difference in fluid (air) temperature between outlet and inlet section, °C

$\delta$ – fin thickness, m
$\delta_f$ – max fin thickness, m
$\delta_t$ – tube thickness, m

$\varepsilon$ – optimization function, °C/m$^3$

$\theta$ – temperature difference between a point on a fin surface (with coordinate r) and the surroundings, °C

$\theta_b$ – temperature difference between a point on a fin base and the surroundings, °C

$\lambda$ – thermal conductivity, W/(m K)

$\xi$ – ratio between the heat removed from the tube/fin component to the tube/fin weight, J/(kg s)

$\rho_s$ – material density for solid (tube and fin), kg/m$^3$

1. Introduction

Heat exchangers have different concerns and requirements depending on commercial applications and are widely used for instance in conditioning systems, radiators and heaters. There are various systems and designs of heat exchangers. They are classified according to the transfer process (direct, indirect), number of fluids, construction features (spiral plates, gasket plate heat exchanger, extended surface, tubular) or heat transfer mechanism (single phase convection, two phase convection). In convective heat exchangers, heat is transferred from fluid to a metallic plate or from a metallic plate to fluid. Fins in heat exchangers are used to increase the heat transfer rate between the primary surface (tube surface) and the surroundings in a large variety of thermal equipment. Annular fins are of great practical importance and are in common use for different heat exchanger constructions. Such equipment is often chosen from the sets of similar exchangers without real design work. Quick selection among the existing solutions can cause that the design does not fulfill the specification or does not consider the proper functionality. One of the important issues that should be defined during the design work is the optimization of the heat efficiency taking in consideration the cost of material. Its optimization is crucial because there is a high pressure from the market and demand for high performance heat exchangers. In the fin, the temperature difference reduces from the fin base to the tip and it allows to make the profile dimensions smaller. It leads to material saving. The criteria for the optimum fin profile optimization are proposed by Schmidt [1]. The optimization of the fin design is based on two approaches [1] [2] [3]:

- for a given fin weight, the fin shape is searched that can dissipate the maximum heat
- for a given amount of heat dissipation, the minimum mass or fin volume is searched

Analytical investigations and search activities that allow to find the optimal profile of the fin are available under assumptions that simplify the problem of heat transfer. These basic assumptions are proposed by Murray (1938) and Gardner (1945), and are called Murray-Gardner assumptions [4]:

- the heat flow in the fin and its temperatures remain constant with time
- the fin material is homogeneous, its thermal conductivity is the same in all directions, and it remains constant
- the convective heat transfer on the faces of the fin is constant and uniform over the entire surface of the fin
- the temperature of the medium surrounding the fin is uniform
- the fin thickness is small, compared with its height and length, so that temperature gradient across the fin thickness and heat transfer from the edges of the fin may be neglected
- the temperature at the base of the fin is uniform
- there is no contact resistance where the base of the fin joins the prime surface
- there are no heat sources within the fin itself
- the heat transferred through the tip of the fin is negligible compared with the heat leaving its lateral surface
- heat transfer to or from the fin is proportional to the temperature excess between the fin and the surrounding medium
In general, the study of extended surface heat transfer compromises the movement of the heat within the fin by conduction and the process of the heat exchange between the fin and the surroundings by convection.

The simple radial fin with a rectangular profile is sketched in Fig 1:

![Radial fin of rectangular profile](image)

For the ideal case, if the convection is considered in a fin heat exchanger and the surrounding temperature is equal to $T_S$, the temperature difference between any point on the fin surface and the surrounding temperature can be written as:

$$\theta = T(r) - T_S$$  \hspace{1cm} (1)

where:

- $T(r)$ is the fin surface temperature that varies from the fin base to the fin tip

The optimized profile of the symmetrical radial fin of least material can be found from the generalized differential equation:

$$f(r) \frac{d^2 \theta}{dr^2} + f(r) \frac{d \theta}{r dr} + \frac{d f(r)}{dr} \frac{d \theta}{dr} - \frac{\alpha \theta}{\lambda} = 0$$  \hspace{1cm} (2)

assuming that the temperature excess changes linearly:

$$\theta = \theta_b \left(1 - \frac{r - r_a}{r_a - r_b}\right)$$  \hspace{1cm} (3)

and resolving above equation with two differential conditions

$$\frac{d \theta}{dr} = -\frac{\theta_b}{r_a - r_b}$$  \hspace{1cm} (4)

$$\frac{d^2 \theta}{dr^2} = 0$$  \hspace{1cm} (5)

the profile function is derived for the radial fin of least material [4]:

$$\frac{\lambda f(r)}{\alpha r_a^2} = \frac{1}{3} \left(\frac{r}{r_a}\right)^2 - \frac{1}{2} \left(\frac{r}{r_a}\right) + \frac{1}{6} \left(\frac{r}{r_a}\right)$$  \hspace{1cm} (6)

Due to the manufacturing problem, the profile described by Equation 6 is not used.

The fin profile and its optimization issue is often the subject of research. Different authors eliminate some of Murray-Gardner assumptions in their investigations that makes the problem more complex.

In paper [2], Ullman and Kalman proceed an optimization for a single fin for a known fin mass. Four different cross-sections are analyzed: constant thickness, constant area of heat flow, triangular and parabolic shapes. They show that the fins with sharp edges and a sharper reduced thickness have lower efficiencies and higher quantities of heat dissipation per a fin mass. The fin that has the best performance is one of parabolic shape.

The efficiency of annular fin when subjected to simultaneous heat and mass transfer mechanism is analyzed by Sharqawy and Zubair [5]. Analytical solutions are obtained for the temperature distribution over the fin surface when the fin is fully wet.
Kundu and Das [6] determine optimum dimensions of plate fins for fin tube heat exchangers considering rectangular and hexagonal profile of fins. Maximum heat dissipation is obtained for a particular value of pitch length or fin thickness for a fixed fin volume.

Rocha et al. [7] analyze the heat transfer in one and two row tubes. They use plate heat exchangers in which fins are in the shape of circular or elliptical section. Two dimensional model allows to study and compare the performance of two configurations with experimental results (published by others). The elliptic fin heat exchangers have better overall performance than circular tubes.

Dul’kin and Garas’ko [8], [9] obtain closed-form solution of 1-D heat conduction problem for a single straight fin and spine of a constant cross-section. The local heat transfer coefficient is assumed to vary as a power function of temperature excess. They also determine the temperature difference between the fin tip and the ambient fluid.

Variable heat transfer coefficient is investigated by Mokheimer [10] for the annular fins. He assumes the natural convection and analyses heat transfer for the heat transfer coefficient that is the function of local temperature. The results show that the application of constant heat transfer coefficient underestimates the fin efficiency.

Two dimensional heat transfer equation is solved analytically to obtain temperature distribution and heat transfer rate by Arslanturk [11] for non symmetric convective boundary conditions. In this work, the fin volume is fixed and the optimum geometry is searched to maximize the heat transfer rate for a given volume. The optimization variables are the fin thickness and ratio of outer radius to inner radius of a fin. The temperature distribution and the heat transfer rate is reached analytically.

The same author, in paper [12], uses the Adomian decomposition method (ADM) to evaluate the efficiency of straight fins with temperature dependent thermal conductivity. He determines the temperature distribution within the fin and also notices that the thermal conductivity parameter has a strong influence over the fin efficiency. The received data are correlated for a wide range of thermo-geometric fin parameters and the thermal conductivity parameter.

Malekzadeh et al. [3] optimize the shape of non-symmetric, convective radiative annular fins based on two nonlinear dimensional heat transfer analysis. The results received by means of the differential quadrature method are compared with those obtained from finite difference method.

The homothropy analysis method (HAM) is used, by Khani et al. [13], to evaluate the analytical approximate solutions and efficiency of the nonlinear fin problem with temperature dependent thermal conductivity and heat transfer coefficient that can change along the surface especially in boiling, condensing or natural convection situations.

Aziz and Fang [14] determine the energy equations for one-dimensional steady conduction in the longitudinal fins of rectangular, trapezoidal, and concave parabolic profile. The temperature and the heat flux is specified at the base of the fin and the temperature distribution in the fins are provided for these conditions. They resolve the problem for a two-point boundary value with one thermal condition, given at the fin base and the other at the fin tip.

2. Problem description and fin heat exchanger models

Fin heat exchangers consist of tubes and fins. If the fin is positioned into an air stream, the flow applies a force from the fin tip surface in the direction of the oncoming flow (drag). The resistance of the body results in a pressure drop. The fin and tube surface orientation also influences on the flow direction and causes the variation of the air streamlines. Described phenomena modify the conditions of the heat exchange between the plate and the fluid having the effect on the heat transfer.

In this paper, the optimized dimension of the fin is defined as the fin profile for which the ratio between the heat removed from the tube/fin component to the tube/fin weight reaches the maximum. All results are calculated considering the fluent flow and its streamline deviations caused by the plate and tube configuration. The main objective of this work is to determine the performance of the heat transfer process in a given heat exchanger for different fin profiles. The fluid flow is one variability that is often neglected in the fin optimization process. The literature includes large numbers of publications dealing with convective heat transfer for different surface geometry, fluid flow type, fluid composition, and thermal boundary conditions but without the fluid flow introduction.

A single row of tubes with fins that forms the heat exchanger is taken into consideration, including the fluid flow, one variability that is often neglected for the fin optimization. A cross flow heat exchanger is shown schematically in Fig. 2.
The heat exchanger characteristic dimensions are written in Table 1:

<table>
<thead>
<tr>
<th>Fin and tube pitches</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_f$ mm</td>
<td>2.6</td>
</tr>
<tr>
<td>$p_t$ mm</td>
<td>80</td>
</tr>
</tbody>
</table>

Three different radial fin profiles are used for the fin shape optimization. Due to the fact that the symmetry occurs in the air flow among the tubes and fins, the heat transfer area is limited to the quarter of one segment. The segment consists of one tube and fin sections. The fin and tube design, applied for profile modifications, is illustrated in Fig. 3

The shapes of fins, used in simulations, are shown in Fig. 4 (a)-(c). All profiles have the same radius $r_a$ and thickness $\delta_f$ at the fin base. The coordinate dimension $r$ is measured from the tube axis. The dimensions of all fins are presented in Table 2.
Each fin operates as a part of the heat exchangers presented in Fig. 2 and the initial thermal as well as flow conditions are the same for all fins:
Based on the described physical boundary conditions, the 3D model is built. The boundary conditions are the same for all models: no slip walls, interface area between fluid and fin/tube surfaces, and symmetry for other surfaces (excluding inlet and outlet). The model sketch, including also an air volume attached to the fin and tube segment, is demonstrated in Fig. 5:

![Model sketch](image)

**Figure 5: Model used for CFD simulation**

where:
(1) - inlet area with constant temperature $T_{IN} = 10^0C$ and unanimous air velocity normal to the section $v_{IN} = 1,5$ m/s
(2) - outlet area
(3) - fin made of steel
(4) - tube made of steel
(5) - inner tube surface with constant temperature $T_T = 90^0C$
(6) - air volume
(7) - model width equal to $p_f$/2

### 3. Results and discussion

The solutions are obtained by means of ANSYS commercial program. The tube material is kept fixed as well as the heat exchanger fin and tube pitches (spacing). No changes are done to the temperature and pressure or flow rate. The shape of the fin and tube is modified to improve heat transfer, reduce the total mass that refer to the cost of the whole heat exchanger.

The heat removed from the fin and tube to the air can be expressed as:

$$Q = m_f \cdot c_f \cdot (T_{OUT} - T_{IN})$$  \hspace{1cm} (7)

and the tube/fin mass can be written as:

$$m_s = \rho_s \cdot V_s$$  \hspace{1cm} (8)

so that the ratio is $\xi$ equal:

$$\xi = \frac{Q}{m_s}$$  \hspace{1cm} (9)

$$\xi = \frac{m_f \cdot c_f \cdot (T_{OUT} - T_{IN})}{\rho_s \cdot V_s}$$  \hspace{1cm} (10)

If the values of $m_f$, $c_f$, $\rho_s$ do not change during the air flow, then the optimization problem can be resolved by finding the maximum value of the optimization function:

$$\varepsilon = \frac{(T_{OUT} - T_{IN})}{V_s} = \frac{\Delta T}{V_s} \rightarrow \text{max}$$  \hspace{1cm} (11)
The temperature difference is found numerically and the solid volume is calculated for different fin profile shapes. The average air temperature value in the outlet section is evaluated according to the formula:

\[ T_{OUT} = \frac{\sum (m_f n T_{fin})}{m_f} \]  

(12)

Taking into account the mesh element size, the average temperature value at fin surface can be written as follows:

\[ T_{fin} = \frac{\sum (a_{fin} T_{fin n})}{A_{fin}} \]  

(13)

The value of the optimization function is found and given in Table 3:

| Table 3: Optimization function and average fin temperature values for different profiles |
|---------------------------------|--------|---------|--------|
| Fin profile | (a) | (b) | (c) |
| ∆T °C | 20,2 | 13,7 | 17,0 |
| V_s, cm³ | 1,19 | 0,89 | 0,83 |
| ε°C/cm³ | 16,9 | 15,4 | 20,4 |
| T_fin °C | 72,2 | 74,0 | 72,0 |

The results illustrate how the fin dimensions influence on the defined optimization function. The best results, among three profiles, are received for profile (c). In that case, value ε is higher than for profile (a) and (b). The average fin temperature has less value and confirms that the flow cools down the fin better, in comparison with the case (a) and (b).

This conclusion may also be reached with flow analysis. Evaluating the streamlines for all models, the influence of fin shape on mass flow distribution is seen. The example is demonstrated in Fig. 6 for fin profile (a) and in Fig 7 for fin profile (c).

![Figure 6. Air streamlines for fin profile (a)](image)

![Figure 7. Air streamlines for fin profile (c)](image)

To confirm the observation, the outlet area is divided into two sections: Outlet 1 and Outlet 2, as explained in Fig. 8.
Then, the mass flow through these specified surfaces are calculated for each of three different fin models. The results for different fin profiles are presented in Table 4, in the reference to the whole mass flow in outlet section (total mass flow in the outlet section is 100%):

<table>
<thead>
<tr>
<th>Fin profile</th>
<th>Outlet 1 %</th>
<th>Outlet 2 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>83,2</td>
<td>16,8</td>
</tr>
<tr>
<td>(b)</td>
<td>85,7</td>
<td>14,3</td>
</tr>
<tr>
<td>(c)</td>
<td>76,4</td>
<td>23,6</td>
</tr>
</tbody>
</table>

The results of mass flow, collected in Table 4, confirm that the heat transfer between the fluid and the heat exchanger is sensitive to different fin profiles. The fin profiles do not cause only the variation of the fin surface temperature in the direction \( r \), but also affect the flow and cause the variation of the air streamlines. The air that comes through Outlet 2 flows over the fin, where the fin temperature is higher (closest the tube, higher the temperature on fin surface), and have a possibility to absorb more thermal energy from the heat exchanger. The profile (b) obstacles the flow to reach more profitable area.

4. Conclusions

The heat exchange optimization parameter is defined as the amount of dissipated heat to the heat exchanger weight for a one raw heat exchanger. The three different models are built to optimize the heat transfer process and calculate the temperature distribution in the air outlet of the fin heat exchanger. The relationship between the difference in the air temperature \( \Delta T \) and the fin profiles is presented in Table 3. Fin geometry affects the heat transfer phenomenon between the plate itself and the air. It is seen that the flow streams vary and change the flow direction depending on fin profile modification that impact on the fin surface temperature. All obtained results show that the optimum fin design should be calculated and chosen assuming also the flow parameters.

References


