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# LONGITUDINAL SHEAR OF A BI-MATERIAL WITH FRICTIONAL SLIDING CONTACT IN THE INTERFACIAL CRACK

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**Key words:** Friction, Tribofatigue, Cyclic loading, Interfacial crack, Energy dissipation.

**Summary:** *This paper proposes a novel technique for obtaining the analytical solution of anti-plane problem (longitudinal shear) for the bi-material with closed interfacial crack accounting for friction between surfaces. Using jump function method the problem is reduced to the solution of singular integral equations with a Cauchy-type kernel for the jumps of displacements and stresses in areas with sliding friction. This solution allows us to obtain explicit expressions for displacements, stress intensity factors and energy dissipation. To take account of cyclical loading multistep incremental method of solution is proposed. The basis of this technique is the idea of consideration at every step of loading previous step stresses and displacements as the residual. Correctness of the obtained solution is justified. Evolution of the slip zone in matrix is considered at different stages of loading. Critical load values for determining the onset of slip are investigated. The effect of friction and loading parameters on the size of slip zone and stress intensity factors is numerically analyzed.*

## 1 INTRODUCTION

Contact problems have received much attention in the literature as a result of their practical importance. The study of contact phenomena considering friction is one of the most pressing problems in engineering [1-7] and others. To a greater or lesser extent, but contact phenomena are always accompanied by friction at both macroscopic and microscopic levels. Mechanical, electrical, thermal, chemical processes and vibration that can simultaneously occur due to friction significantly affect the degradation of materials, duration of their wave processes, reliability and durability of structural elements etc. Effect of friction can be both harmful and helpful, when causing dissipation of the accumulated strain energy in the body and thus reduce stress.

However, the problem of contact interaction between adjacent surfaces of a crack has no sufficient attention. Major achievements in this area include the study of the theory of cracks at the interface of two media assuming the elimination of the physically incorrect oscillating features singularity by widely used model of local contact directly near the vicinity of the crack end [1, 8, 9]. A wide class of problems on the effect of friction forces on the contact

stresses between the half-planes was examined in [10, 11].

The importance of this research increases when solids with the possibility of the sliding friction are under multistep or cyclic loading. Note that the frictional slippage is essentially incremental process and therefore the solution of the contact problem depends on the load history [12-14].

This paper proposes a novel technique for obtaining the analytical solution of anti-plane problem (longitudinal shear) for the bi-material with closed interfacial crack accounting for friction between surfaces under the multistep or cyclic loading. Therefore, all the characteristics of the stress-strain state, such as displacements, stresses, energy dissipation, slip zone size etc., are exactly calculated. We assume that the magnitude and direction of the external force factors that generate longitudinal shear change quasi-statically (so slowly that it is not necessary to consider the inertial member) by a certain law, which may be different.

## 2 FORMULATION OF THE PROBLEM

Consider the infinite isotropic matrix consisting of two half-spaces with the elastic constants $E_k, \nu_k$ $(k=1,2)$. Half-spaces are mutually pressed to the interface by external normal stresses $\sigma_{yy}(x)<0$. Here *Oxyz* are the Cartesian coordinates and *xOz* is the contact plane of half-spaces.

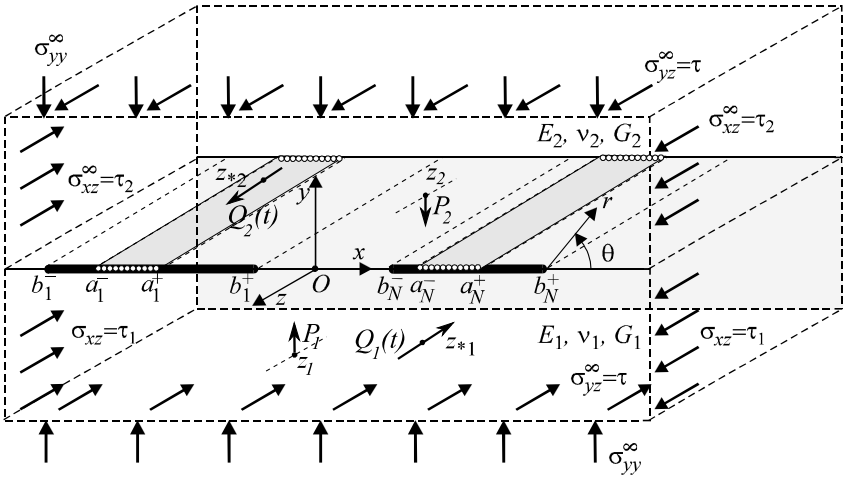


Figure 1: The geometry and loading scheme of problem.

We'll study the stress-strain state (SSS) of the body section by the plane $xOy$ perpendicular to the direction $z$ of its longitudinal shear. This section form two half-planes $S_k$ $(k=1,2)$, and the interface between them corresponds to the *x*-axis $L$. Under the action of the applied loads cracks may slip at intervals forming line $L'=\bigcup_{n=1}^{N} L_n' = \bigcup_{n=1}^{N}[b_n^-;b_n^+]$ as indicated in Fig. 1. The normal stress in the body is generated by uniform compression at the infinity $\sigma_{yy}^{\infty}<0$ and two balanced concentrated forces $P_k=\mp iP$ at the points $z_k \in S_k$. The same traditional notation for axis $z$ and a complex variable $z=x+iy$ should not cause misunderstanding in the solution of the problem.

Suppose that the shear external loading increase or decrease monotonically and consist of

step-by-step sequences of uniformly distributed at the infinity shear stress $\sigma_{yz}^{\infty} = \sum_p \tau_{(p)}(t)$, $\sigma_{xz}^{\infty} = \sum_p \tau_{k(p)}(t)$, concentrated forces with magnitude $Q_k(t) = \sum_p Q_{k(p)}(t)$, screw dislocations with Burger's vector $b_k(t) = \sum_p b_{k(p)}(t)$ at the points $z_{*k} \in S_k$ $(k = 1, 2)$, ( $p$ ) in parentheses denotes loading step number, $t \in \left[t_{(p-1)};\, t_{(p)}\right]$ denotes time duration of the loading step. According to Eq (20.5) [15] stresses at the infinity must always satisfy the conditions $\tau_{2(p)}(t)G_1 = \tau_{1(p)}(t)G_2$ that provides straightness of matrix interface at the infinity.

Contact between the half-spaces is assumed mechanically perfect omitting $L'$ where it is mechanically perfect until the points of time $t_{(p)}^{st}$ when relative sliding of the crack surfaces may start on ( $p$ ) step of loading in some areas $\gamma_{n(p)} \subset L_n'$ [12-14].

Thus, the multistep problem of longitudinal shear with possible slip in the interface cracks under the action of the inhomogeneous distribution of compressive normal stresses and frictional forces on the surfaces of contact (line section $L$ ) is formulated. These forces may cause in slip zones heat emission, energy dissipation, wear, etc [16-19].

Thus, we formulate the problem of longitudinal shear with possible slip in the interfacial cracks under the action of the inhomogeneous distribution of compressive normal stresses and frictional forces on the surfaces of contact (line section $L$ ). These forces may cause in these apriority unknown slip zones heat emission, energy dissipation, wear, etc.

## 3 THE PROBLEM SOLUTION

The presence of such slip zones in the cracks can be simulated by the jumps of tractions and displacements at $L_n'$ [13 – 15, 20]:

$$\left[\sigma_{yz}\right]_{L_n'} \equiv \sigma_{yz}^{-} - \sigma_{yz}^{+} = f_{3(p)}^{n}(x,t), \tag{1}$$

$$\left[\frac{\partial w}{\partial x}\right]_{L_n'} \equiv \frac{\partial w^{-}}{\partial x} - \frac{\partial w^{+}}{\partial x} = \left[\frac{\sigma_{xz}}{G}\right]_{L_n'} \equiv \frac{\sigma_{xz}^{-}}{G_1} - \frac{\sigma_{xz}^{+}}{G_2} = f_{6(p)}^{n}(x,t),\ x \in \gamma_{n(p)} \subset L_n' \tag{2}$$

$$f_{3(p)}^{n}(x,t) = f_{6(p)}^{n}(x,t) = 0, \text{ if } x \notin \gamma_{n(p)} \tag{3}$$

where $f_{3(p)}^{n}, f_{6(p)}^{n}$ are the jumps of tractions and displacements; $t$ – time, as a formal monotonically increasing parameter associated with the variability of the force. Hereinafter the following notation is used: $[\varphi]_L = \varphi(x,-0) - \varphi(x,+0)$, $\langle\varphi\rangle_L = \varphi(x,-0) + \varphi(x,+0)$; symbol "+" and "-" correspond to a threshold function on the top and bottom edges of the line $L$.

The boundary conditions at $L_n'$ on ( $p$ ) step of loading provide that slipping starts at some zones $\gamma_n = [a_n^{-}; a_n^{+}] \subset L_n'$ when reaching the tangent stress $\sigma_{yz}$ of certain critical value

$\tau_{yz}^{\max}$, moreover, this threshold shear stress $\sigma_{yz}$ can not exceed $\tau_{yz}^{\max}$. Confining with the classic Amonton's law of friction [3-5] consider the contact problem which states that everywhere in $\gamma_n$ shear stresses (friction force) are equal

$$\sigma_{yz(p)}^{\pm} = -\operatorname{sgn}([w]_{(p)})\tau_{yz}^{\max}(x) = -4\alpha \operatorname{sgn}([w]_{(p)})\left\{-\frac{\sigma_{yy}^{\infty}}{4} + \sum_{k=1}^{2} E_j \eta_k \operatorname{Re}\frac{N_k}{x - z_k}\right\}, \tag{4}$$

$$N_k = \left(\frac{P_k}{e_{jk}} - \frac{\kappa_k \overline{P}_k - P_k}{e_{kj}}\right),\ \kappa_k = 4 - 3\nu_k,\ \eta_k = 1/8\pi(1-\nu_k),\ e_{kj} = 2\frac{G_k + \kappa_k G_j}{(1-\nu_1)(1-\nu_2)},$$

$$\theta_k = \frac{1}{e_{jk}} + \frac{\kappa_k + 1}{e_{kj}},\ \gamma^{+} = E_2\eta_1\theta_1 + E_1\eta_2\theta_2,$$

where $\alpha$ denotes the coefficient of sliding friction. Outside the domains $\gamma_n$ there is no slippage and the magnitude of shear stresses does not exceed the maximum allowable level. Sign (direction of action) of shear stresses is chosen depending on the sign of the difference in mutual displacement $[w]$ at the source point of $L_n'$.

Amonton's law of friction in the classical form (3) provides, of course, simplifying the boundary conditions for basic problem, but using of more complex models of friction [1, 3, 12, 20], including taking into account the wear, does not essentially complicate the process of solving.

Using jump function method [15] the solution of this problem was reduced to a system of $2N$ singular integral equations (SIE) with a Cauchy-type kernel for the jumps of displacements and stresses in areas with sliding friction in the case of monotonically increasing loading (first step of loading)

$$\begin{cases} f_{3(1)}^{n}(x,t) = 0, \\ g_{6(1)}^{n}(x,t) = \dfrac{1}{2C}\left(\left\langle \sigma_{yz(1)}^{0}(x,t)\right\rangle + 2\operatorname{sgn}[w]_{(1)}\tau_{yz}^{\max}(x)\right), \end{cases} \tag{5}$$

whose solution is known.

Hereinafter the following notations [13-15, 20] are used:

$$\sigma_{yz(1)}(z,t) + i\sigma_{xz(1)}(z,t) = \sigma_{yz(1)}^{0}(z,t) + i\sigma_{xz(1)}^{0}(z,t) + ip_k g_{3(1)}^{n}(z,t) - Cg_{6(1)}^{n}(z,t), \tag{6}$$

$$\sigma_{yz(1)}^{0}(z,t) + i\sigma_{xz(1)}^{0}(z,t) = \tau_{(1)}(t) +$$
$$+ i\left\{\tau_{k(1)}(t) + D_{k(1)}(z,t) + \left(p_k - p_j\right)\overline{D}_{k(1)}(z,t) + 2p_k D_{j(1)}(z,t)\right\},$$

$$g_{r(p)}^{n}(z,t) \equiv \frac{1}{\pi}\int_{L_n'} \frac{f_{r(p)}^{n}(x,t)\,dx}{x - z},\ p_k = \frac{G_k}{G_1 + G_2},\ C = G_{3-k}p_k,$$

$$D_{k(p)}(z,t) = -\frac{Q_{k(p)}(t) + iG_k b_{k(p)}(t)}{2\pi(z - z_{*k})} \qquad (z \in S_k,\, k = 1,2;\ j = 3-k;\ r = 3,6).$$

Superscript "0" denote the corresponding values in the solid body model without heterogeneity (cracks) under the same external loading (homogeneous solution).

Such solution technique allows us to obtain explicit expressions for displacements, stress intensity factors and energy dissipation [15]. But analyzing of the elasticity problem involving friction under the variable (cyclic) loading requires consideration of the history of loading and is more complicated.

We propose the next method to solve such a problem with consideration of the history of loading. The idea of the method is as follows: obtained on the first (initial) step at the point of time $t_{(1)}$ completion value of the SSS in matrix will be considered as a residual in the second step of loading (additional load or unload). Therefore, assuming that the sign of applied loading at each step changes to the opposite, and its relative value monotonically increases from 0 to the maximum value we can apply the following method for solving this problem.

We consider that the formulation of the problem at the second step of loading is different from the previous (initial, first) step only by the presence of the jumps of displacements and stresses caused by the previous step of loading. Thus, the stress field has the form

$$\sigma_{yz}(z,t) + i\sigma_{xz}(z,t) = \sigma_{yz(1)}\left(z,t_{(1)}\right) + i\sigma_{xz(1)}\left(z,t_{(1)}\right) + \sigma^0_{yz(2)}(z,t) +$$

$$+ i\sigma^0_{xz(2)}(z,t) + ip_k g_{3(2)}(z,t) - Cg_{6(2)}(z,t)\ \left(z \in S_k;\ k = 1,2;\ j = 3-k\right) \tag{7}$$

where the component $\sigma_{yz(1)}\left(z,t_{(1)}\right)$ is residual.

The displacements and stresses must satisfy the boundary conditions (4) based on the direction of the load. Then using (7) we can formulate the local problem for a second step:

$$\sigma_{yz(2)}(z,t) + i\sigma_{xz(2)}(z,t) = \left\{\sigma_{yz}(z,t) + i\sigma_{xz}(z,t)\right\} -$$

$$-\left\{\sigma_{yz(1)}\left(z,t_{(1)}\right) + i\sigma_{xz(1)}\left(z,t_{(1)}\right)\right\}\ \left(z \in S_k;\ k = 1,2;\ j = 3-k\right) \tag{8}$$

with boundary conditions

$$\sigma^{\pm}_{yz(2)}(x,t) = \begin{cases} -\operatorname{sgn}([w]_{(2)})\tau^{\max}_{yz}(x) + \operatorname{sgn}([w]_{(1)})\tau^{\max}_{yz}(x), & x \in \gamma_{n(2)} \subset \gamma_{n(1)}; \\ -\operatorname{sgn}([w]_{(2)})\tau^{\max}_{yz}(x) - \sigma^{\pm}_{yz(1)}\left(x,t_{(1)}\right), & x \in \gamma_{n(2)} \setminus \gamma_{n(1)}. \end{cases} \tag{9}$$

As it is shown in (9) the abovementioned assumption does not require proof when the loading at the next step changes the sign (direction). When the first step local load reaches extreme values (in their growth - maximum) at the time of completion $t_{(1)}$, the slip zone become fixed, the slipping stopped and the contact surfaces stick together. The resulting stress-strain state is considered further as the residual. Then, with the beginning of the next step the absolute values of the total load begins to decrease and, quite similar to the process of unloading plastic material, new slip does not occur until the condition (4) arise somewhere at the $L'_n$ at some point of time $t^{st}_{(2)}\left(t_{(1)} < t^{st}_{(2)} \le t_{(2)}\right)$. This slip zone size in the start of

second step is always smaller than the size at the finish of the previous step. Therefore, using the similar to the initial step reasoning, we obtain SIE

$$\begin{cases} f^{n}_{3(2)}(x,t)=0, \\ g^{n}_{6(2)}(x,t)=\dfrac{1}{2C}\left(\left\langle\sigma^{0}_{yz}(x,t)\right\rangle+2\left(\mathrm{sgn}[w]_{(2)}-\mathrm{sgn}[w]_{(1)}\right)\tau^{\max}_{yz}(x)\right), \end{cases} \tag{10}$$

to determine the local (as to achieved at point of time $t_{(1)}$ SSS) jumps of displacements and tractions under local (this step) load

$$\tau_{(2)}(t)=\tau(t)-\tau_{(1)}(t),\ \ \tau_{k(2)}(t)=\tau_{k}(t)-\tau_{k(1)}(t),$$

$$b_{k(2)}(t)=b_{k}(t)-b_{k(1)}(t_{(1)}),\ Q_{k(2)}(t)=Q_{k}(t)-Q_{k(1)}(t_{(1)})\ \ (k=1,2;\ t>t_{(1)}) \tag{11}$$

which differs from initial one only by the presence of term $-2\tau^{\max}_{yz}(x)\mathrm{sgn}[w]_{(1)}$ on the right side. Arguing similarly for all subsequent steps under the alternating monotonically varying loading one can get a local problem for ( $p$ )-th step

$$\sigma_{yz(p)}(z,t)+i\sigma_{xz(p)}(z,t)=\left\{\sigma_{yz}(z,t)+i\sigma_{xz}(z,t)\right\}-$$

$$-\sum_{i=1}^{p-1}\left\{\sigma_{yz(i)}\left(z,t_{(i)}\right)+i\sigma_{xz(i)}\left(z,t_{(i)}\right)\right\}\ \left(z\in S_{k};\ k=1,2;\ j=3-k;t>t_{(p-1)}\right) \tag{12}$$

with boundary conditions

$$\sigma^{\pm}_{yz(p)}(x,t)=-\mathrm{sgn}([w]_{(p)})\tau^{\max}_{yz}(x)-\sigma^{\pm}_{yz(p-1)}\left(x,t_{(m)}\right)=$$

$$=-\tau^{\max}_{yz}(x)\left(\mathrm{sgn}[w]_{(p)}-\mathrm{sgn}[w]_{(p-1)}\right),\ \ x\in\gamma_{n(p)}\subset L'_{n}\ \ (n=\overline{1,N}), \tag{13}$$

and its corresponding SIE, which solution has the known form [15].

As a result, the totals of energy dissipation, stress, strain, displacement and their jumps etc. after ( $p$ )-th step can be represented as a superposition of such.

For more detailed illustration of the mentioned approach for solving this problem consider the special case when the half-spaces compression is performed by symmetrically centered balanced forces $P_k=\mp iP$ at the points $z_k=\pm ih\in S_k$, ( $k=2,1$) and alternating symmetrical about the vertical axis ( $z_{*k}=\pm id$ ) shear loading the matrix with one crack $L'_1=[-b;b]$ by concentrated forces. Then within domain $L'_1$ at each step only symmetric slip zone $\gamma_{1(p)}=\left[-a_{(p)};a_{(p)}\right]\left(a_{(p)}\le b\right)$ can occur.

In this case the solution of the SIE and the rest of the parameters SSS after calculating the corresponding integrals have the exact form on the $\left(x\in[-a_{(p)};a_{(p)}]\right)$

$$f_{6(p)}(x,t)=\frac{1}{\pi C\sqrt{a^{2}_{(p)}-x^{2}}}\left\{\pi\left(\tau_{(p)}(t)-(\mathrm{sgn}[w]_{(p)}-\mathrm{sgn}[w]_{(p-1)})\alpha\sigma^{\infty}_{yy}\right)x+\right.$$

$$+\sum_{k=1}^{2} p_{3-k}\,\mathrm{Im}\left(\left(Q_{k(p)}(t)+iG_k b_{k(p)}(t)\right)\left(\frac{\sqrt{z_{*k}{}^2-a_{(p)}^2}}{x-z_{*k}}+1\right)\right)\Bigg\}+$$

$$+\frac{4\alpha(\mathrm{sgn}[w]_{(p)}-\mathrm{sgn}[w]_{(p-1)})}{C\sqrt{a_{(p)}^2-x^2}}\sum_{k=1}^{2} E_j\eta_k\,\mathrm{Re}\left(N_k\left(\frac{\sqrt{z_k{}^2-a_{(p)}^2}}{x-z_k}+1\right)\right). \qquad (14)$$

As a result, using (14) one can obtain the exact expressions for the energy dissipation and jump of displacements by integrating

$$W_{(p)}^{d}(t)=-\int_{L'}\left|\tau_{yz}^{\max}(x)[w]_{(p)}(x,t)\right|dx\,,\ [w]_{(p)}(x,t)=\int_{-a_{(p)}}^{x} f_{6(p)}(s,t)\,ds,\quad x\in\gamma_{n(p)} \qquad (15)$$

To illustrate this technique, we assume that at the point $z_{*2}=id$ of the upper half-space has acted only one alternating variable concentrated force of intensity $Q_2(t)=\sum_p Q_{2(p)}(t)$, growing at odd and descending at even ( $p$ ) (other aforementioned cases of loading may be investigated similarly). This allows for alternating variable load to fulfill the requirement $\gamma_{n(2)}(t)\subset\gamma_{n(1)}(t_{(1)})\left(t_{(2)}^{st}=t_{(1)}\right)$.

To calculate the slip zone size it is necessary to obtain first an expression for the stress intensity factor (SIF)

$$K_{3(p)}^{\pm}(t)=\frac{1}{\sqrt{\pi a_{(p)}}}\int_{-a_{(p)}}^{a_{(p)}}\sqrt{\frac{a_{(p)}\pm x}{a_{(p)}\mp x}}\sigma_{yz}(x,t)dx=$$

$$=\frac{1}{\sqrt{\pi a_{(p)}}}\int_{-a_{(p)}}^{a_{(p)}}\sqrt{\frac{a_{(p)}\pm x}{a_{(p)}\mp x}}\left\{\sum_{m=1}^{p-1}\sigma_{yz(m)}\left(x,t_{(m)}\right)+\sigma_{yz(p)}^{0}(x,t)+\mathrm{sgn}[w]_{(p)}\tau_{yz}^{\max}(x)\right\}dx \qquad (16)$$

or, taking into account the abovementioned assumptions and given the fact that with increasing load $\mathrm{sgn}[w]=-1$ using (16) we obtain

$$K_{3(1)}(t)=\sqrt{\frac{a_{(1)}}{\pi}}\frac{p_1 Q_{2(1)}(t)}{\sqrt{a_{(1)}^2+d^2}}-\sqrt{\pi a_{(1)}}\frac{4\alpha P\gamma^{+}}{\sqrt{a_{(1)}^2+h^2}}. \qquad (17)$$

Thus, at the initial step the condition when slippage appears for the first time and corresponding critical value of loading force $Q_{2(1)}^{*}$ take the form

$$a_{(1)}(t)=\sqrt{\frac{h^2d^2(Q_{2(1)}(t)^2-Q_{2(1)}^{*}{}^2)}{h^2Q_{2(1)}^{*}{}^2-d^2Q_{2(1)}(t)^2}}\,,\quad Q_{2(1)}^{*}=\frac{4\gamma^{+}P}{h}\frac{\pi\alpha d}{p_1} \qquad (18)$$

Consider the second step is unloading. Whereas $\mathrm{sgn}[w]_{(2)}=1$, the expression (16) results

$$K_{3(2)}(t)=\sqrt{\frac{a_{(2)}}{\pi}}\frac{p_1 Q_{2(2)}(t)}{\sqrt{a_{(2)}^2+d^2}}-\sqrt{\pi a_{(2)}}\frac{8\alpha P\gamma^+}{\sqrt{a_{(2)}^2+h^2}}. \tag{19}$$

Thus, condition for starting the slippage again and the size of the new zone slip equals

$$\left|Q_{2(2)}(t)\right|>Q_{2(2)}^*=\frac{8\pi\alpha\gamma^+ Pd}{hp_1}=2Q_{2(1)}^*,\quad a_{(2)}(t)=\sqrt{\frac{h^2d^2(Q_{2(2)}(t)^2-Q_{2(2)}^{*\,2})}{h^2Q_{2(2)}^{*\,2}-d^2Q_{2(2)}(t)^2}} \tag{20}$$

Note that all reasoning about the SSS and phases of slip development at the first step are valid for the next steps of loading. Continuing the analysis of the next steps alternating loads we discover that the starting slippage conditions, critical load values coincide with the values for the second step and are twice over the initial step values.

Now we prove the suitability of the proposed multistep approach for the case where applied load does not change sign at the next step. Since the point of time $t_{(1)}$ loading at the first step reaches a maximum and then continues to increase at the second step, the slip zone after the first step continues to grow without delay to the maximum in the second step - $\gamma_{n(2)}(t)\supset\gamma_{n(1)}(t_{(1)})$ $\left(t_{(2)}^{st}=t_{(1)}\right)$. It was analyzed that the total solution after the second step obtained using formulas (10) − (14) coincides with the solution of this problem for single step, but under the cumulative load of two steps. Thus, the proposed to multistep sequence of SSS approach remains suitable for arbitrary incorporation multistep loading-unloading. However, to simplify the procedure for solving advisable successive steps of loading in one direction combine into one step. Then the problem can be easily solved using the above method for alternating load.

## 4 NUMERICAL ANALYSIS

Using the abovementioned incremental approach we determine the dependence of the slip zone size, shape and incremental evolution of displacement jump in $L_1'$, energy dissipation and SIF from basic parameters of SSS (distance and magnitude of the applied force, friction coefficient, ratio of the elastic properties) in the case of cyclic loading $Q_{2(1)}^{***}\to -2Q_{2(1)}^{***}\to 2Q_{2(1)}^{***}\to -2Q_{2(1)}^{***}\to\ldots$.

To apply formulas (15) − (20) we introduce into consideration dimensionless values: the size of the slip zone $a_{(p)}/b$, the coordinate and distance from the crack points of application of the shear and pressing forces respectively − $x/b$, $d/b$, $h/b$; $Q_{2(p)}(t)/Q_{2(1)}^{***}$ and $Q_{2(p)}(t)/Q_{2(1)}^{*}$ − the absolute and relative intensity of shear magnitude; $[w]_{(p)}(x)C/2\alpha\gamma^+P$, $W_1^d(t)C/8\pi\gamma^{+2}P^2$ and $K_{3(p)}\sqrt{b}/4\sqrt{\pi}\alpha\gamma^+P$ − the displacement jump, energy dissipation and SIF respectively.

The size $a_{(p)}/b$ dependence on the absolute magnitude of the applied force $Q_{2(p)}(t)/Q_{2(1)}^{***}$ on $p$-th step is shown in Fig. 2 using different values $d/b$, $h/b$. It is noticeable that the growth rate of size increases when $d/b$ approaching to $h/b$. Growing distance from the crack position of the coordinate $h/b$ also leads to an increase in the rate of growth $a_{(p)}/b$.

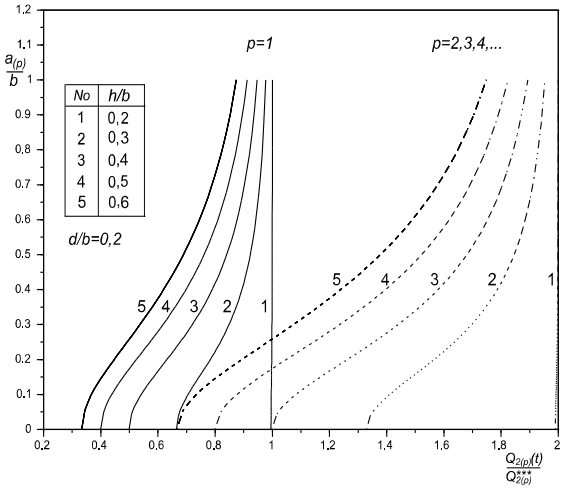


Fig. 2. Size of slip zone dependence on coordinates of applying concentrated forces during the cycle.

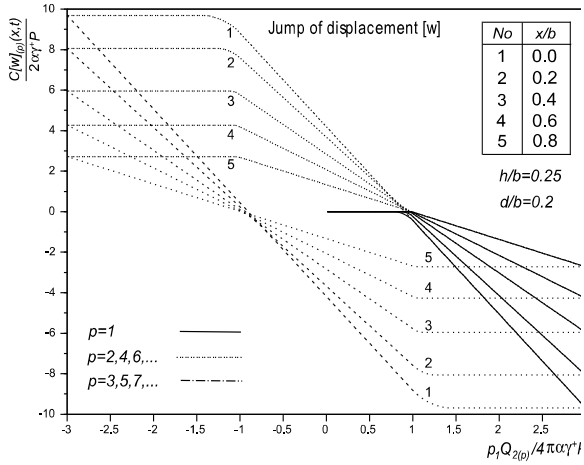


Fig. 3. Hysteretic behavior of displacement jump in symmetric cyclic loading.

Fig. 3 illustrates the hysteretic nature of total displacement jump at different points of the slip zone depending on the load in the symmetrical alternating cycle. It is evident that such evolution of shape of the displacement jump is inherent to all points of the slip zone, not only to the center point.

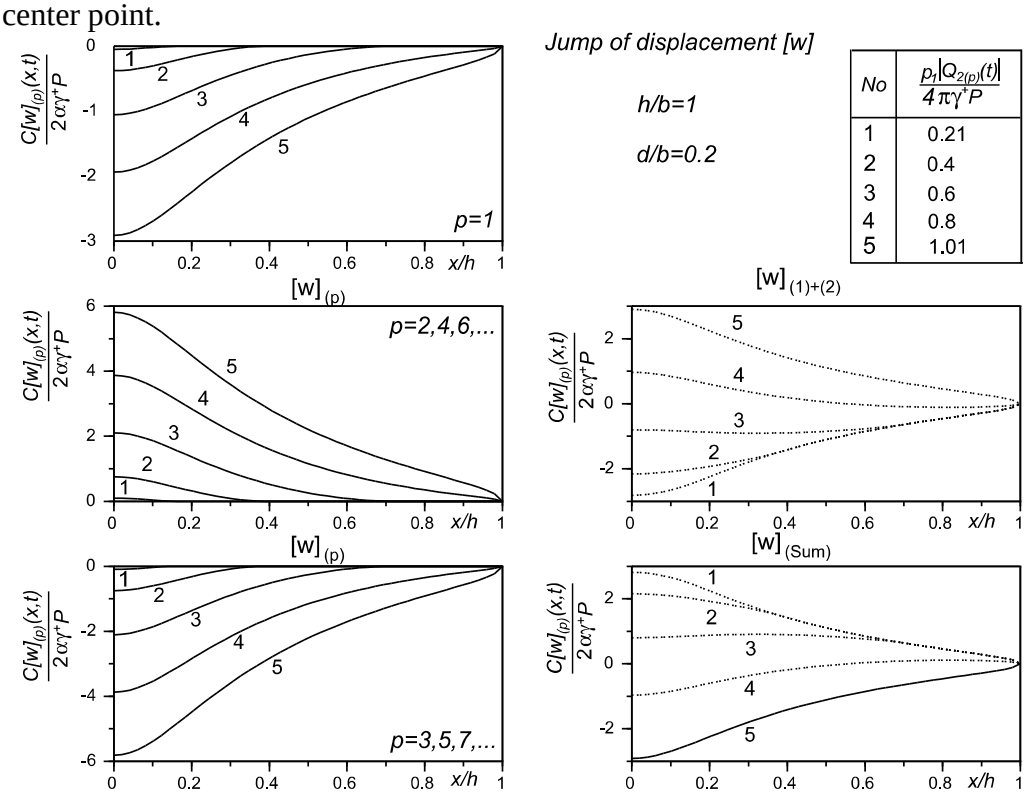


Fig. 4. Evolution of shape of displacement jump at symmetric cyclic loading

The evolution of shape both locally and total displacement jump $[w]_{(p)}(x)C/2\alpha\gamma^{+}P$ depending on $x/h$ and different values of loading parameters is shown in Fig. 4. It is noticeable that crack surfaces do not return to the original position after a full cycle of loading, retaining some residual displacement jump. The greatest sensitivity to changes in shape of displacement jump is observed when the distance from point of application of the

shear forces to the crack vanishes. Approaching the points of application of normal forces less effect on the displacement jump, but significantly changes its amplitude.

## 5 CONCLUSIONS

Using incremental approach we build the effective analytical solution for the antiplane problem of bi-material under cyclic loading with closed interfacial crack, where the sliding friction is possible. Amonton's law of friction in the classical form provides, of course, simplifying the boundary conditions for basic problem, but using of more complex models of friction including taking into account the wear does not essentially complicate the process of solving.

This solution allows obtaining of explicit expressions for displacements, stress intensity factors and energy dissipation. To take account of cyclical loading multistep method of solution is proposed. The basis of this technique is the idea of consideration at every step of loading previous step stresses and displacements as the residual. Correctness of the obtained solution is justified. Contact zone size dependence from the loading parameters at different stages is analyzed. The critical load values for determining the onset of slip at every step of loading are investigated.

We numerically analyze the effect of friction and loading parameters on the size of slip zone and stress intensity factors at every step of loading. It is discovered that the slip zone appears and grows fastest when it pressing normal stresses are minimal. Growth rate of slip zone also promotes increasing the distance of application point of concentrated power factors from her. Energy dissipation becomes more intense when the point of force application is closer to the sliding zone.

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