Dynamic Analysis of Planar Multibody Systems with Fully Cartesian Coordinates

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Introduction: Multibody systems are described as a collection of bodies interconnected by kinematic pairs and acted upon by external forces. Despite each system intrinsic specifications, all without exception require the use of a set of parameters that uniquely defines the position and orientation of each component during the period of analysis. These parameters are referred to as generalized coordinates and, depending on the type of formulation adopted in the modeling procedure, different model solutions may result when discretizing a given multibody system. The selected type of coordinates directly interferes on the number of coordinates necessary to model the system, as well as on the number and degree of non-linearity of the kinematic constraint equations required to express coordinates dependency, when such is necessary. Hence, the choice of coordinates has direct influence on the complexity of the multibody formulation adopted, on its computational performance and systematization ability, and, equally important, on the intuitiveness and easiness of its utilization.

Ideally a multibody formulation should be able to represent a given multibody system with a minimum set of coordinates, it should generate kinematic constraints with a low degree of non-linearity that should be simple and fast to evaluate, it should present a systematic approach to model general multibody systems and its use should be intuitive. It's not possible to conciliate all these characteristics in a single formulation and therefore compromises had to be found. Classical formulations, based on analytical dynamics, tend to use reduced sets of coordinates to analyze small size problems. These approaches generate highly non-linear equations and require considerable user expertise to discretize the system and achieve a solution. Formulations based on computational dynamics, on the other hand, are able to systematize the modeling process and describe medium and large size multibody systems, although requiring larger sets of variables and equations to solve.

Multibody formulations with Cartesian Coordinates are a good example of the latter case. This type of coordinates led the way in this type of approach as these are able to ally systematization with intuitiveness of use. However, due to the type of coordinates utilized to describe angular degrees of freedom, Cartesian coordinates generate kinematic constraint equations with transcendental terms that are computationally expensive to evaluate. In order to circumvent this problem, Natural coordinates were proposed and, although requiring, on average, less coordinates than Cartesian coordinates generating kinematic constraint equations that are quadratic or linear in nature, this formulation, although its simplicity, is less systematic than the one with Cartesian coordinates and its use far less intuitive.

With Natural coordinates, angular variables are not used at all and the angular degrees of freedom of the system are described, for planar systems, using solely the Cartesian coordinates of points located at relevant positions of the system like joints or extremities. This is in fact the reason why these coordinates were originally referred to as fully Cartesian coordinates as no angular variables are used. The term Natural coordinates arose solely from the fact that, with the purpose of reducing the number of coordinates required to model a given system, points can belong to more than one rigid body, i.e. can be shared by different bodies, and doing so several kinematic joints, such as the revolute in 2D cases, would appear 'naturally' without the need of an explicit joint definition, that is to say, without the use of specific joint kinematic constraint equations.

Moving generalized coordinates to relevant points presents the reported advantages but it also introduces some intricacies such as the fact that rigid bodies structure now varies depending on the specific structure of relevant points of a body, and the fact that, with shared points, not only system matrices become coupled but also, since there is no explicit joint definition, reaction forces cannot be calculated directly from the solution of the equations of motion of the system.

Having all this in consideration, in the present work a novel approach to modeling planar (2D) multibody systems with Fully Cartesian coordinates is proposed. In this approach the two major characteristics of this formulation are kept, i.e., multibody systems are still described only with Cartesian coordinates and kinematic constraint equations are still quadratic or linear, but the reported disadvantages are eliminated, i.e., generalized coordinates are no longer located on relevant points such as joints or extremities, and points are no longer shared between rigid bodies. This is accomplished by introducing the concept of general rigid body, which presents a general, predetermined, kinematic structure. As represented in Fig. 1, the kinematic structure of the general body is defined by one point (point *i* located at its center of mass) and one unit vector (vector u). The two Cartesian coordinates of point *i* are used to describe rigid body translations and the two Cartesian coordinates of unit vector *u* are used to describe its orientation. Since *u* is a unit vector, a unit module condition is introduced to relate these two coordinates. Hence, the general rigid body has a constant kinematic structure described with four generalized coordinates that correspond to the Cartesian coordinates of the point *i* and vector *u*.

The introduction of the general rigid body presents several advantages regarding the formulation and systematization, as it will be seen later in the paper. Additionally, the adoption of a constant rigid body structure brings the modeling approach closer to the one adopted with Cartesian coordinates and familiar to most users.

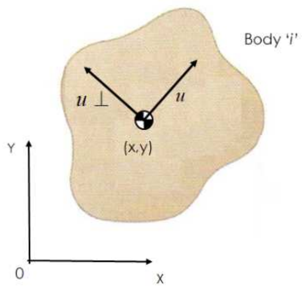

Figure 1. Body 'i' local reference frame.

To test the validity and efficiency of the proposed approach with Fully Cartesian Coordinates two planar mechanisms (a simple pendulum and a slider crank) were selected and modelled with Python 2.7.10, according to the specifications of the collection of benchmark problems available on the Library of Computational Benchmark Problems[4]. In both models the Newton Raphson iterative method and the appended driving constraint method were used to solve the position problem during the kinematic analysis. Each mechanism kinematic and dynamic outcomes were collected and compared with similar results available from the database of the Library of Computational Benchmark Problems. Statistical analysis were performed using Bland Altman Limits of Agreement methodology[5] and Intra Class Correlation (ICC)[6] and the results obtained discussed in face of the modeling assumptions and approach described previously.

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