Optimal Control of Flexible Multibody Systems using the Adjoint Method

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Optimal control problems of multibody systems are often defined for mechanical systems, as e.g. industrial robots, in order to follow a specific trajectory or to increase the overall performance. Modern robot design will include promising lightweight techniques in order reduce mass and energy consumptions in production lines. Therefore, optimal control problems have to be defined for flexible multibody systems in which the flexible components have to be able to describe large deformations during dynamic analysis.

In the present study, the absolute nodal coordinate formulation (ANCF), which has been developed particularly for solving large deformation problems in multibody dynamics [1], is utilized. In contrast to classical nonlinear finite elements in literature, the ANCF does not use rotational degrees of freedom and therefore does not necessarily suffer from singularities emerging from angular parameterizations. The benefits of the ANCF are as well the isoparametric approach and the existence of a consistent displacement field. Moreover, the most essential advantage of the ANCF is the fact that the mass matrix remains constant with respect to the generalized coordinates during the entire dynamic simulation.

The equations of motion of the constrained flexible multibody system can be expressed as a system of differential algebraic equations including the nonlinear elastic force terms in the ANCF. A beam finite element described in the ANCF with bending, axial and shear deformation properties is used which accounts for cross section deformation in order to avoid locking. This proposed element is available and tested extensively in literature, see e.g. [2, 3]. In general, the optimal control problem could be defined as an optimization task described by minimizing a cost function. The gradient of this cost function can be computed very efficiently also in complex multibody systems when incorporating the adjoint method, see e.g. [4] for a detailed derivation of the adjoint equations deduced from the system of differential algebraic equations in index 3 notation. Due to the fact that the ANCF includes a constant mass matrix with vanishing derivative, the equations reduce to a simpler form, also pointed out in [5]. There, as well a gradient-based optimization approach using adjoint equations for flexible ANCF bodies has been presented [5], with special focus on sensitivity analysis. A first and second order adjoint sensitivity analysis in the framework of the ANCF is as well studied in [6] and [7], respectively. In [6], the effect of Young's modulus on elastic deformation of a planar single pendulum is presented. In [7] the dramatically decrease in computational costs for a large number of design variables is shown when comparing the adjoint method and the direct differentiation method. The direct differentiation method and the adjoint method for sensitivity analysis is as well compared in [8]. Moreover, sensitivity analysis for multibody systems formulated on a Lie group using the direct differentiation and the adjoint method can be found in [9].

In contrast to the mentioned literature above, in which the adjoint method is derived for sensitivity analysis for multibody systems with flexible components, in the present paper, the adjoint gradient computation is derived for optimal control problems for flexible multibody systems. The multibody system consisting of rigid and flexible bodies, forces and constraints acting between these bodies can be described by equations of motion in the following form:

$$
\begin{aligned}
\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} &= \mathbf{Q}_{ext}(\mathbf{q},\dot{\mathbf{q}},\mathbf{u},t) + \mathbf{Q}_{elast}(\mathbf{q},t) - \mathbf{C}_{\mathbf{q}}^{\mathsf{T}}(\mathbf{q})\boldsymbol{\lambda} \\
\mathbf{C}(\mathbf{q}) &= \mathbf{0}
\end{aligned}
\tag{1}
$$

Here, $\mathbf{q}$ denotes the vector of generalized coordinates. They are subject to the holonomic constraints $\mathbf{C}(\mathbf{q}) = \mathbf{0}$, which enter the equations of motion via the constraint Jacobian $\mathbf{C_q}$ multiplied by the vector of Lagrange multipliers $\boldsymbol{\lambda}$. The force vector $\mathbf{Q}_{ext}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{u}, t)$ describes the external force vector incorporating a control $\mathbf{u}(t)$ and $\mathbf{Q}_{elast}(\mathbf{q}, t)$ is the elastic force vector accounting for the elastic deformation of the flexible bodies. In the optimal control problem the goal is to determine the control $\mathbf{u}(t)$ which minimizes a cost functional. A cost function can be specified in the general form

$$J = \int_{t_0}^{t_1} h(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{u}, \boldsymbol{\lambda}, t)\, dt \tag{2}$$

in which t_0 and t_1 are the initial and final time of dynamic simulation, respectively. To evaluate the gradient of this cost function the adjoint method is used in this paper following the derivation in [4]. In addition to the derivation in [4], elastic components will be included here and the elastic force vector will be defined in the ANCF following [2].

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