Application of ANCF Beams in Buckling Analysis

Jia Wang[1], Hongsheng Zhang[1], Marko K. Matikainen[2], Aki M. Mikkola[2]

[1] *School of Machtronics, Harbin Institute of Technology, {wangjia, zhanghs}@hit.edu.cn*
[2] *School of Energy Systems, Lappeenranta University of Technology, { marko.matikainen, aki.mikkola }@lut.fi*

Beam elements based on the absolute nodal coordinate formulation (ANCF) have developed for several decades since first contribution of Shabana[1], its performance in various static and dynamics problem with large rotation and deflection have been validated. Yet, to the authors acknowledgment, the application of the ANCF formulation in structure buckling problems to determinate the critical load has not been thoroughly discussed. The main intention of the present paper is to validate the accuracy and usability of ANCF beams in determining the critical load of extensible and shear-deformable beams under a compressive force, and the influence of extensional and shear stiffness on the critical load will be discussed. Afterward, the geometrically nonlinear equilibrium path is traced with Arc-length control methods, the buckled configuration is obtained.

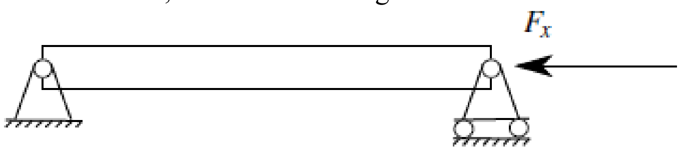

Fig.1 Simple supported beam under axial force

A simple supported beam under axial force F_x is illustrated in fig.1, as the axial force increase to the critical force F_{cr}, the beam becomes unstable and lateral deformation occurs and possibly collapse subsequently. For simple structures as shown in Fig.1, exact solutions of the critical loads can be find in vast of textbooks include the solutions such as [2], while the influence of the shear deformation is not a trivial thing and need to be discussed in detail. Hummer[3] revisited the buckling and post-buckling of beams taking account both the influence of axial compressibility and shear deformation, based on Reissner's geometrically exact relations for plane deformation of beams, the exact solution of the critical load and post-buckling configuration for some classical boundary conditions is presented, it is shown that both buckling and the post-buckling behavior turn out to be affected by shear not only quantitatively, but also qualitatively. For complicate structures with multi-components, finite element method becomes an effective and efficient way to determinate the critical load and the post-buckling behavior while the performance of finite element method in the element level must be validated in the most first.

To determinate the critical load with FEM method, the energy criteria is employed, when the structure becomes unstable, the seconder variation equals to zero, which can be written as

$$\delta^2 U_{elastic} = 0 \tag{1.1}$$

Where $U_{elastic}$ is the strain energy of the deformed structure.

equation (1.1) is equivalent to

$$|\mathbf{k_t}| = 0 \tag{1.2}$$

Which means the determinate of the tangent stiffness of the structure is zeros. For traditional FEM element such as BEAM188 in ANSYS, the tangent stiffness can be written as

$$\mathbf{K_t} = \mathbf{K_0} + \mathbf{K_g} \tag{1.3}$$

Where $\mathbf{K_0}$ is constant value and $\mathbf{K_g}$ is linear function of axial force. In this case, equation (1.2) can be solved with eigenvalue solver. For ANCF beams, $\mathbf{K_t}$ is not linearly dependent on the axial force, but a nonlinear function of the nodal coordinate of the elements, dichotomy method is employed to search the solution of equation (1.1) with increasingly axial force.

To elucidate the application of ANCF formulation in buckling analysis, several ANCF beams proposed by pioneer researcher were implemented again. The first element was proposed by Omar and Shabana[4], whose elastic forces follows from continuum mechanics based formulation for the arbitrary deformation of a mechanical solid, using fully nonlinear Green strain tensor and the second Piola-Kirchhoff stress tensor. The second element was proposed by Nachbagauer[5], the work of elastic forces is derived from a structural mechanics based formulation, which is defined according to Simo and is based on generalized strains. The work of elastic force of the third element is derived from structural mechanical based formulation, while the shear deformation is taken into account via independently interpolated linear terms [6]. These ANCF beam elements was firstly used to determinate the first three critical load of the simple supported beam with changing dimensionless parameters which implies the change of the shear stiffness with respect to axial stiffness, the performance of the ANCF beams were validated by compare with both exact solution and solutions obtained with commercial software, then the ANCF beams was use to trace the post-buckling configuration with arc-length method.

References

[1] A. Shabana. “Definition of the Slopes and the Finite Element Absolute Nodal Coordinate Formulation”, Multibody System Dynamics,1(3): 339-348, 1997.

[2] S. Timoshenko and J. Gere. “Theory of Elastic Stability”, New York: McGraw-Hill, 1961.

[3] A. Hummer. “Exact solutions for the buckling and postbuckling of shear-deformable beams”, Acta Mech, 224: 14931525, 2013.

[4] M. Berzeri and A. Shabana. “Development of Simple Models for the Elastic Forces in the Absolute Nodal Co-ordinate Formulation”, Journal of Sound and Vibration, 235(4): 539-565, 2001.

[5] K. Nachbagauer, S. Pechstein, H. Irschik and J. Gerstmayr. “A new locking-free formulation for planar, shear deformable, linear and quadratic beam finite elements based on the absolute noddal coordinate formulation”, Multibody System Dynamics, 26(3): 245-263, 2011.

[6] V.V. T. Hurskainen, M. K. Matikainen, J. Wang and A. M. Mikkola. “A planar beam finite element formulation with individually interpolated shear deformation”, 12(4), 2017.