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INTERVAL ANALYSIS METHODOLOGY IN THE VIBRATION CONTROL OF A SMART STRUCTURE WITH LAMINATED COMPOSITE UNCERTAINTIES

Marcos D. F. Awruch[*] , Herbert M. Gomes[†]

[*]Mechanical Engineering, Federal University of Rio Grande do Sul
Rua Sarmento Leite, 425, 90050-170, Porto Alegre, RS, Brazil
marcos.dfa@gmail.com

[†] Mechanical Engineering, Federal University of Rio Grande do Sul
Rua Sarmento Leite, 425, 90050-170, Porto Alegre, RS, Brazil
herbert@mecanica.ufrgs.br

Key words: Vibration Control, Smart Materials, Possibilistic method, Interval analysis, Uncertainties quantification, Laminated Composite.

Summary: *This works applies a possibilistic interval analysis methodology associated with a Particle Swarm Optimization algorithm to search for input parameters to minimize and maximize different system outputs in order to build their envelopes. This methodology is applied to a smart structure of a laminated composite material with attached piezoelectric patches and controlled by Linear Quadratic Regulator. System's interval outputs like, natural frequency, mechanical and electrical energies and electrical potential peaks are investigated taking into account uncertainties in material properties, ply angles and layer thickness. It is concluded that these uncertainties may cause significant modifications in the dynamic behavior and should be accounted in the design stage.*

1 INTRODUCTION

In the past few years, substantial attention has been paid to active vibration control of smart and lightly damped flexible structures in several fields of civil, mechanical and aerospace engineering, in applications such as tower structures, motion control of robotic systems, satellite solar panels, and many others. In order to satisfy precision control and lightweight requirements, smart materials such as piezoelectric and shape memory alloys are frequently integrated into laminated composite structures as sensors or actuators. The usage of piezoceramic material is a field with ongoing investigation and application and its advantages include low-power consumption, fast response time, wide variety of shapes and sizes, and easy implementation.

Laminated composite structures are known for their challenges to deal with some uncertain properties that can arise from the manufacturing process as well as material defects such as interlaminar voids, fiber misalignment, residual stresses, variation in ply thickness, and others [1, 2]. To deal with the uncertainty in a project, it is possible to use a probabilistic approach, but in that case it is necessary to have enough and reliable information on the random variables, such as mean values, moments and distribution type [3]. Usually, in many engineering applications, there are not enough measurements or knowledge about the uncertain parameters, or even they were measured with insufficient accuracy. When statistical data cannot be obtained or the information is imprecise, the possibilistic approach is preferable. Possibilistic methods deal with the extreme scenarios, or the problem output boundaries, giving no information about their probabilistic distribution.

There are some new studies where the focus is the analysis of uncertainties in composites, such as [4], where the first order reliability method (FORM) is used with a high fidelity shear deformable laminated model, where uncertainties are associated with fiber orientation and ply thickness. Lopez *et al.* [5] presents a comparison between the FORM and the polynomial chaos representation for the reliability analysis, where loads, strength properties and orientation angles of layers are considered as random variables. In Goyal and Kapania [6] paper, angles and properties of the laminate are considered as having an uncertainty degree, so the problem reliability is analyzed as well. There are also experimental studies trying to take into account sources of uncertainties, as Lekou *et al.* [7] study where they try to obtain the estimation of measurement uncertainties in the properties of composite materials. A good survey on methodologies used for uncertainty evaluation in composite structures can be found in Sing and Grover [8].

This works applies an interval analysis methodology associated with a heuristic optimization algorithm to search for the output boundaries for a case example, quantifying its uncertainty. In this work the PSO (Particle Swarm Optimization) algorithm is used as the tool for the minimum and maximum boundary search. Herein we work with a laminated composite plate with embedded piezoelectric actuators controlled by Linear Quadratic Regulator (LQR). The output performance parameter analyzed is the integral over time of the kinetic and potential energy of the controlled beam. The uncertainties are considered present in the material properties, ply angles and ply thickness.

Some comparisons related to structural displacements and control for each analyzed case are presented and depicted, highlighting the role and main features of the uncertain parameters.

2 PROBLEM FORMULATION

The problem consists in a laminated composite plate with embedded piezoelectric patches placed as collocated actuators used for the device's vibration control. The global coupled equation of motion of this type of system can be cast as follows [9]:

$$\begin{bmatrix}\mathbf{M} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{q}}\\ \ddot{\boldsymbol{\phi}}\end{bmatrix}+\begin{bmatrix}\mathbf{D_d} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{q}}\\ \dot{\boldsymbol{\phi}}\end{bmatrix}+\begin{bmatrix}\mathbf{K_{qq}} & \mathbf{K_{q\phi}}\\ \mathbf{K_{\phi q}} & \mathbf{K_{\phi\phi}}\end{bmatrix}\begin{bmatrix}\mathbf{q}\\ \boldsymbol{\phi}\end{bmatrix}=\begin{bmatrix}\mathbf{F}_{\mathrm{q}}\\ \mathbf{F_{\phi}}\end{bmatrix} \tag{1}$$

where $\mathbf{q}$ and $\boldsymbol{\phi}$ are displacement and potential electrical fields, respectively, $\mathbf{M}$ is the mass matrix, $\mathbf{D_d}$ is the damping matrix, $\mathbf{F_q}$ and $\mathbf{F_\phi}$ are the mechanical and electrical force vectors, $\mathbf{K_{qq}}$ is the mechanical stiffness, $\mathbf{K_{\phi\phi}}$ is the electric stiffness and, finally, $\mathbf{K_{\phi q}} = \mathbf{K_{q\phi}}^{\mathrm{T}}$ are the electro-mechanical coupling stiffness matrix.

Eq. (1) may be rewritten in a single equation:

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{D_d}\dot{\mathbf{q}}+\left(\mathbf{K_{qq}}-\mathbf{K_{q\phi}}\mathbf{K_{\phi\phi}^{-1}}\mathbf{K_{\phi q}}\right)\mathbf{q}=\mathbf{F}_{\mathrm{q}}-\mathbf{K_{q\phi}}\mathbf{K_{\phi\phi}^{-1}}\mathbf{F_\phi} \tag{2}$$

or

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{D_d}\dot{\mathbf{q}}+\mathbf{K}\mathbf{q}=\mathbf{F_q}+\mathbf{K_e}\mathbf{F_\phi} \tag{3}$$

where $\mathbf{K}=\left(\mathbf{K_{qq}}-\mathbf{K_{q\phi}}\mathbf{K_{\phi\phi}^{-1}}\mathbf{K_{\phi q}}\right)$ and $\mathbf{K_e}=-\mathbf{K_{q\phi}}\mathbf{K_{\phi\phi}^{-1}}$.

The composite laminated plate is modeled using the Classical Lamination Theory [10] so it can be treated as shell or plate elements and it holds Kirchhoff plate hypothesis. Other constraints are also considered such as a perfect bonding between layers; the resin between plies is infinitesimally thin; and each layer has a uniform thickness. Furthermore, the modifications in the angle-ply configurations could induce bending–stretching and bending-twisting coupling effects depending on the generated asymmetry of the laminate [11].

Piezoelectric elements might behave nonlinearly at high voltages, so it is desired to remain at lower levels and use the linear constitutive relations defined by IEEE standards [12] and commonly considered as a good representation for those materials.

$$\begin{gathered}\boldsymbol{\sigma} = \mathbf{C_L}\boldsymbol{\varepsilon} - \mathbf{e}^{\mathrm{T}}\mathbf{E} \\ \mathbf{D}_{\boldsymbol{e}} = \mathbf{e}\boldsymbol{\varepsilon} + \boldsymbol{\xi}\mathbf{E}\end{gathered} \tag{4}$$

where $\boldsymbol{\sigma}$ is the stress vector, $\mathbf{D}_{\boldsymbol{e}}$ denotes the electric displacement vector, $\boldsymbol{\varepsilon}$ is the strain vector, $\mathbf{E}$ is the electrical field vector, $\mathbf{C_L}$ is the elastic tensor. Finally, $\mathbf{e}$ is the piezoelectric constants matrix and $\boldsymbol{\xi}$ represents the dielectric constants.

For the numerical simulations it is used the GPL-T9 element. This is a triangular element, applicable for plates and shells, where it takes into account the coupling of membrane and bending effects, having 6 DoF for each node and an extra DoF for each piezoelectric layer [13].

3 OPTIMAL CONTROL

Modern control theory is usually applied in MIMO (multi-input multi output) systems, using time domain instead of frequency domain as the classical control theory. The optimal control objective [14] is to determine the control signal that will make a process be controlled and at the same time optimize a performance index.

To reduce the problem order of multiple degrees of freedom, especially for complex structures in finite element, it is usual to work with a truncated modal model, where only the most important modes are considered in the simulation (usually lower modes of vibration are the most easily excitable). The transformation to the modal form is performed starting from the separation hypothesis:

$$\mathbf{q} = \boldsymbol{\Phi}\boldsymbol{\eta} \tag{5}$$

where $\boldsymbol{\eta}$ is the modal coordinates vector and $\boldsymbol{\Phi}$ the modal matrix, found solving the eigenvalue problem:

$$(\mathbf{K} - \boldsymbol{\Omega}\mathbf{M})\boldsymbol{\Phi} = \mathbf{0} \tag{6}$$

and $= diag\{{\omega_i}^2\}$, ω_i being the natural frequencies of the structure and each column of $\boldsymbol{\Phi}$ its correspondent eigenvector.

Taking into account Eq. (3) and Eq. (5), the following equation of motion is obtained:

$$\mathbf{I}\ddot{\boldsymbol{\eta}} + \boldsymbol{\Lambda}\dot{\boldsymbol{\eta}} + \boldsymbol{\Omega}\boldsymbol{\eta} = \boldsymbol{\Phi}^{\mathrm{T}}\mathbf{F} + \boldsymbol{\Phi}^{\mathrm{T}}\mathbf{K_e}\mathbf{F}_{\boldsymbol{\phi}} \tag{7}$$

where $\mathbf{I}$ is the identity matrix and $\boldsymbol{\Lambda} = diag\{2\xi_i\omega_i\}$ with ξ_i being the damping ratio of the i$^{\mathrm{th}}$ mode.

In the next step it is convenient to work in the space state model to reduce a second order problem, Eq. (3), to a first order one. Defining the state space vector $\mathbf{x}$:

$$\mathbf{x} = \begin{bmatrix}\mathbf{x_1} \\ \mathbf{x_2}\end{bmatrix} = \begin{bmatrix}\boldsymbol{\eta} \\ \dot{\boldsymbol{\eta}}\end{bmatrix} \tag{8}$$

It is possible to get the following system of equations:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bu} + \widetilde{\mathbf{B}}\mathbf{f} \\ \mathbf{y} &= \mathbf{Cx} \end{aligned} \tag{9}$$

where $\mathbf{u}$ is the control input force and $\mathbf{f}$ some external mechanical disturbance. The state matrices are defined as:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{\Omega} & -\mathbf{\Lambda} \end{bmatrix}$$
$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{\Phi}^{\mathrm{T}}\mathbf{K}_{\mathbf{e}}\mathbf{F}_{\mathbf{\phi}} \end{bmatrix} \tag{10}$$
$$\widetilde{\mathbf{B}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{\Phi}^{\mathrm{T}}\mathbf{F} \end{bmatrix}$$

and $\mathbf{C}$ is the identity matrix in the case of a full state feedback.

In this work it is used the Linear Quadratic Regulator (LQR) from the modern control theory where the main objective it is to minimize the performance index J:

$$J = \frac{1}{2}\int_0^{\infty} (\mathbf{x}^{\mathrm{T}}\mathbf{Qx} + \mathbf{u}^{\mathrm{T}}\mathbf{Ru})dt \tag{11}$$

where $\mathbf{Q}$ is semi positive definite and $\mathbf{R}$ is strictly positive definite, both defined by the designer of the system through some criteria.

It is known [15] that the minimum for J in full state feedback cases is using control forces that are proportional to the space state vector, Eq. (12), so the objective is to find the gain vector $\mathbf{G}$. It is possible to know the control force needed:

$$\mathbf{u} = -\mathbf{Gx} \tag{12}$$

It can be shown that the solution to obtain the gain is

$$\mathbf{G} = \mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{S} \tag{13}$$

where $\mathbf{S}$ is the Riccati matrix, defined by the solution of the Algebric Riccati Equation

$$\mathbf{SA} + \mathbf{A}^{\mathrm{T}}\mathbf{S} - \mathbf{SBR}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{S} + \mathbf{Q} = 0 \tag{14}$$

4 INTERVAL ANALYSIS METHODOLOGY

The main objective of interval analysis is to find a specific combination among the possibilities considering the input variables interval (or uncertainty) in order to predict the possible extreme outputs. This means the interval that encompass all the related output possibilities.

Assuming a system defined as $= f(\boldsymbol{x})$, where $\boldsymbol{x} = [x_1, x_2, \dots, x_n]$ is the input vector with *n* variables, and z= $[z_1, z_2, \dots, z_m]$ the corresponding output vector with *m* considered outputs, the proposed methodology can be defined as a multiple optimization problem as follows:

$$\begin{gathered} \text{Find } \underline{\boldsymbol{x}} \text{ to } \min(z_j) \text{ where } j = 1, 2, \dots, m \\ \text{subject to } \boldsymbol{x} \subset [\boldsymbol{x}^{min} \ \ \boldsymbol{x}^{max}] \\ \text{and} \\ \text{Find } \overline{\boldsymbol{x}} \text{ to } \max(z_j) \text{ where } j = 1, 2, \dots, m \\ \text{subject to } \boldsymbol{x} \subset [\boldsymbol{x}^{min} \ \ \boldsymbol{x}^{max}] \end{gathered} \tag{15}$$

The optimization might be performed for the desired outputs j which envelope is desired, resulting in the interval [z_j^{min} z_j^{max}]. This interval analysis methodology consists in a double step optimization for each output. First of all, it is defined the constraints and parameters in the composite properties that are uncertain. The next step is the definition of the objective function which envelope or interval are pursued. For instance, the sum of kinetic and potential energies is one of the elected output parameter to check the control performance under the input variability. Finally, the optimizations are performed looking for the input parameter combinations in the allowed interval that maximizes and minimizes this objective.

4.1 Particle Swarm Optimization

As a heuristic optimization algorithm, the particle swarm optimization (PSO) was chosen due its simple implementation and robustness to find global optimum. This algorithm was introduced by Kennedy and Eberhart [16] and it was inspired by the observation of social behavior of beings, such as fish schooling, insects swarming and birds flocking. The method is based on the social influence and social learning, so the exchange of information between individuals may lead them to solve complex problems. As stated by Li *et al*. [17], it involves a number of particles, which have a defined position and velocity, and they are initialized randomly in a multidimensional search space of a cost function (a modification of the objective function to handle with constraints in the problem). Each particle represents a potential solution for the problem and the measure of suitability is the cost function value. The set of particles is generally referred as “swarm”. These particles “fly” through the multidimensional space. They have three essential reasoning capabilities: inertia, the memory of their own best position and knowledge of the global or neighborhoods best position.

The basic parameters, position and velocity, are updated throughout each iteration by the following equations:

$$v_{i,j}^{k+1} = \varpi v_i^k + \lambda_1 r_1\left(xlbest_{i,j}^k - x_{i,j}^k\right) + \lambda_2 r_2\left(xgbest_j^k - x_{i,j}^k\right) \tag{16}$$

$$x_{i,j}^{k+1} = x_{i,j}^k + v_{i,j}^{k+1} \tag{17}$$

where $v_{i,j}^{k+1}$ e $v_{i,j}^k$ are the updated velocity and actual one, respectively, of particle i with respect to design variable j. In the same way $x_{i,j}^{k+1}$ and $x_{i,j}^k$ are the particle position. $xlbest_i^k$ is the best position found so far from self-historical path by the particle i while $xgbest^k$ is the best position found by anyone in the swarm until that iteration k. r_1 e r_2 are random numbers between 0 and 1, while λ_1 e λ_2 are cognitive parameters that represent the confidence in its own results or the swarm best results. Finally, ϖ is the inertia factor, introduced in the original PSO by Shi e Eberhart [18], and is the importance of the current speed on the searching procedure.

5 EXAMPLES

Being a multidisciplinary field, smart materials might present a variety of uncertainty sources, from materials properties to circuitry and controlling features. This analysis focus on the composite material properties uncertainty and the resulting variability in structural behavior, more specifically its envelope. In the example it is studied a cantilever composite plate consisted of 4 layers with 30 cm in length and 4.5 cm in width embedded with PZT patches to control free vibration due a suddenly applied force of 0,1 N on all free nodes at the tip of the beam for a short period (5 ms) as shown in Fig. (1).

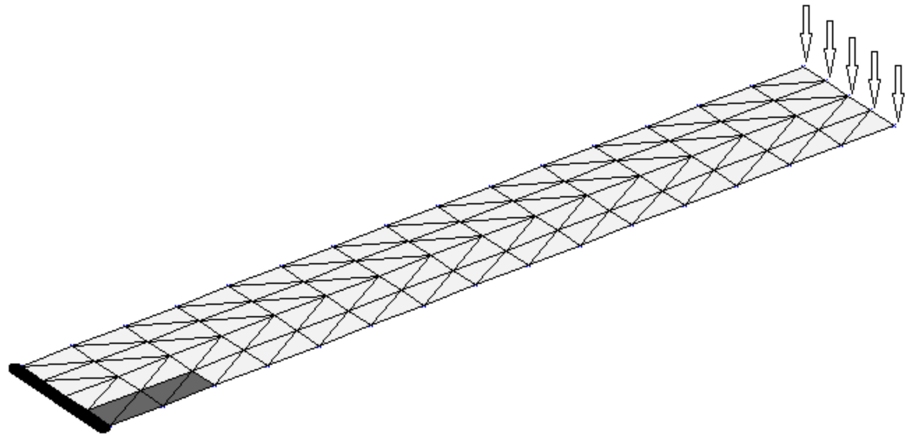

Figure 1: Finite element mesh of the cantilever composite plate with location of the collocated piezoelectric actuators (dark elements) and the location of applied forces.

The nominal properties of composite material plate and piezoelectric layer properties are given in Table 1. To define the control parameters **Q** and **R**, Eq. (11), an optimization was made looking for their balance that should result in less mechanical energy (potential and kinetic) in the cantilever plate, not exceeding the piezoceramic electrical potential of linearity of ±200 V. In this configuration, the first 4 vibration modes have been selected to be controlled, and for simulation purposes, the modal FE model includes the first 10 vibration modes. Fig. (2) shows the time response for the transversal displacement of a node at the free tip of the plate and the control force applied in that case. It was imposed a frequency-independent weakly modal damping ratio for all modes with value $\boldsymbol{\xi}$= 2%.

	Composite laminate	**PZT**
Stacking sequence	[45º/-45º/-45º/45º]	
Density	$\rho = 1600$ kg/m³	$\rho = 7600$ kg/m³
Layer thickness	h = 0.5 mm	h = 0.25 mm
Young's moduli	E_1 = 172.5 GPa , E_2 = 6.9 GPa	$E_1 = E_2$ = 63.0 GPa
Shear moduli	G_{12} = 3.45 GPa	G_{12} = 24.6 GPa
Poisson's ratio	ν_{12} = 0.25	ν_{12} = 0.28
Piezoelectric constant		$e_{31} = e_{32}$ = 10.62 C/m^2
Electrical permissivity		ξ_{33} = 0.1555×10^{-7} F/m
Electrical potential limit		±200 V

Table 1: Composite laminate and PZT piezoceramics properties.

5.1 Uncertainty case 1

For this example, it is considered an independent uncertainty of ±3º for each layer of the laminated composite plate fiber orientation. The intention is to build a displacement envelope for the extreme possibilities where it is maximized and minimized for each instant of time. This type of analysis might be computationally expensive, since two optimizations are performed for each period where it is intended to know the interval.

Analyzing Fig. (3) it is possible to note that in the beginning of the transition, most configurations have a similar path and the interval (lower and upper bounds) follows the nominal behavior. However, as time goes by that interval becomes smooth. That effect is due to different vibration phases for different ply configurations that fulfill the wave gaps presented in the beginning. It is concluded that even with a relatively low variation of ±3º in the fiber orientation, the resulting mechanical vibrations might change, especially its phase after transient period. This displacement envelope might be interesting on robotic applications for positional error analysis

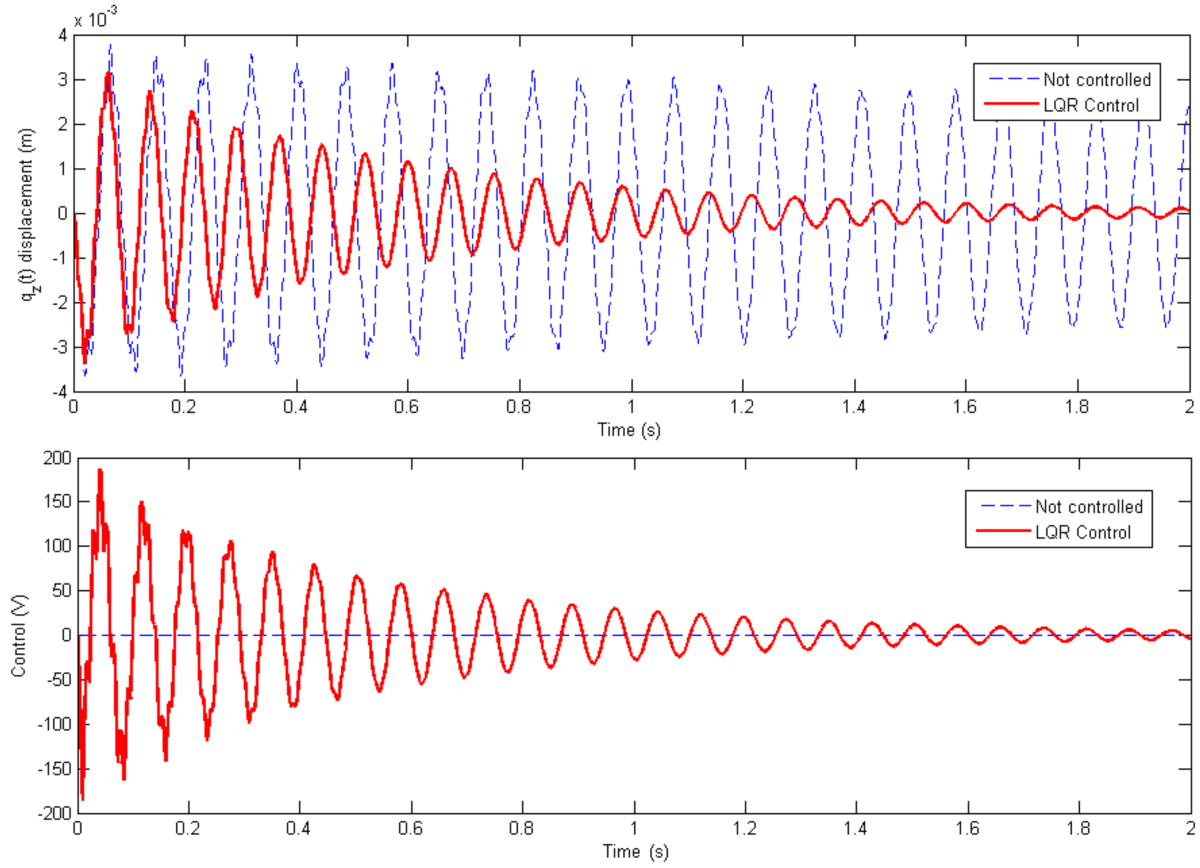

Figure 2: Time response for tip displacement and control force.

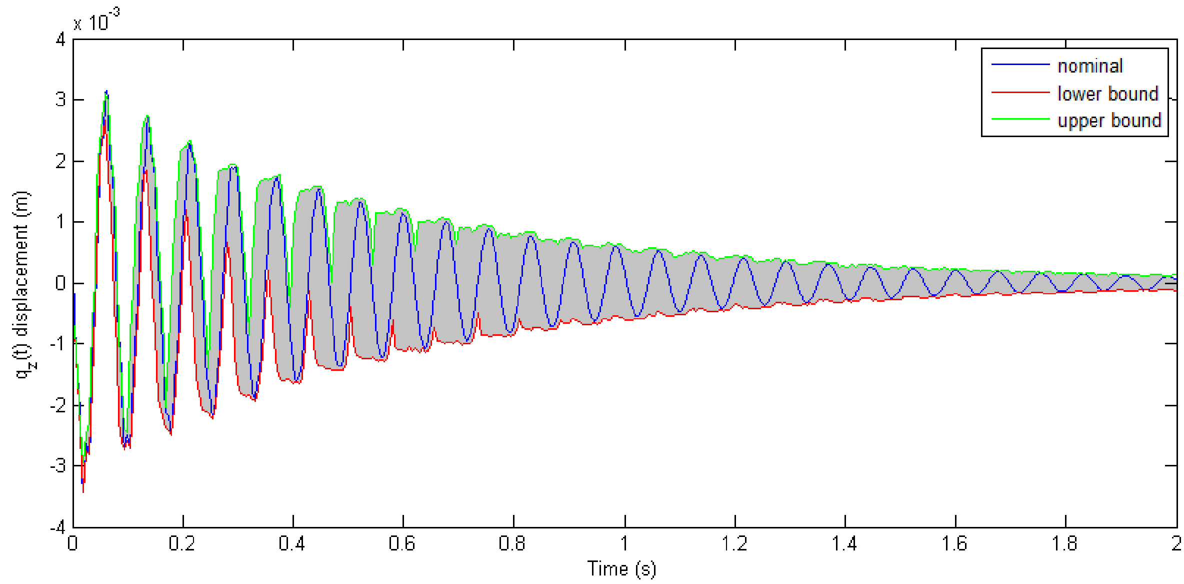

Figure 3: Time response for tip displacement.

The interval between lower and upper boundaries over time is shown in Fig. (4). The interesting point is the larger interval at instant 0.3 s, meaning that at this time the range on the variability due to uncertainty is more pronounced. Other point of interest might be the interval of variability at the dynamic peak at 0.02 s just after the loading application.

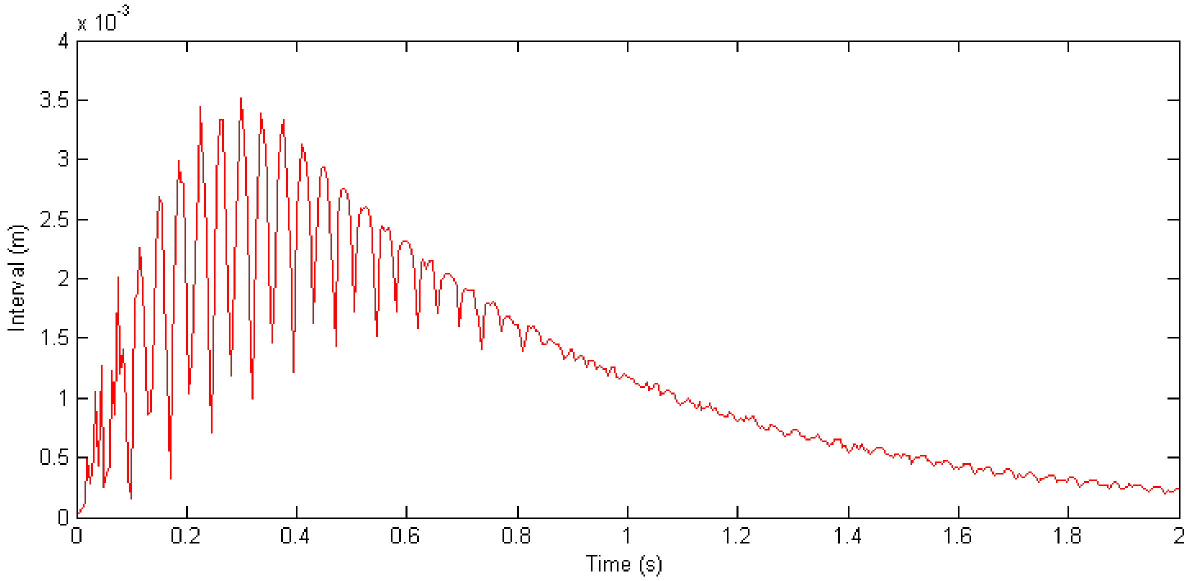

Figure 4: Interval between lower and upper boundaries over time.

5.2 Uncertainty case 2

Depending on the process quality the final product might have less uncertainty in material properties. In this case, it is considered for the composite material proprieties ±1% uncertainty from the nominal values of E_1 , E_2, G_{12} , thickness of each layer and ±1.5º of variation on each fiber angle for each lamina. The interval analysis to quantify that uncertainty is shown in Table 2 where the total mechanical energy (sum of kinetic and potential energies from the controlled modes), the spillover energy (energy from the modes being simulated, but not controlled by the LQR), the input energy (electrical energy), all integrated over the time, the maximum electrical potential applied and the frequency of the first mode are the outputs. For each one of those a different optimization with the PSO is performed, so each result might have a singular property configuration.

	Nominal	Minimum	Maximum	Min %	Max %
Mechanical energy (J.s)	1.04E-4	9.63E-5	1.11E-4	-7.05%	7.07%
Spillover energy (J.s)	7.06E-7	6.59E-7	1.42E-6	-6.61%	100.65%
Input energy (J.s)	3.08E-5	2.75E-5	3.42E-5	-10.72%	11.18%
Maximum potential (V)	199.47	186.28	214.98	-6.62%	7.77%
First mode freq. (Hz)	12.98	12.80	13.71	-1.36%	5.63%

Table 2: Composite laminate and PZT piezoceramics properties.

There are 12 variables in this process, each one varying ±1% from the nominal value and ±1.5º for the fiber angles. Those outputs are analyzed individually, meaning each one has its own combination of uncertain input parameters that might not be valid for the others. Usually when uncertainties are considered in the project the focus is in the worst cases scenarios. Starting with the first output, those uncertainties, in this example, might result in an overall increase of 7.07% of the total mechanical energy, resulting in a more vibrating structure. Next, the spillover energy has a significant increase (doubled) from the nominal value, but fortunately, in this example it does not represent a significant percentage of the total energy for the first four controlled modes. However, that is not the case for general systems, which might be sensitive to spillover effects becoming unstable. In the case of input energy, it reached 11.18% and that might be a significant amount depending on the application. For instance, satellites might have limited resources to operate and the energy it collects should be used wisely. For the maximum potential output, it is shown that certain configuration might exceed the PZT limit of 200 V, which would result in damages to the piezoceramic. Finally, the interval analysis of the first mode was considered where the values ranged from 12.80 Hz to 13.71 Hz. Some frequencies might be of interest depending on the project, for instance, for flutter analysis the uncertainties on an airfoil should be considered.

6 CONCLUSIONS

Smart materials with piezoelectric control are applied in different fields of engineering. The uncertainty propagation can be used to estimate the final envelope for structural behavior. The extreme values could be found using the interval analysis together with a heuristic algorithm approach resulting in accurate solutions and computational savings. Considering only uncertainties on the composite cantilever plate, it was shown that the system’s output values presented a considerable interval of variation. The spatial analysis of interval displacement might be interesting for robotic applications in positional error analysis. For the second example, as previously cited, those outputs might be interesting for aerospace applications like flutter studies and vibration control, so their variability should be accounted for in the project design.

In view of practical applications, this interval analysis method is not intrusive and may well adapt to several existing codes and software, allowing to deal with a wide range of problems. Material property uncertainties are a reality and may cause significant modifications in the dynamic behavior of structures. Therefore, it is advisable to investigate the design performance under uncertain parameters.

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