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OPTIMUM DESIGN OF LAMINATED COMPOSITES FOR MAXIMUM FATIGUE LIFE

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Summary: *Composite structures especially used in the applications such as airplanes, wind turbine rotor blades, leisure boats, bridges are subjected to significant cyclic fatigue loads throughout their service life, which may lead to catastrophic failure. Therefore, fatigue is an important parameter that must be considered in calculations during design processes. Fatigue strength (fatigue life), thus structural performance of laminated composites, can significantly be increased through design optimization. However, there are no adequate studies in the literature on the optimum design of laminated composites under fatigue loading. Hence, in this study, optimum fibre orientation designs of laminated composites under various in-plane loading conditions are searched to obtain maximum fatigue strength. For this purpose, a fatigue assessment model termed as "Failure Tensor Polynomial in Fatigue (FTPF)" is used to predict the fatigue life of the laminates. A hybrid algorithm composed of genetic algorithm and generalized pattern search algorithm is used as the search algorithm in optimization. The first ply failure approach is implemented to the study using Hashin-Rotem failure criterion index in order to ensure the reliability of the designs. The validity and effectiveness of the model are investigated using experimental data available in the literature, and an experimental correlation is presented. A number of problems including different design cases are solved, and the best fibre angle orientations selected from a set of discrete angles by the algorithm and the corresponding failure indexes are proposed to discuss. A comparison study is also performed with selected design cases to demonstrate potential advantages of using non-conventional fibre angles in design. In addition, the performance of the hybrid algorithm is shown for some design cases by comparing to single performances of genetic and generalized pattern search algorithms.*

1 INTRODUCTION

In the last few decades, fibre-reinforced composites have increasingly been used instead of conventional metallic materials in industries such as aviation, aerospace, marine and automotive due to their superior strength, stiffness and/or lightness properties. Besides, composite materials are constitutionally less fatigue-sensitive compared to metallic ones. As a further advantage, composite structures offer great design flexibility since they enable material tailoring in different ways.

The durability of a structure is provided by preserving its mechanical performance as long as possible throughout the service life. Just as in metals, in composite materials under cyclic loads, failure (crack) formation and propagation finally lead to fatigue failure which may cause the loss of structural integrity. The use of composite materials in a wide range of applications, especially in the structures that must bear significant cyclic fatigue loads during operation, such as airplanes, wind turbine rotor blades, leisure boats, bridges etc. obliged researchers to consider fatigue when investigating a composite material and engineers to realize that fatigue is an important parameter that must be considered in calculations during design processes [1].

Fatigue strength, thus structural performance of laminated composites, can significantly be increased through the optimum selection of parameters such as fibre and matrix materials, fibre angle orientations, and layer thicknesses. Although there are many studies related to various design optimizations of laminated composites subjected to static loadings (i.e., weight minimization, buckling strength maximization, etc.) in the literature, there are very few published studies on the optimum design of laminated composites under fatigue loading [2-4]. For example; Adali [2] optimized a symmetric angle-ply laminate under in-plane cyclic loads and determined the maximum failure load. Walker [3] presented a procedure to minimize the thickness of laminated composite plates under cyclic loads by using a specific limit for fatigue life as a constraint. In the previous studies on fatigue design optimization, the researchers used fatigue models that were valid only for limited laminate configurations and specific loading conditions. Mainly, more general layups and loading conditions need to be considered in design optimization for typical applications. In this regard, the study of Ertas and Sonmez [4] showed that the optimum designs of laminated composite plates under in-plane cyclic loading for maximum fatigue life can theoretically be obtained for more general layups.

A large number of fatigue theories and methodologies for the fatigue life prediction of composite materials and structures have been developed, based on empirical, phenomenological modelling or on the quantification of specific damage metrics, such as the residual strength and/or stiffness of the examined material or structural element. Moreover, the investigation of fatigue behaviour of composite plates under multiaxial loadings is more important for the applications subjected to real complex loading conditions [5]. Some considerable multiaxial composite plate fatigue theories and their modelling can be found in detail in the literature [6-10].

2 FATIGUE LIFE PREDICTION MODEL

Design optimization of composite laminates for maximum fatigue strength requires a reliable fatigue assessment model that accounts for the factors affecting fatigue life. The fatigue strength of a given fibre-reinforced composite laminate mainly depends on the type of constituent layers and the fibre orientation angle (θ) with respect to the loading directions, and magnitude of the cyclic stress as cyclic stress ratio. Besides these mentioned requirements, the option of the method to use laminate properties instead of lamina properties to predict laminate behaviour enhances the applicability of the criterion for unidirectional (UD) and multidirectional lay-ups made up of any type of composite, e.g., UD, woven or stitched layers. Furthermore, the method needs few empirical relations as reference for the predictions. In these respects, FTPF [9, 11] seems to be a promising model to use in a design optimization study when considering its applicability to various in-plane loading states and arbitrary fibre orientations requiring little experimental effort.

A modification of the quadratic version of the failure tensor polynomial for the prediction of fatigue strength under complex stress states was introduced by Philippidis and Vassilopoulos [9] and termed as Failure Tensor Polynomial in Fatigue (FTPF). The theory is based on the Tsai-Hahn tensor polynomial [12] and adapted for fatigue by substituting the failure tensor components with the corresponding S-N curves. Therefore, if a fibre-reinforced composite plate subjected to in-plane loading (Figure 1) is considered, the FTPF criterion can be expressed in the material coordinates 1 and 2, under plane stress by:

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + 2F_{12}\sigma_1\sigma_2 + F_1\sigma_1 + F_2\sigma_2 + F_{66}\sigma_6^2 - 1 \le 0 \tag{1}$$

with the components of the failure tensors given by:

$$F_{11} = \frac{1}{XX'},\ F_{22} = \frac{1}{YY'},\ F_{66} = \frac{1}{S^2},\ F_1 = \frac{1}{X} - \frac{1}{X'},\ F_2 = \frac{1}{Y} - \frac{1}{Y'} \tag{2}$$

where X, Y and S represent the fatigue strengths of the material along the longitudinal, the transverse directions and under shear loading, respectively. The prime (′) is used for compressive fatigue strengths. The term F_{12} used in the criterion (Eq. (1)) can also be expressed as

$$F_{12} = -\frac{1}{2}\sqrt{F_{11}F_{22}} \tag{3}$$

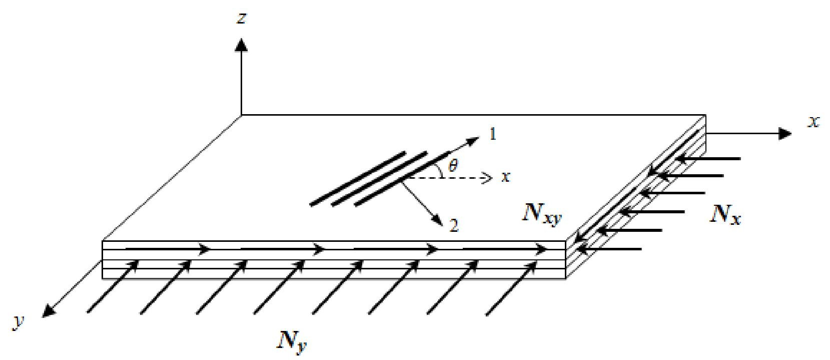

Figure 1: Representative plate geometry showing in-plane loading and principal coordinates.

2.1 Fatigue criterion

Failure tensor polynomial in fatigue has the same general form as in Equation (1):

$$F_{ij}\sigma_i\sigma_j + F_i\sigma_i - 1 \le 0, \quad i,j = 1,2,6 \tag{4}$$

where the components of failure tensors F_{ij}, F_i are functions of the number of cycles N, stress ratio R and the frequency ν, of the loading since the failure stresses have been substituted with the S–N curves. Thus;

$$F_{ij} = F_{ij}(N,R,\nu), \quad F_i = F_i(N,R,\nu) \tag{5}$$

Equations (2) and (3) are still valid for the calculation of tensor components but the failure stresses X, X', Y, Y', S are replaced by the S-N curves of the material along the same directions and under the same conditions. More clearly, static strength X is replaced by the S-N curve under tension-tension fatigue loading along direction 1 of material symmetry coordinate system (fibre direction), while failure stress X' is replaced by the corresponding compressive-compressive S-N curve. In the same manner, Y, Y' and S are replaced by the transverse direction S-N curves under tension-tension, compression-compression and shear fatigue loadings, respectively.

Thus, the failure stresses X, X', Y, Y', S can be now given as functions of number of cycles, stress ratio and frequency. If the S-N curves of the material are assumed in the general form:

$$S = A + B \log N \tag{6}$$

then, the expressions of the fatigue failure stresses can be written as:

$$\begin{aligned} X(N,R,\nu) &= A_X + B_X \log N \\ X'(N,R,\nu) &= A_{X'} + B_{X'} \log N \\ Y(N,R,\nu) &= A_Y + B_Y \log N \\ Y'(N,R,\nu) &= A_{Y'} + B_{Y'} \log N \\ S(N,R,\nu) &= A_S + B_S \log N \end{aligned} \tag{7}$$

It is reported that assuming $X' = X$ and $Y' = Y$, only $X(N,R,\nu)$, $Y(N,R,\nu)$ and $S(N,R,\nu)$ from the above fatigue failure stresses are sufficient to yield satisfactory predictions using the failure tensor polynomial in fatigue for the material and test conditions considered.

When only these three S-N curves are used, the failure tensor components are given by:

$$F_{11} = \frac{1}{X^2(N,R,\nu)},\ F_{22} = \frac{1}{Y^2(N,R,\nu)},\ F_{66} = \frac{1}{S^2(N,R,\nu)},\ F_1 = F_2 = 0 \tag{8}$$

and by substituting these, Equation (4) takes the form of the criterion as:

$$\frac{\sigma_1^2}{X^2(N)} + \frac{\sigma_2^2}{Y^2(N)} - \frac{\sigma_1\sigma_2}{X(N)Y(N)} + \frac{\sigma_6^2}{S^2(N)} - 1 \le 0 \tag{9}$$

where the fatigue failure stresses are shown as functions of the number of cycles *N*, only. Nevertheless, the criterion can be used in the form of Equation (9) for any stress ratio R, and frequency ν, provided the basic S-N curves are also known for the same R and ν values.

2.2 Experimental correlation of the model

Philippidis and Vassilopoulos [9] demonstrated the applicability of the FTPF criterion on a wide range of materials under various loading conditions by comparing the predictions of their model with the experimental data available in the literature. They showed that the agreement between the model's predictions and experimental data was satisfactory.

In this study, the model will be applied to multidirectional laminates consisting of different fibre alignments. Hence, proper modelling of multiaxial fatigue behaviour of the laminates is important for design optimization. In this regard, first, the fatigue behaviour of unidirectional E-glass/epoxy laminae is predicted using the FTPF criterion model for various fibre off-axis angles and compared with the experimental data. Input data for theoretical derivations are taken directly from [6]:

$$X = 1414.98 - 138.60 \log N \tag{10}$$

$$Y = 36.11 - 3.26 \log N$$

$$S = 35.95 - 3.65 \log N$$

The prediction curves of the various off-axis angles with the experimental data are presented as stress amplitude (σ_a) versus logarithmic fatigue life (logN) in Figure 2.

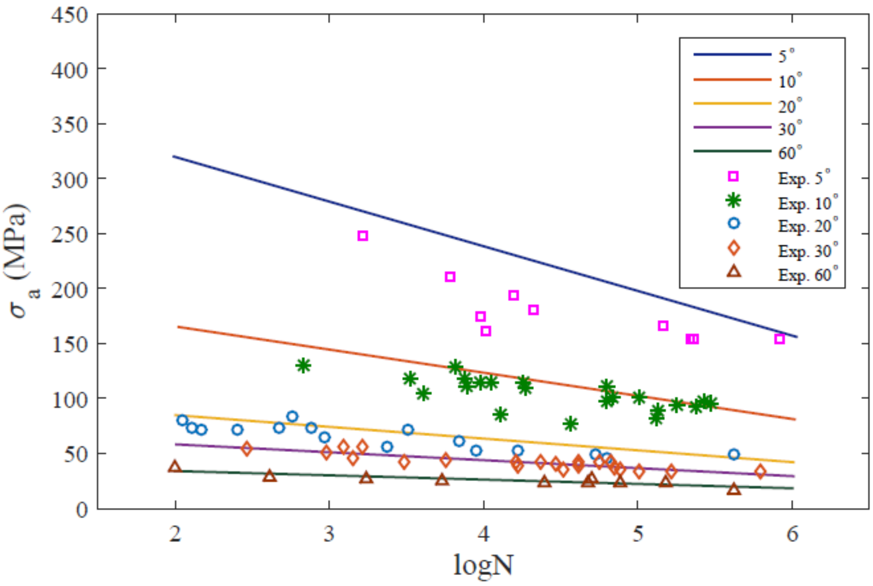

Figure 2: Predicted S-N curves and experimental data for various off-axis flat coupons.

It is seen from the figure that the predictions of the FTPF criterion model are found to be in good agreement with the experimental data. Particularly, the prediction curves for the off-axis 30° and 60° are in a very good agreement.

Secondly, the fatigue behaviour of a multiaxial composite laminate, quasi-isotropic $[0/\pm 45/90]_{2s}$ graphite/epoxy, is predicted using the model. Experimental data are taken from the study of Rotem and Nelson [13]. The prediction plot is shown in Figure 3.

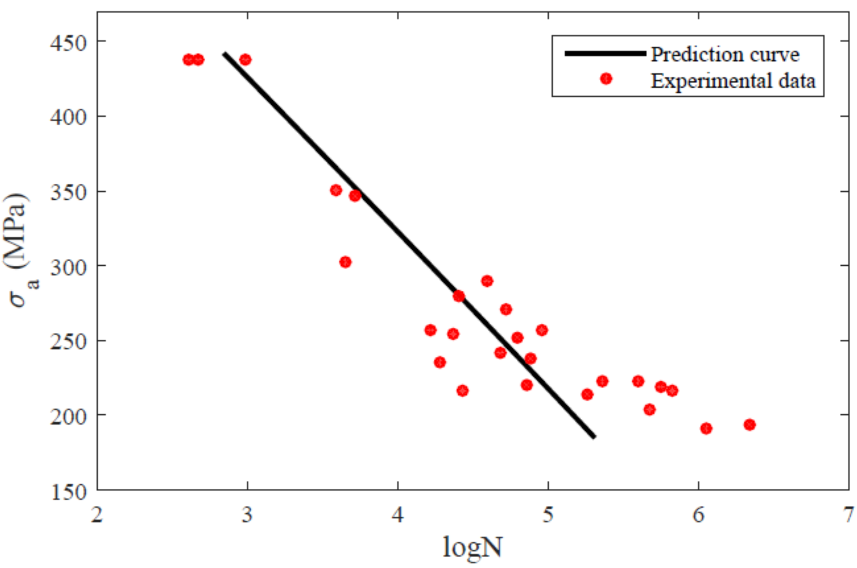

Figure 3: Predicted S-N curve of quasi-isotropic $[0/\pm 45/90]_{2s}$ graphite/epoxy laminate [13].

As seen from Figure 3, the prediction of the criterion is found to be in a good agreement with the experimental data. However, the model underestimates the fatigue life of the laminate. The modelling approach used to obtain the fatigue life is based on the criterion that the fatigue life of each lamina is calculated using classical lamination theory and FTPF model; then the lowest is taken as the fatigue life of the laminate, which is that of 90° lamina in this situation. This correlation study shows that the FTPF criterion model is appropriate to be used in the optimum design efforts of multiaxial composite plates.

3 OPTIMIZATION

Composite laminate design problems cannot be handled with the traditional nonlinear optimization techniques because their derivative calculations or approximations are almost impossible to achieve and requires great computational effort. On the other hand, the modern or non-traditional optimization methods have come up as powerful and popular methods for solving complex engineering optimization problems such as composite laminate design problems in recent years. Some of these optimization methods commonly used are genetic algorithm, pattern search algorithm, simulated annealing, particle swarm optimization, ant colony optimization, neural network-based optimization, and fuzzy optimization.

Standard optimization algorithms, when used individually, may have the disadvantages of slow convergence rates, local optima and the lack of cooperation between solution steps. In order to overcome these problems, some hybrid methods have emerged [14]. Hence, in this study, the combination of genetic algorithm and generalized pattern search algorithm is considered as a hybrid algorithm to achieve a high accuracy and a moderate computational effort in our results. In the following subsections, brief information about the algorithms is given.

3.1 Genetic algorithm

Genetic algorithm (GA) is a class of evolutionary algorithms inspired by evolutionary biology and is obtained by John Holland [15] through modelling the Darwin's theory. GA is based on the natural selection process which ends up with the evolution of organisms best adapted to the environment.

Genetic algorithm begins its search with a population of random individuals. Each member of the population holds a chromosome which encodes certain characteristics of the individual. The algorithm methodically analyses each individual in the population of designs according to set specifications and assigns it a fitness rating which represents the designer's aims. This fitness rating is then used to identify the structural designs which perform better than others. Thus, it enables the genetic algorithm to determine the designs which are weak and must be eliminated using the reproduction operator. After this step, the remaining, more desirable genetic material is utilized to create a new population of individuals. This part is carried out by applying two more operators similar to natural genetic processes, which called gene crossover, gene mutation and replacement. The process is iterated over many generations in order to obtain optimal designs [16].

3.2 Generalized pattern search algorithm

Generalized Pattern Search Algorithm (GPSA) has been developed for derivative-free unconstrained optimization of functions by Torczon [17] and later extended to cover nonlinear constrained optimization problems. GPSA is a direct search method which finds a sequence of points that approach the optimal point. Each iteration consists of two phases: the search phase and the poll phase. In the search phase, the objective function is evaluated at a finite number of points on a mesh. The main task of the search phase is to find a new point that has a lower objective function value than the best current solution which is called the incumbent. In the poll phase, the objective function is evaluated at the neighbouring mesh points in order to see whether a lower objective function value can be obtained [18]. GPSA includes some collection of vectors that form the pattern and also has two commonly used positive basis sets; the maximal basis with 2N vectors and the minimal basis with N+1 vectors [19].

4 PROBLEM DEFINITION

In this study, optimum fibre orientation angles of laminated composites subjected to in-plane loads are investigated for maximum fatigue life. The number of distinct laminae *n* and the thickness of the laminae t_0 are given. The orientation angle θ_k in each lamina will be determined in design process. Accordingly, the number of design variables is *n*. The orientation angles take discrete values of 0°, 45°, −45°, 90° which are conventional in industry.

The material and experimental fatigue parameters are taken from the study of Hashin and Rotem [6] and problem is taken from [4] except loading conditions. Material is a unidirectional 32-layer E-glass/epoxy fibre-reinforced composite. The material and strength properties are given in the following table.

Material Properties	**Strength Properties**
$E_{11} = 181\,\text{GPa}$	$X_t = -X_c = 1235.64\,\text{MPa}$
$E_{22} = 10.3\,\text{GPa}$	$Y_t = -Y_c = 28.44\,\text{MPa}$
$G_{12} = 7.17\,\text{GPa}$	$S = 37.95\,\text{MPa}$
$\nu_{12} = 0.28$	

Table 1: Properties of the laminates used in the study [4].

The ply thickness t_0 is 0.127 mm. It is reported that R ratio is 0.1 and frequency (ν) is 19 Hz. The hybrid optimization algorithm is applied to composite laminates subjected to different types of in-plane loadings.

4.1 Formulation of the objective function

Derivation of the objective function and the optimization strategy applied will be addressed here. The expressions of the fatigue failure stresses (*X*, *Y*, *S*) directly taken from [6] and given with Equation (10) are substituted into the expression of the FTPF criterion defined in Equation (9). Afterwards, the fatigue life, $\log(N)$, is obtained in polynomial form through some mathematical calculations. Hence, the objective function is formulated as

$$f(n,\theta_k) = -\log(N) \;\mid \{k = 1,\ldots,n = 32\} \tag{11}$$

where n is the number of plies, and θ_k is the fibre orientation angle of each ply. Since the search algorithm is normally constructed to minimize the objective function, the logarithmic fatigue life of the laminate, $\log(N)$ is taken as negative in the objective function to be able to maximize. The laminates are subjected to symmetry and balance geometric constraints. In addition, Hashin-Rotem failure criterion [20] is used to check whether or not the first ply failure occurs in the laminates.

Consequently, the optimization problem can be defined as

Find: $\{\theta_k, n\},\ \theta_k \in \{0_2, \pm 45, 90_2\},\ k = 1, \ldots, n = 8$

Maximize: $\log(N)$

Constraints: Hashin-Rotem failure criterion $\{FI_{fibre} \leq 1, FI_{matrix} \leq 1\}$

Symmetry

Balance

where the number of design variables (θ_k) becomes 8 due to balance and symmetry. The fibre angles will be used as ply stacks of 0_2, ± 45, 90_2 for the design cases. FI_{fibre} and FI_{matrix} represent fibre and matrix failure indexes of the laminate, respectively, and they must be smaller than 1 to avoid any ply failure.

In order to determine *N*, fatigue life of each lamina is calculated using Equation (11) and the smallest one of the obtained fatigue lives is chosen as the fatigue life of the laminate, *N*. Thus, the first-ply failure approach is typically involved in this study. A laminate configuration is considered to be more fatigue-resistant than another if the fatigue life estimated by the fatigue model is longer than that of the other even if the applied cyclic stress is less than their endurance limits and actually they both have infinite fatigue life.

5 RESULTS AND DISCUSSIONS

Composite plates are optimized by using a hybrid algorithm of genetic and generalized pattern search algorithms to obtain the best fibre stacking sequences which enables the maximum fatigue life. The study consists of two parts. At first, optimization problems are solved using the specified discrete fibre angles as defined. Then, an optimization study is performed using integer fibre angle values between $-90°$ and $90°$ to compare with the conventional stacking sequence designs. MATLAB Optimization Toolbox [21] is used to solve the problems. In order to increase the efficiency and reliability of the algorithm, at least 50 independent searches are performed for each case. Before starting the optimization, a preliminary study was executed to determine the most appropriate options of the sub-algorithms. The optimum stacking sequences of laminates, the corresponding fatigue lives, and the number of global optima found for various in-plane cyclic loads are presented in Tables 2 - 4. Failure indexes of the laminates determined according to Hashin-Rotem failure criterion is also specified for each design case in the tables.

Table 2 shows the results for only tension in-plane cyclic loads. One should note that the objective function of the optimization is not affected by the stacking sequence for in-plane loads. In other words, there are multiple global optima. For this reason, alternative stacking sequences are not shown in the tables. As the results indicate, the fatigue life is found to be sensitive to the level of stress. Fatigue life decreases with the increase of loading. There are many global optima in various laminate configurations in almost all cases. Failure indexes indicate that all the laminate configurations are reliable against static failure.

Table 3 shows the effect of the existence of shear stress on the optimum designs. It is seen that the fatigue life dramatically changes according to the shear stress level and the type of loading applied. The fatigue life decreases when the shear stress level is increased and the type of loading is changed. For instance, the design of 5/2.5/2.5 ($\times 10^2$ N/mm) loading case is more critical than the one of 5/2.5/5 ($\times 10^2$ N/mm) loading case. While different stacking sequences are obtained, in some cases the same global designs are obtained. Even if the laminates are safe against static loading, they are seen to have short fatigue lives under dynamic fatigue loading.

Loading ($\times 10^2$ N/mm) $N_{xx}/N_{yy}/N_{xy}$	One of stacking sequence	No. of global optima	Fatigue life (cycles)	FI_{fibre}	FI_{matrix}
5/0/0	$[0_{16}]_s$	1	4.704×10^8	0.0996	0.0001
5/2.5/0	$[0_4/\pm 45_3/0_2/\pm 45_2]_s$	25	5.503×10^7	0.1420	0.1849
5/5/0	$[(0_2/90_2)_2/90_2/0_4/90_2]_s$	20	4.660×10^6	0.1858	0.3350
5/7.5/0	$[90_2/0_6/90_8]_s$	32	2.979×10^5	0.2374	0.5387
5/10/0	$[90_2/\pm 45/90_4/\pm 45_4]_s$	22	3.175×10^4	0.2839	0.7398
10/7.5/0	$[\pm 45_6/0_2/\pm 45]_s$	6	2.058×10^3	0.3823	0.9882

Table 2: Optimum stacking sequence designs and the corresponding fatigue lives for various in-plane tension-tension cyclic loads.

Loading ($\times 10^2$ N/mm) $N_{xx}/N_{yy}/N_{xy}$	One of stacking sequence	No. of global optima	Fatigue life (cycles)	FI_{fibre}	FI_{matrix}
0/0/5	$[\pm 45_8]_s$	1	3.987×10^6	0.1912	0.1196
5/0/2.5	$[(\pm 45/0_2)_4]_s$	19	4.344×10^4	0.1990	0.1940
5/0/5	$[0_2/\pm 45_2/0_4/\pm 45_3]_s$	21	46	0.3157	0.4352
5/2.5/2.5	$[(0_2/\pm 45)_2/\pm 45_4]_s$	14	2.642×10^5	0.2491	0.3838
5/2.5/5	$[\pm 45/0_2/\pm 45_5/0_2]_s$	11	932	0.3734	0.7102
5/5/2.5	$[\pm 45_8]_s$	1	8.551×10^4	0.2814	0.5651
5/5/5	$[\pm 45_8]_s$	1	962	0.3770	0.8550
10/0/2.5	$[\pm 45_2/0_{12}]_s$	14	73	0.3565	0.5709

Table 3: Optimum stacking sequence designs and the corresponding fatigue lives for various in-plane tension and shear cyclic loads.

Loading ($\times 10^2$ N/mm) $N_{xx}/N_{yy}/N_{xy}$	One of stacking sequence	No. of global optima	Fatigue life (cycles)	FI_{fibre}	FI_{matrix}
5/-2.5/0	$[(0_2/90_2)_2/0_4/90_2/0_2]_s$	27	2.833×10^7	0.1552	0.0903
5/-5/0	$[0_2/90_4/0_6/90_4]_s$	36	3.987×10^6	0.1912	0.1196
5/-7.5/0	$[90_4/0_4/90_6/0_2]_s$	20	2.709×10^5	0.2479	0.2099
5/-10/0	$[90_4/0_4/90_4/0_2/90_2]_s$	22	1.573×10^4	0.3103	0.3611
-5/-5/0	$[90_4/0_2/90_2/0_4/90_2/0_2]_s$	14	4.660×10^6	0.1858	0.3350
10/-2.5/0	$[0_2/90_4/0_{10}]_s$	18	2.009×10^5	0.2615	0.2767
10/-5/0	$[0_4/90_4/0_4/90_2/0_2]_s$	23	1.573×10^4	0.3103	0.3611
10/-10/0	$[0_4/(0_2/90_4)_2]_s$	27	311	0.3824	0.4784
-10/-7.5/0	$[0_2/\pm 45_7]_s$	2	2.058×10^3	0.3823	0.9882

Table 4: Optimum stacking sequence designs and the corresponding fatigue lives for various in-plane tension and compression cyclic loads.

Table 4 shows the optimum results for tension-compression and compression-compression in-plane cyclic loadings. Similarly with the previous cases, multiple global optima are found and fatigue life decreases with the increase of loading. It can be seen that for compression-compression loading cases, the same optimum design results are obtained as in the previous tension-tension loading cases. For example, the results of the loading case of 5/5/0 are the same with the case of -5/-5/0. Different global optima are presented intentionally in these cases. It is also seen that the laminates are within the safe zone considering the Hashin-Rotem failure indexes.

Table 5 shows the comparison results of conventional (with 0, +45, −45, 90 discrete fibre angle values) and non-conventional (with integer fibre angle values between $-90°$ and $90°$) optimizations for various in-plane cyclic loadings. Stacking sequences and fatigue lives corresponding to related loadings and angle types are presented in the table.

Loading ($\times 10^2$ N/mm) $N_{xx}/N_{yy}/N_{xy}$	Angle type	Stacking sequence	Fatigue life (cycles)
5/0/0	Con.	$[0_{16}]_s$	4.704×10^8
	Non-con.	$[0_{16}]_s$	4.704×10^8
5/2.5/0	Con.	$[0_4/\pm 45_3/0_2/\pm 45_2]_s$	5.503×10^7
	Non-con.	$[\pm 35_8]_s$	6.373×10^7
5/10/0	Con.	$[90_2/\pm 45/90_4/\pm 45_4]_s$	3.175×10^4
	Non-con.	$[\pm 55]_s$	8.230×10^4
5/-2.5/0	Con.	$[(0_2/90_2)_2/0_4/90_2/0_2]_s$	2.833×10^7
	Non-con.	$[0_2/90_2/0_4/(90_2/0_2)_2]_s$	2.833×10^7
5/-7.5/0	Con.	$[90_4/0_4/90_6/0_2]_s$	2.709×10^5
	Non-con.	$[(90_2/0_2)_3/90_4]_s$	2.709×10^5
0/0/5	Con.	$[\pm 45_8]_s$	3.987×10^6
	Non-con.	$[\pm 45_8]_s$	3.987×10^6
5/5/2.5	Con.	$[\pm 45_8]_s$	8.551×10^4
	Non-con.	$[\pm 45_8]_s$	8.551×10^4
5/2.5/5	Con.	$[\pm 45/0_2/\pm 45_5/0_2]_s$	932
	Non-con.	$[\pm 36]_s$	3243

Table 5: Comparison of conventional (Con.) and non-conventional (Non-con.) fibre angle optimizations for various in-plane cyclic loadings.

As seen from the results, optimum designs providing considerable higher fatigue lives could be obtained in some non-conventional cases, whereas the same maximum fatigue lives are obtained for some conventional design cases even if different global optima are found.

In addition, a study is performed so as to demonstrate the efficiency of the hybrid algorithm (HA) used in the optimization procedure. For this purpose, four design cases are chosen among the specified problems and the performances of GA and GPSA are compared to the HA in terms of number of global optima. Both GA and GPSA are run 50 times. Table 6 shows the results of the comparison.

Loading ($\times 10^2$ N/mm) $N_{xx}/N_{yy}/N_{xy}$	One optimum design	Number of global optima		
		GA	GPSA	HA
5/2.5/0	$[0_4/\pm 45_3/0_2/\pm 45_2]_s$	22	1	25
5/10/0	$[90_2/\pm 45/90_4/\pm 45_4]_s$	18	1	22
5/0/5	$[0_2/\pm 45_2/0_4/\pm 45_3]_s$	20	1	21
10/-5/0	$[(90_2/0_2)_2/0_2/90_2/0_4]_s$	21	1	23

Table 6: Comparison of performances of optimization algorithms.

It is seen that HA works more efficiently for finding global optimum or optima in repeated runs than that of GA and GPSA. GA seems to run faster, but finds fewer global optima. GPSA unpredictably finds one global optimum in all of the runs. This is likely to result from the behaviour of GPSA to mathematical structure of the fatigue optimization problem.

6 CONCLUSIONS

A procedure is proposed to find the optimum fibre orientations of laminated composites under various cyclic loads in order to achieve maximum fatigue strength. For this purpose, a fatigue life prediction model, Failure Tensor Polynomial in Fatigue (FTPF) is used to determine the fatigue lives of the laminates. A hybrid algorithm composed of genetic and generalized pattern search algorithms is used as search algorithm in the optimization procedure. An experimental correlation is performed by using experimental data from the literature to demonstrate the effectiveness of the model. The correlation study shows that the FTPF criterion model is valid and appropriate to be used in the optimum design efforts of multiaxial composite plates.

A number of problems including several design cases are solved. The search algorithm turns out to be consistent and reliable in finding the optimum designs. The optimization process is repeated at least 50 times starting from random initial configurations for each loading case. In almost all of these runs, multiple global optimum designs are found even with a large number of optimization variables. Fatigue life is found to be very sensitive to stress level. Increasing the stress level considerably shortens the fatigue life. Therefore, one should be more cautious in designing structures against fatigue failure. Besides, the comparison study of conventional and non-conventional optimizations presents potential advantages of using non-conventional fibre angles in design. All these outcomes demonstrate the necessity and importance of design optimization practice of laminated composites for fatigue life advance. In this manner, the study shows that in general, fatigue life of laminated composites can seriously be improved by the appropriate fibre stacking sequence designs.

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