A model-based corrector approach for explicit co-simulation using subspace identification

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Multiple coupling algorithms specialized in handling strongly coupled systems in explicit co-simulation have been proposed in recent years (e.g. NEPCE [1] with feed-through correction [2], LIS [3], PLC [4]). Almost all of them are based on model knowledge in form of directional derivatives and therefore require a linear representation of the coupled system in the current time step. In principle the FMI for co-simulation 2.0 standard allows the calculation of these directional derivatives [5], but only a few tools commonly used in the automotive industry currently support this feature.

In this paper a new model-based corrector approach for handling stiff coupling loops in explicit co-simulation problems is presented, where one or more subsystems may have direct feed-throughs, but do not form an algebraic loop. In a first step the algorithm is developed assuming exact model knowledge in form of directional derivatives as provided by the FMI 2.0 standard. In a second step the necessary model information is identified at runtime via a MOESP (Multivariable Output Error State Space) subspace identification approach [6] directly form the observed input and output data of the subsystems. Both algorithm types, the one based on exact model knowledge as well as the identification based, are applied to a dual mass oscillator with linear and non-linear spring-damper characteristic [7]. The gained results are compared to several other coupling algorithms for evaluation purposes. It is shown that the model-based corrector approach produces high quality results even for relatively large macro-steps, while having minimal requirements in terms of model interface and tool capabilities.

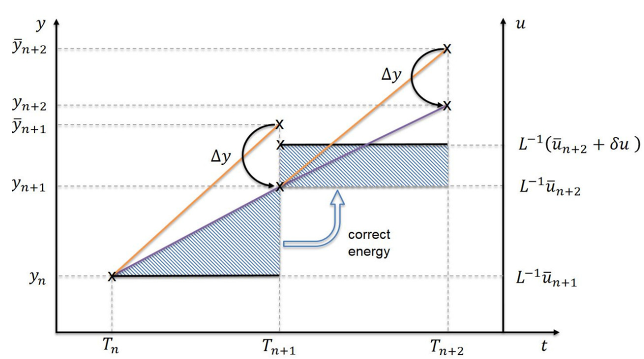

Fig. 1: Basic principle of the model-based corrector approach used with zero-order-hold extrapolation.

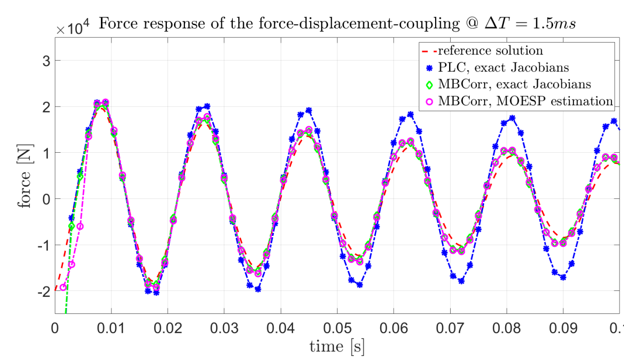

Fig. 2: Performance of the model-based corrector approach compared with PLC for the linear dual mass oscillator with exact and estimated Jacobians.

The basic two step procedure of the model-based corrector approach is depicted in Fig. 1:

1. After every macro-step, estimate the local error Δy in the biased outputs $\bar{y}$ due to the error in the estimated inputs $\bar{u}$ and calculate the corrected outputs $y = \bar{y} + \Delta y$.
2. Account for the error in the subsystem states via an energy correction scheme (see NEPCE [1]) during the next macro-step by applying an offset to the estimated inputs $\bar{u}_{corr} = \bar{u} + \delta u$.

Starting from the coupled system represented as a combined continuous-time state space model

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}, \mathbf{u}), \qquad \mathbf{y} = \mathbf{g}(t, \mathbf{x}, \mathbf{u}), \qquad \mathbf{u} = \mathbf{L} \cdot \mathbf{y} \tag{1}$$

with e.g. the states of all subsystems combined as $\mathbf{x} = \left(\mathbf{x}_1^T, \mathbf{x}_2^T, \ldots\right)^T$ (see [8]), an estimate for the local error of the outputs is derived, which together with the coupling equation can be used to directly calculate the corrected outputs as

$$\mathbf{y}(T_{n+1}) \approx \left[\mathbf{I} - (\mathbf{C}\mathbf{B}_d + \mathbf{D})\,\mathbf{L}\right]^{-1} \left(\overline{\mathbf{y}}\,(T_{n+1}) - (\mathbf{C}\mathbf{B}_d + \mathbf{D}) \cdot \overline{\mathbf{u}}\,(T_{n+1})\right) \tag{2}$$

where $\mathbf{C}$, $\mathbf{D}$ denote the Jacobians $\frac{\partial \mathbf{g}}{\partial \mathbf{x}}$, $\frac{\partial \mathbf{g}}{\partial \mathbf{u}}$ at time T_n, $\mathbf{L}$ denotes the coupling matrix, $\mathbf{I}$ denotes the identity matrix and $\mathbf{B}_d$ denotes the zero order hold discrete time equivalent of $\frac{\partial \mathbf{f}}{\partial \mathbf{u}}$ at time T_n. The necessary input related corrective offset to account for the error in the subsystem states results in

$$\delta\mathbf{u}\,(T_{n+2}) = \alpha \left[\mathbf{L}\left(\frac{\mathbf{y}\,(T_n) + \mathbf{y}\,(T_{n+1})}{2}\right) - \overline{\mathbf{u}}\,(T_{n+1})\right] \tag{3}$$

where α denotes a scaling factor (see [2]). If the mandetory Jacobians in eqn. (2) are not provided they can be estimated using a MOESP subspace identification approach which works on a small batch of previously observed input and output data. The algorithm can either work on a set of micro-steps if supported by the subsystem or fall back to interpolated macro-steps. In contrast to the more common prediction error methods, e.g. least squares estimation of an ARX model [9], subspace identification methods avoid a priori parameterization like number of poles, zeros and dead time, which gets especially tricky for MIMO systems. In addition subspace techniques are inherently MIMO capable, can handle unstable systems and non-zero initial states and are based on robust numerical operations like QR and singular value decomposition (SVD) [10], which makes them ideal for an automated and user friendly coupling algorithm. Apart from the approach presented in this paper, this technique can also be used to support all other model-based coupling algorithms when no model knowledge is available.

As demonstrated in Fig. 2 both versions of the model-based corrector approach (exact and MOESP est. Jacobians) outperform e.g. parallel linear correction (PLC). Furthermore a detailed analysis of the local and global error over a range of macro-step sizes will be outlined, showing that the model-based corrector is especially suited for relatively large macro-steps compared to the coupling signal frequency. In addition the stability region of the model-based corrector approach for variations of the coupling stiffness and damping is compared to other coupling algorithms with similar tool requirements. In this analysis the model-based corrector approach performs best, especially for high stiffness and low damping scenarios.

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