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Structural optimization of high tension towers

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Abstract
Optimal design has not been yet generalized in industrial applications since it is very difficult to define CAD tools that allow to define and to state general optimization procedures that can be applied to a wide variety of structural problems.
Thus, it is necessary to propose algorithms and models to obtain the optimal design of specific problems. The definition and the analysis of these models is advisable when the benefits obtained by applying optimization techniques may be profitable.
According to this idea, optimization techniques are oriented to solve great singular projects that involves a very large cost (for example, a dam, a dyke of a harbour, a long span bridge,) or specific problems that can be used repeatedly in a large number of applications or projects (automotive and aeronautical industry, small structures in civil engineering,). The structural optimization of electric transmission towers for large and high tension lines corresponds to the second group of optimization problems since a transmission line usually requires hundreds or even thousands of transmission towers.
In this paper, the authors propose a general formulation to obtain the optimal design of latticed high tension towers. The formulation is devoted to obtain the most common objective in engineering (minimum cost) by considering the limitations imposed in actual laws and norms for this kind of structures. The system conforms to the corresponding Spanish Standard.
The shape optimization problem is solved by means of an improved sequential linear programming algorithm with quadratic line search.
The system performs the sensitivity analysis by means of analytical techniques.
Finally, some application examples that analyse real high tension towers are solved in order to check the benefits obtained by using the optimization techniques proposed.
Keywords: High tension towers, Structural optimization, minimum cost, FEM.

1. Parametrization

Fuerzas Eléctricas de Cataluña S.A. (FECSA) was one of the major electric power companies operating in the northeastern region of Spain during the second half of the 20th century. Nowadays FECSA is part of the Endesa Group, that is one of the most important supplier of electricity for Spain and one of the world's largest utility holding companies.
The GL-110 KV family is one of the basic types of steel latticed towers that has most frequently been built by FECSA to support its high tension electric power lines.

1.1. Structural Elements

The towers of the GL-110 KV family are basically made up by linking together several elements of certain standardized types. These are:

- the top unit (see Fig. 1), with 9 nodes and 16 bars,
- the cross unit (see Fig. 2), with 8 nodes and 12 bars,
- the double cantilevered unit (see Fig. 3), with 10 nodes and 32 bars,
- the X-block or super X-block unit (see Fig. 4), with 12 nodes and 25 bars,
- the K-block unit (see Fig. 5), with 12 nodes and 25 bars,

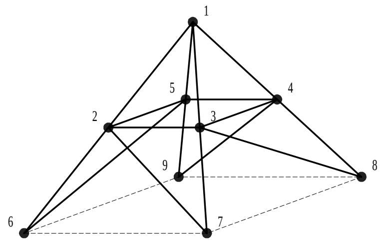


Figure 1: Top unit for high tension tower type FECSA/GL-110KV
(9 nodes and 16 bars of 2 different sections).

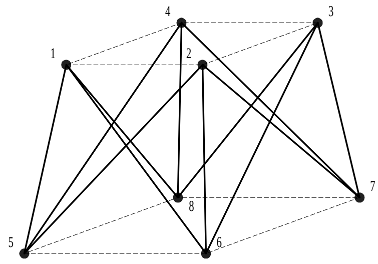


Figure 2: Cross unit for high tension tower type FECSA/GL-110KV
(8 nodes and 12 bars of 3 different sections).

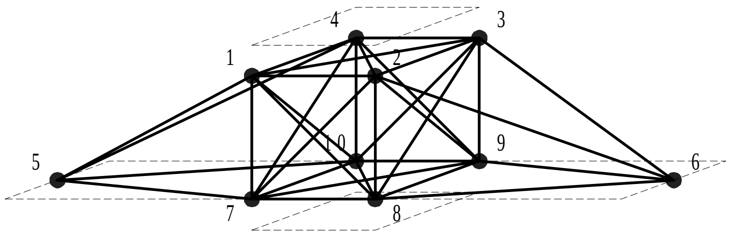


Figure 3: Double cantilevered unit for high tension tower type FECSA/GL-110KV
(10 nodes and 32 bars of 5 different sections).

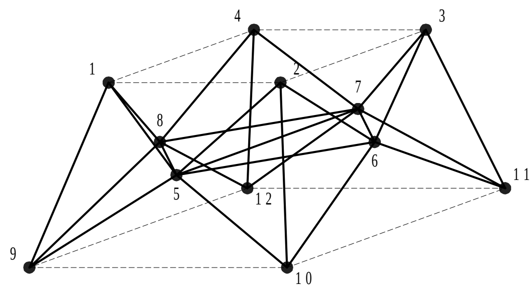


Figure 4: X-block unit for high tension tower type FECSA/GL-110KV
(12 nodes and 25 bars of 4 different sections).

Some of these units have an internal secondary system of bars (not detailed in this paper) which structural contribution is insignificant, since its purpose is just to prevent the local buckling [1]. Therefore, these additional bars are not considered in the structural analysis, but they are taken into account

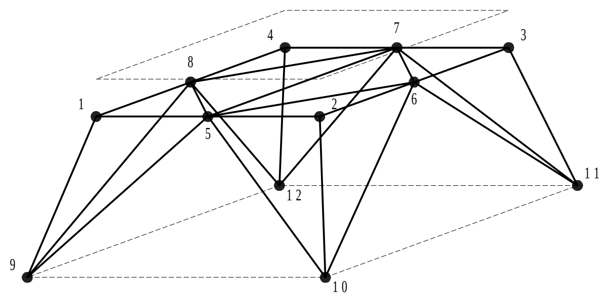


Figure 5: K-block unit for high tension tower type FECSA/GL-110KV
(12 nodes and 25 bars of 4 different sections).

in the definition of the wind load as much as in the statement of the buckling constraints. In fact, the above mentioned super X-block unit is just a reinforced version of the X-block unit (designed for cases in which the height of the block is over 4 m) which only difference with the standard version is the thicker secondary system of bars.
All of the elements of a tower are assembled with standardized rolled steel sections type angle (also called L–shaped sections), selected from the Spanish Standard tables [2]. The joints are bolted.
Because of symmetry, some of the bars of each unit should have the same section. Furthermore, the number of different sections that might be used to assemble each type of element is limited for practical reasons. Thus, the bars of each type of element are classified into groups (A, B, etc.) in accordance with the significance of their structural function, all of the members of each group having the same section. Then, each bar of each type of element is said to be an A-type section, or a B-type section, etc., depending on the group it belongs to.
The base of all of the elements is always square (lower polygon represented in dashed lines in Figs. 5–1) in the examples solved. However, more sophisticated formulations with rectangular bases can be easily addressed by defining additional design variables. Since the units have been designed to be sequentially linked together, the top of all of the elements (except for the top unit) is square too (upper polygon represented in dashed lines in Figs. 5–2). Therefore, the basic geometry of each element is defined by giving the height of the element, the length of the side of its top and the length of the side of its base. The double cantilevered elements need another geometric parameter to be given, that is the length of the lateral cantilevers that support the suspended conductors.
More recently, additional block units have been included in the model. Some of these units corresponds to reinforced versions of the previous ones. The rest of them allow to apply the optimization techniques proposed to a wide range of structures and problems by defining useful and frequent configurations in practice. Some of these blocks are

- the double line top unit (see Fig. 6), with 26 nodes and 72 bars,
- the lambda unit (see Fig. 7), with 8 nodes and 8 bars,
- the super-lambda unit (see Fig. 8), with 8 nodes and 12 bars,
- the super-cross unit (see Fig. 9), with 8 nodes and 16 bars,
- the left/right cantilevered unit (see Fig. 11), with 13 nodes and 32 bars,
- the reinforced double cantilevered unit (see Fig. 10), with 34 nodes and 96 bars,

1.1. Definition of a Tower
Now, the topology of any tower of the GL-110 KV family can be defined by just giving the suitable sequence of elements to be linked together. The optimum design system automatically connects the square top of each element to the square base of the next one in the assembling sequence.
By using the block units proposed in figures 6, 7, 8, 9, 11 and 10, a number of different cable configurations can be analyzed (figure 13).

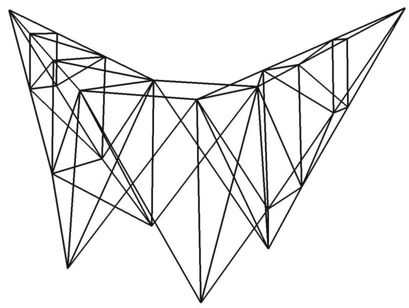


Figure 6: double line top unit for high tension tower (26 nodes and 72 bars of 6 different sections).

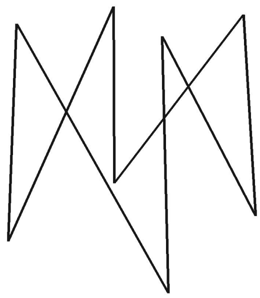


Figure 7: Lambda unit for high tension tower (8 nodes and 8 bars of 2 different sections).

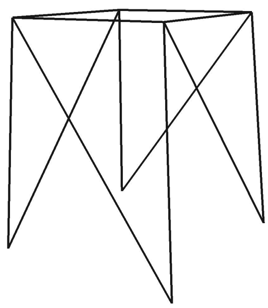


Figure 8: Super-Lambda unit for high tension tower (8 nodes and 12 bars of 3 different sections).

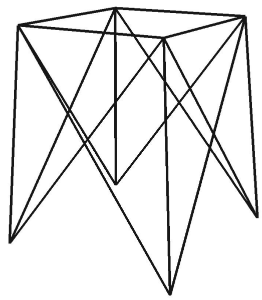


Figure 9: Super-cross unit for high tension tower (8 nodes and 16 bars of X different sections).

2. Structural Analysis

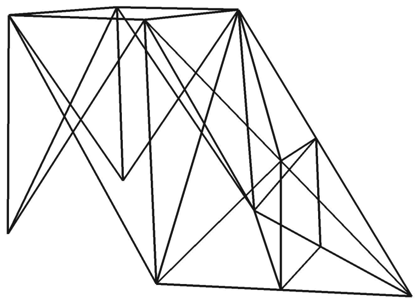


Figure 10: Left/right cantilevered unit for high tension tower
(13 nodes and 32 bars of 6 different sections).

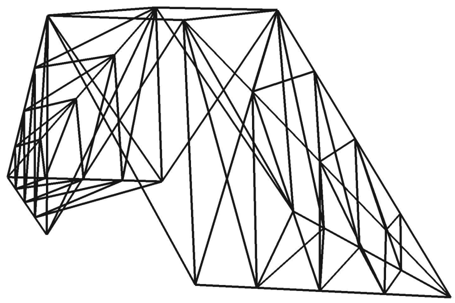


Figure 11: Reinforced double cantilevered unit for high tension tower
(34 nodes and 96 bars of 6 different sections).

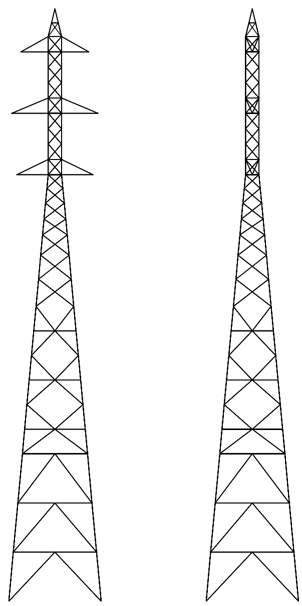


Figure 12: High tension tower
FECSA/GL-110KV
(Elev. and side view).

Fig. 12 shows the elevation and the side view of an example. This tower is made up by linking together (from bottom to top) three K-block units, three X-block units, seven cross units, one double cantilevered unit, three cross units, one double cantilevered unit, three cross units, one double cantilevered unit and the top unit.
The geometry of the tower is defined by giving the heights and the lengths of the sides of the bases of all of the elements, as well as the lengths of the arms of the double cantilevered elements. Obviously, the length of the side of the top of each element must be equal to the length of the side of the base of the next one in the sequence, what reduces the overall number of free parameters to be given.
On the other hand, the mechanical properties of the bars are defined by listing the codes of the standardized rolled steel sections (type angle) that will be used for each group (A, B, etc.) of bars of each element, and the corresponding Young modulus E, elastic limit σ^e and specific weight ρ of the steel being used.

2.1. Load Cases
A tower of the GL-110 KV family is considered to be a so-called "support of transmission line within

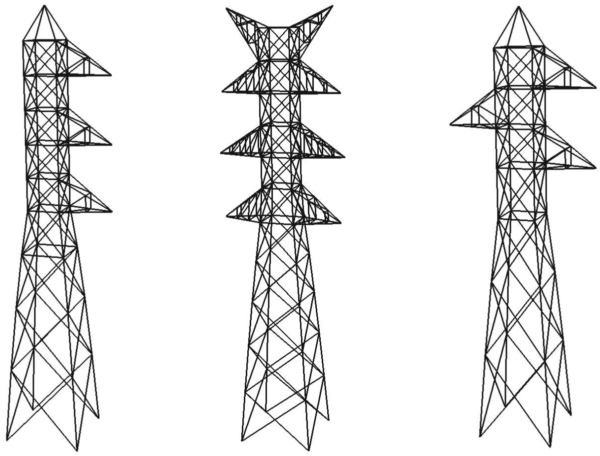


Figure 13: Different cable configurations in hight tension towers: all cables in one side, two cables on the top unit and left/right alternate cables distributions.

B-type zones" (*apoyo de alineación en zona B*) according to the Spanish Standard [3]. For these cases, the Spanish Standard requires to take into account the following four typified load cases (see Fig. 14):

S1) **wind load** (on the structure and on the conductors).

S2) **ice load** (on the conductors).

F1) **tension imbalance** with ice load.

F2) **failure of one cable** with ice load.

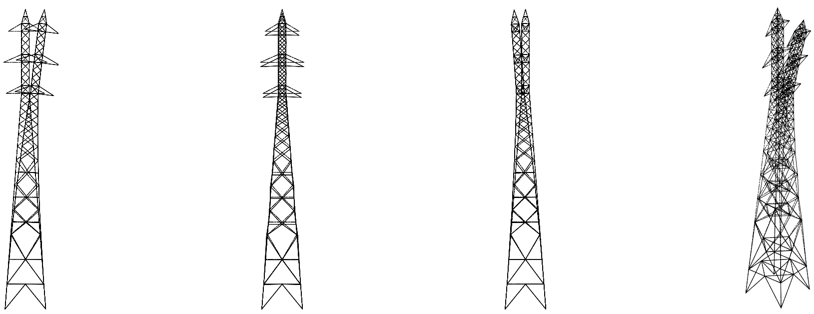


Figure 14: Load cases considered (from left to right): S1) wind load; S2) ice load; F1) tension imbalance; F2) failure of one cable.

All of the above listed load cases include (as normal loads) the self weight of the tower, the self weight of the conductors and the expected thermal effects on the conductors. Load cases S1 and S2 are related to service conditions, while load cases F1 and F2 are related to failure conditions. Specific details on how to deal with all of these load cases can be found in [3, 1].
At this point, it should be mentioned that the number of load cases to be considered in practice is higher. For example, the typified load case F2 leads in our case to four specific load cases, since the broken conductor could be the grounding conductor (that is suspended from the top of the tower), or any of the conductors that are suspended from the end of the arms of each one of the three double cantilevered elements.

2.1. The Structural Analysis Model

The structures herein considered behave mostly as three-dimensional pin-jointed trusses, while the maximum stress is not allowed to exceed the elastic limit and the displacements are assumed to be small [4, 1].

For these reasons, we can use a standard linear elasticity matrix formulation for 3D pin-jointed frames to solve the structural analysis problem [1]. Therefore, we have to state and solve a set of linear systems type

$$\bar{K}\bar{u}_\ell = \bar{f}_\ell, \tag{1}$$

for each one of the load cases to be considered (ℓ). In the above statement $\bar{K}$ is the so-called global stiffness matrix, that depends on the topology of the truss, the position of the nodes (joints), and the sections and the mechanical properties of the bars. On the other hand, $\bar{f}_\ell$ is the so-called global force vector, that depends on the equivalent external nodal forces that correspond to each load case. Finally, $\bar{u}_\ell$ is the so-called global displacement vector. Linear system (1) also includes some corrections that are required for dealing with the prescribed displacements at the supports [1].
Then, for each bar i we can compute the internal axial force $N_{i\ell}$ corresponding to each load case ℓ as

$$N_{i\ell} = \bar{S}_i\bar{u}_\ell, \tag{2}$$

being $\bar{S}_i$ the so-called stress matrix of the bar.
We assume that the reader is familiar with the matrix theory of linear structural analysis and, thus, we do not need to go deeper into the details of the formulation, that can be found in [1].

3. The Optimum Design Problem

3.1. Objective Function and Constraints

In this terms, the optimum design problem is stated as

$$\begin{array}{lll} \textit{minimize} & W = \sum_i \rho_i A_i L_i, & \\ \textit{verifying} & N_{i\ell}\varphi_\ell - \sigma_i^e A_i \leq 0 & \forall i,\ \forall \ell, \\ & -\ N_{i\ell}\varphi_\ell - \frac{\sigma_i^e A_i}{\kappa_i} \leq 0 & \forall i,\ \forall \ell, \end{array} \tag{3}$$

where (for each bar i) ρ_i and σ_i^e are the specific weight and the elastic limit of the steel, A_i is the area of the cross-section, L_i is the length of the bar, and κ_i is the buckling coefficient of the bar [1, 2], and (for each load case ℓ) φ_ℓ is the security coefficient that corresponds to the load case. The buckling coefficient κ_i of each bar is automatically computed by the optimum design system, taking into account the system of seconday bars of the corresponding element [1].
Therefore, it is clear that the objective function (to be minimized) is the overall weight of the structure, while the constraints limit the maximum stress and preclude the local bukcling to occur. These constraints are specifically imposed by the Spanish Standard [3]. The Standard also imposes other constraints that apply to the distances between the conductors, and to the distances between the conductors and the tower. These constraints are automatically fulfilled for the designs considered in this paper. Otherwise, they could be easily implemented.

3.2. Design Variables

Some of the parameters that define the geometry of the tower can now be declared as design variables, while the others are fixed (and thus declared as design constants). Normally, additional side constraints should be imposed on the design variables to avoid meaningless or simply unwanted results.
More frequently, one should prefer to define how these parameters vary with a relatively small group of design variables that control the shape of the tower. In the examples presented in this paper, the heights of the elements that compose the tower are kept constant, while the shape of the tower is controlled by two design variables: the length of the side of the base of the lower K-block (x_1) and the length of the side of the base of the top (x_2). The lengths of the sides of the bases of the upper elements are kept equal to x_2. The lengths of the sides of the bases of the lower elements (down from the lower double cantilevered element) vary linearly between x_1 and x_2, depending on the height above the base of the first K-block (see Fig. 12). It seems clear that additional side constraints should also be imposed on these design variables, for the same above stated reasons. Since the range of variation of these design variables is continuous, we are dealing in this case with a problem of continuous shape optimization.
But one could as well wish the optimum design system to select the optimal sections for each group of bars of each structural element. The selection should be made among certain available standardized rolled steel sections (angles in our case). Since the range of variation of these design variables is discrete,

we should be dealing with a much more cumbersome problem of discrete sizing optimization.

4. The Optimum Design System

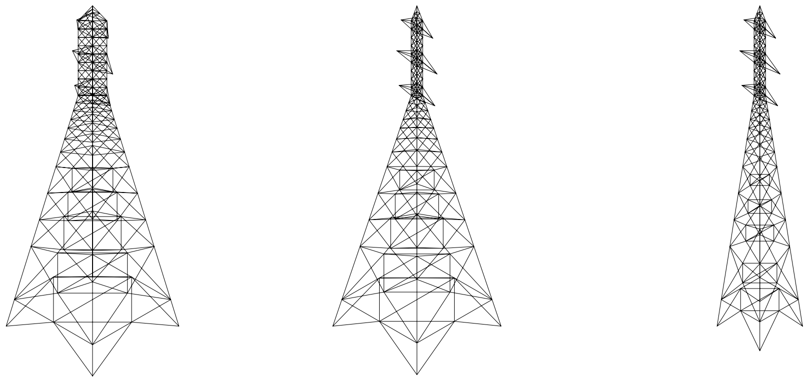
A specific optimum design system has been developed for the GL-110 KV family of high tension towers. The system is organized according to the general methodology that was previously proposed by the authors [5]. The sensitivity analysis techniques and the improved SLP algorithm with quadratic line-search that have been implemented in the system were previously proposed by the authors too [5, 6].
The improved SLP algorithm can be described as a classical descent method. Basically, an approximated linear problem with aditional side constraints is stated and solved by Linear Programming in each iteration. Therefore, a full first order sensitivity analysis is required in each iteration. The solution to this linear problem defines a search direction. Then, the objective function and the non linear constraints are quadratically approximated in the search direction, and a subsequent line-search is performed. Therefore, a directional second order sensitivity analysis is required in each iteration for the search direction in the design space. We notice that this kind of algorithms can not deal with discrete design variables. Thus, this algorithm can not be used to select the optimal sections for the groups of bars.
One could think that selecting the optimal sections is just a subsidiary stage in the main shape optimization process. Hence, the section that corresponds to each group of bars could be selected from the given steel tables according to the computed axial forces, right after each iteration. If necessary, the axial forces could be recalculated, and the assignment of sections could be revised, until convergence. In the limit (if the range of available sections becomes continuous) this approach stalls the optimization process right after each iteration, since the bars are dimensioned at maximum stress and any further shape modification should either increase the weight or violate the constraints. In practice, the range of available sections is not continuous, what provides some looseness. However, the approach still trends to stall the optimization process before reaching the optimum.
In the proposed approach, the optimum design system is programmed to solve the shape optimization problem without modifying the previous assignment of sections. Then, new sections are selected from the given steel tables according to the computed axial forces. If necessary, the axial forces could be recalculated, and the assignment of sections could be revised, until convergence. If necessary, the whole process is repeated until convergence. This approach can not guarantee that the optimum will be reached, since the continuous and the discrete design variables are not allowed to be modified simultaneously, but it has proved to be quite effective in improving pre-existing designs, as we will see in the following examples.

5. Application Examples

The tower herein used is taken from a real case with the following sequence of elements: one K-block unit, one super X-block unit, three X-block units, seven cross units, one double cantilevered unit, three cross units, one double cantilevered unit, three cross units, one double cantilevered unit and the top unit, being $x_1 = 6.55\ m$ and $x_2 = 1.25\ m$. The weight of the real tower is $W = 4349.93\ Kp$.
Examples S1 and S2 are intended to show the reliability, robustness and efficiency of the improved SLP algorithm to solve shape optimization problems. Hence, the sections of all of the bars are fixed (L100.8). While the initial design of example S1 (see Fig. 15) is largely over-dimensioned ($x_1 = 15.00\ m$, $x_2 = 2.50\ m$), the initial design of example S2 (see Fig. 16) is highly unfeasible ($x_1 = 1.00\ m$, $x_2 = 1.00\ m$). Figures 15 and 16 show the design evolution for these significantly different initial designs. The same solution ($x_1 = 7.43\ m$, $x_2 = 1.00\ m$) is reached in a small number of iterations in both cases.
The initial design of example T1 is the final solution found in examples S1 and S2. Now the sections are not fixed. The final design ($x_1 = 6.71\ m$, $x_2 = 1.00\ m$) is obtained after a few cycles being the final weight $W = 3892.79\ Kp$ (10.5% less than the real tower). In example T2 we use the same initial geometry but slightly different initial sections (L100.7 instead of L100.8). The final design ($x_1 = 8.25\ m$, $x_2 = 1.00\ m$) is obtained after a few cycles being the final weight $W = 3753.91\ Kp$ (13.7% less than the real tower). We notice that the final solution is not the same. Finally, in example T3 the two lower X-blocks are substituted by one super X-block. The initial design is given by $x_1 = 9.30\ m$ and $x_2 = 1.00\ m$ and the initial sections are L100.7. The final design ($x_1 = 8.43\ m$, $x_2 = 1.00\ m$) is obtained after a few cycles, being the final weight $W = 3611.41\ Kp$ (17.0% less than the real tower). The final designs are compared in Figure 17.

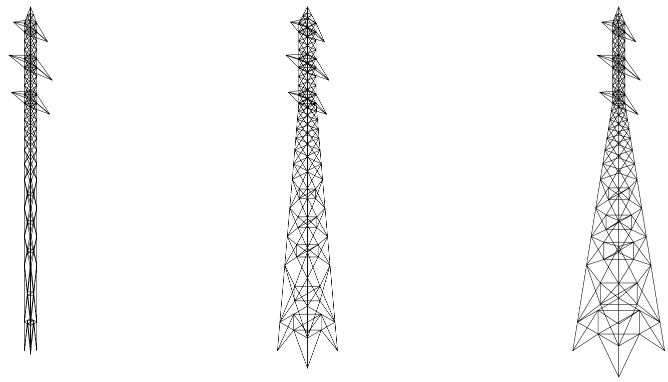
6. Conclusions

In this paper we have presented an optimum design system for a particular family of high tension towers.



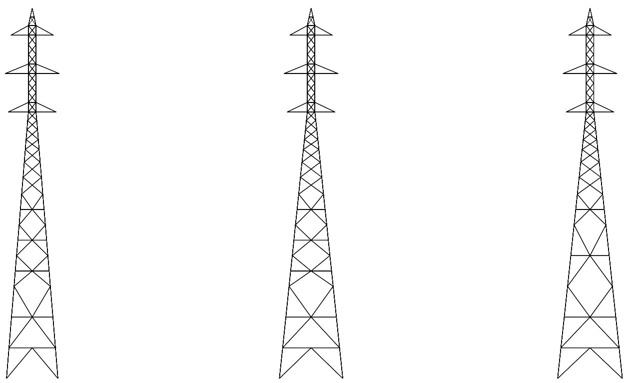
Initial Design Iteration 10 Iteration 19 (OPT.)

Figure 15: Example S1. Evolution of the structural design



Initial Design Iteration 5 Iteration 11 (OPT.)

Figure 16: Example S2. Evolution of the structural design



Example T1 Example T2 Example T3

Figure 17: Examples T1, T2 and T3. Comparison between the final designs.

The system conforms to the corresponding Spanish Standard. The topology of any tower of the family can be defined by just specifying a suitable sequence of structural elements of certain standardized types. The shape optimization problem is solved by means of an improved sequential linear programming algorithm with quadratic line search. The sections that should be used for each group of bars of each structural element are selected from the given steel tables, according to the stress state of the corresponding bars. Several application examples have been presented. These examples show the reliability, robustness and efficiency of the shape optimization process. The examples also show that the proposed techniques produce weight reductions over a 10% in pre-existing real designs. The system can be easily modified to optimize other families of steel latticed structures and/or to conform to any other standard of structural design.

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