A Modified Rayleigh-Ritz Method for Analyzing Flexible Multibody Systems and its Applications

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In the eigenvalue problem, for structures where the parameternon-uniformities are not particularly pronounced and/or geometries are not complicated, the assumed mode substructure synthesis method (AMSSM) has a great capability of yielding superior accuracy for the same number of degrees of freedom (DOF) or the same accuracy with fewer DOF than finite element method (FEM) [1]. However, different from FEM, AMSSM needs more laborious procedures to satisfy compatibility conditions of the system [2]. In this paper, an efficient AMSSM withsimplified procedures for satisfyingcompatibility conditionsis presented. To eliminate a serial kinematics expression in the conventional AMSSM, nodal displacement (ND) isintroduced. These ND form a bridge between local deformations in the members of the structure. This paper also shows that AMSSM has excellent convergence performance in the large deflection static analysis of multi beam structures. Some numerical examples are introduced and results like displacement and/or stress are investigated. The results are validated with those obtained from commercial Nonlinear FEM (NFEM) code and convergence performance is also compared.

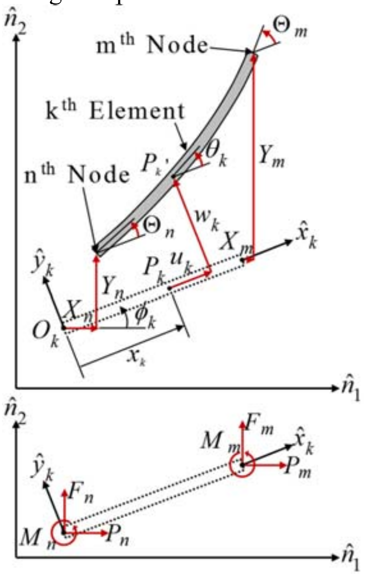
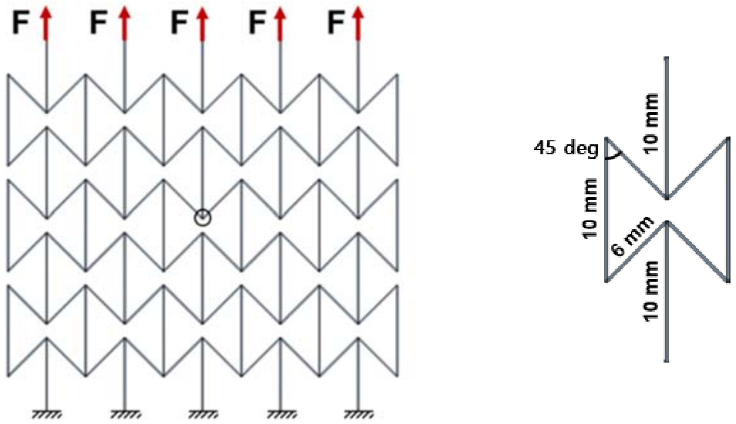


Figure 1. Configuration of a beam before and after deformation



(a) (b)

Figure 2. (a) Configuration of the structure and (b) geometry of the unit structure

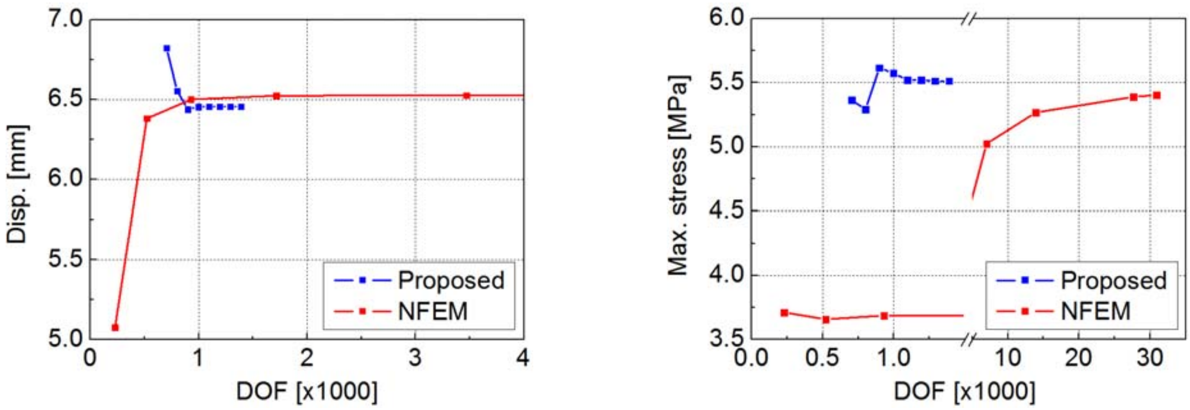
The following assumptions are employed. The beam is made of homogeneous and isotropic material. The beam has a slender shape so that shear and rotary inertia effects are neglected. The beam is in-extensible and only in-plane 2-D motion occurs. Fig. 1 shows the configuration of a cantilever beam before and after deformation occurs. Here, $(\hat{n}_1, \hat{n}_2)$ is an inertial frame and $(\hat{x}_k, \hat{y}_k)$ is a reference frame attached to the k^{th} beam. The point O_k lies on the fixed end and neutral axis of the k^{th} beam. The initial angle between $\hat{n}_1$ and $\hat{x}_k$ is denoted as ϕ_k and length of the beam is L_k . Before deformation, the generic point on the k^{th} beam is denoted as P_k and the distance from O_k to P_k in the direction of $\hat{x}_k$ is x_k . After deformation occurs, P_k moves to P_k' . The displacement of the generic point can be expressed as $u_k \hat{x}_k + w_k \hat{y}_k$. (X_n, Y_n, Θ_n) and (X_m, Y_m, Θ_m) are ND of the n^{th} and m^{th} node, respectively. (P_n, F_n, M_n) and (P_m, F_m, M_m) are a horizontal force, a vertical force, and a moment applied to the n^{th} and m^{th} node, respectively.

In the present work, the assumed mode method is used to approximate the following deformation variables.

$$u_k(x_k,\, t) = \sum_{i=1}^{\mu_1} \Psi_i^k(x_k)\, q_{1i}^k(t),\; w_k(x_k,\, t) = \sum_{i=1}^{\mu_2} \Phi_i^k(x_k)\, q_{2i}^k(t),\; \theta_k(x_k,\, t) = \sum_{i=1}^{\nu} \Phi_{i,x}^k(x_k)\, p_{2i}^k(t), \tag{1, 2, 3}$$

Here, q_{1i}^k, q_{2i}^k, and p_{2i}^k are generalized coordinates for the deformation variables and numbers μ_1, μ_2, and ν are the numbers of generalized coordinates used for the deformation variables. $\Psi^k(x_k)$ and $\Phi^k(x_k)$ are mode functions for approximating the variables. Applying Hamilton's principle, static equilibrium equations are derived but they are omitted for lack of space.

An auxetic structure [3, 4] is given in the Fig. 2. The structure is subjected to the vertical forces having the same magnitude F. Parameters of this example are given in the Table 1. Fig. 3 shows convergence of the displacement of the vertex circled in the Fig. 2 (a) and maximum stress. In both methods, displacement converges at the similar DOF. On the other hand, stress result obtained with the proposed method converges much faster than that obtained with NFEM.



(a) (b)

Figure 3. The convergence of (a) the displacement and (b) maximum stress obtained with the proposed method and NFEM

In this study, AMSSM considering geometric nonlinearity is developed and convergence performances of the proposed method and NFEM are compared each other. In conclusion, only 1/30 DOF of NFEM model is sufficient to obtain the converged stress result in the proposed method.

Acknowledgments

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