Stability-Limit Analysis of Time-Delayed Systems

Dominik Hamann and Peter Eberhard
Institute of Engineering and Computational Mechanics, University of Stuttgart, Germany, dominik.hamann@itm.uni-stuttgart.de

Machining is an essential and critical part of production processes in industry. Designing efficient processes, reduced cycle times and consequently reduced costs can be attained for instance by high material removal rates. High material removal rates in turn may yield high amplitude vibrations resulting in unacceptable surface quality of the workpieces and additional tool wear or even yield unstable processes resulting in damaged cutting tools and machine tools.

Time-delayed systems are used for mathematical modeling of the surface regeneration of the workpiece which may lead to an instability called chatter. For milling processes these systems are periodic and frequently in addition non-smooth. Periodicity of systems can cause dynamic instabilities as well. Based on Floquet theory, the stability can be analyzed by an eigenvalue analysis of an approximated transition matrix by time discretization. If all eigenvalues of the discrete system are located inside of the unit circle in the complex plane, the system is stable.

Stability lobe diagrams, representing the stability limit in dependency of machining parameters such as spindle speed Ω and axial immersion and the specific cutting-force coefficient H respectively, are employed for the choice of appropriate operating points. For this purpose, the magnitude of the critical eigenvalue λ_1 can be evaluated on a full grid and, subsequently, the implicitly defined stability limit can be interpolated.

Efficient calculation schemes for stability lobe diagrams in the literature are based on bisection [1] or curve tracking as marching squares [2], both acting on regular grids, or continuation methods based on circular search [3, 4], for instance. If the time-delay system is smooth, software packages are available, see [5, 6]. These software packages are capable of continuation methods using Newton's iteration.

There are typical characteristics in stability limits which complicate the calculation: singular points and near-branch zones where the stability limit almost intersects with itself, see Fig. 1 and for a detailed description [4]. However, using the semi-discretization method (SDM) for stability analysis, which is capable of handling non-smoothness, these algorithms in literature are exclusively based on the magnitude of the critical eigenvalue

$$f_1(\Omega,H) = |\lambda_1(\Omega,H)| - 1 = 0 \tag{1}$$

Figure 1 depicts a typical stability lobe diagram of machining processes. Here exemplary a single-degree-of-freedom model is applied with full-immersion, for detailed information see [7]. Beginning on the right-hand side, a Flip lobe can be identified, followed by three Hopf lobes, a Flip lobe and so on. At the stability limit of a Flip lobe, the critical eigenvalue can be located at $\lambda_1 = -1$, in case of a Hopf lobe complex conjugated eigenvalues are located at $|\lambda_{1,2}| = 1$ with $\Im(\lambda_{1,2}) \neq 0$.

At singular points, where two lobes intersect, at least two eigenvalues are located at the unit circle in the complex plane, thus, these points may be defined by two linearly independent equations using for several eigenvalues or the real and the imaginary component of the critical eigenvalue.

In addition, eigenvalue analysis provides eigenvalues and their associated eigenvectors, hence, a specified eigenvalue can be tracked during an operation. For this purpose, the eigenvalue to be tracked can be identified by its associated eigenvector using the modal assurance criterion, for instance.

Furthermore, derivatives of eigenvalues of the approximated transition matrix obtained by semi-discretization can be calculated resulting in

$$\Delta f_i(\Omega,H) = \begin{bmatrix} \frac{\partial\lambda_i(\Omega,H)}{\partial\Omega} & \frac{\partial\lambda_i(\Omega,H)}{\partial H} \end{bmatrix} . \tag{2}$$

Consequently, the calculation of the implicitly defined stability limit

$$F = \{\ (\Omega,H) \mid f(\Omega,H) = 0\ \} \tag{3}$$

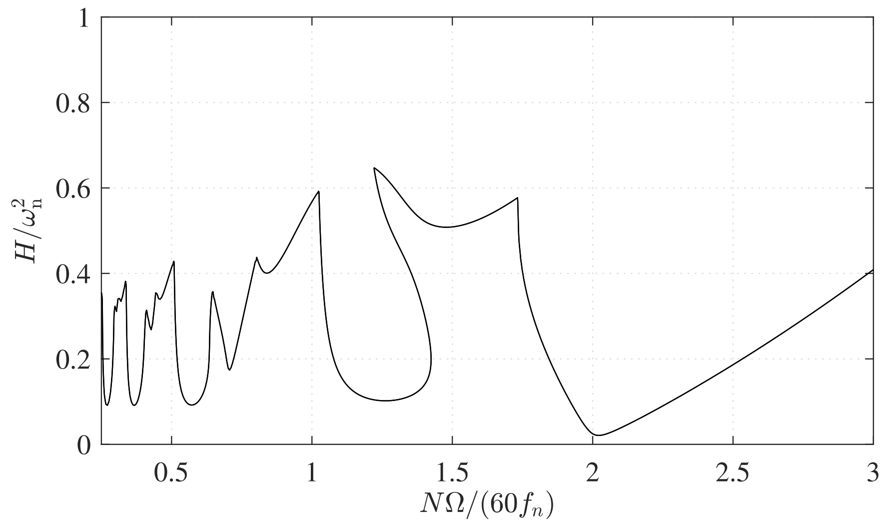

Fig. 1: Stability lobe diagram of full-immersion milling with a single-degree-of-freedom model in the space of the technological parameters spindle speed Ω and specific cutting-force coefficient H, for details see [7]

can be enhanced by the additional information or can be done by another class of methods such as continuation methods using derivatives, for example based on the implicit function theorem [8]. Furthermore, approaches in uncertainty quantification of machining processes, see [9], based on optimization schemes may be enhanced as well.

References

[1] D. Bachrathy and G. Stépán, Bisection Method in Higher Dimensions and the Efficiency Number. Periodica Polytechnica - Mechanical Engineering, Vol. 56, No. 2, pp. 81-86, 2012.

[2] R. Chandler, A Tracking Algorithm for Implicitly Defined Curves. IEEE Computer Graphics and Applications, Vol. 8, No. 2, pp. 82-89, 1988.

[3] J. Morgado and A. Gomes, A Derivative-Free Tracking Algorithm for Implicit Curves with Singularities. Computational Science - ICCS 2004, Lecture Notes in Computer Science, Vol. 3039, pp. 221-228, 2004.

[4] C. Henninger and P. Eberhard, A New Curve Tracking Algorithm for Efficient Computation of Stability Boundaries of Cutting Processes. Journal of Computational and Nonlinear Dynamics, Vol. 4, No. 2, pp. 360–365, 2007.

[5] K. Engelborghs, T. Luzyanina and G. Samaey, DDE-BIFTOOL v. 2.00: a Matlab package for bifurcation analysis of delay differential equations, Technical Report TW330, Department of Computer Science, K.U. Leuven, Leuven, Belgium, 2001.

[6] D. Roose and R. Szalai, Continuation and bifurcation analysis of delay differential equations, in: B. Krauskopf, H.M. Osinga, J. Galan-Vioque (Eds.), Numerical Continuation Methods for Dynamical Systems, New York: Springer, 2007.

[7] T. Insperger and G. Stépán, Semi-Discretization for Time-Delay Systems: Stability and Engineering Applications. New York: Springer, 2011.

[8] S. Krantz and H. Parks, The Implicit Function Theorem: History, Theory, and Applications. Boston: Springer Science & Business Media, 2012.

[9] D. Hamann, N.-P. Walz, A. Fischer, M. Hanss and P. Eberhard, Fuzzy Arithmetical Stability Analysis of Uncertain Machining Systems. Mechanical Systems and Signal Processing, Vol. 98, pp. 534-547, 2018