# Generating realistic trajectories for robotic hippotherapy from 3D captured horseback motion 

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Riding a horse is a complex movement that is cognitively and physically demanding. It is established as a rehabilitation method for improving postural stability and locomotion performance, also known as hippotherapy. It has been shown that there is some significant similarity between horseback riding and human walking with the riding motion providing sensory stimuli that have positive rehabilitational effects [1]. To facilitate the application of this treatment in clinical environments or to avoid the necessity of a real horse, robotic hippotherapy systems have recently been developed. A therapy outcome equivalent to classical hippotherapy requires the robotic horse trajectories to be as close to reality as possible.

In order to generate smooth, cyclic and realistic trajectories effectively executable by robotic systems, this work focuses on the synthesis of the horseback motion during typical horse gaits based on 3D motion capture data. The task consists in finding the translation and rotation of the horseback, regarded as a rigid body representing the saddle of a robotic hippotherapy system, during motion. Raw 3D measurements of marker coordinates were acquired with a marker-based motion capture system for the horse gaits walk, trot and gallop. Marker data of the horse in steady standing position were used as reference defining the shape of the horseback as a rigid body. In a pre-processing step the raw data of the standing horse are aligned with the axes of a reference coordinate frame, defining the reference pose of the horseback. Considering the marker placement on the horses it is assumed that, while the horse is moving, some markers have a bigger relative movement with respect to their respective reference position than others, which is contrary to the idea of modeling the horseback as a rigid body. Therefore a marker selection based on the positional variance of the markers during the respective horse movement is performed. The result for each horse gait is the data set $\left\{\mathbf{r}_{1}, \ldots, \mathbf{r}_{n}\right\}$ of $n$ marker positions in the steady reference pose of the horse and a sequence of $N$ data sets $\left\{\mathbf{p}_{1, l}, \ldots, \mathbf{p}_{n, l}\right\}, l=1, \ldots, N$ (one for each of the $N$ snapshots during the respective gait measurement). Figure 1 shows the markers attached to the horse with their respective positional variance for the data set of the walking motion, the reference marker positions, as well as the resulting selected markers (filled circles).


Fig. 1: (a) Arrangement of the 3D markers (colored dots) on the horse and positional variance of the individual markers (dark color indicates high variance, light color indicates low variance) for the walking motion and (b) selected (filled) and ignored (empty) markers based on the variance and a threshold value of this data set.

In the next step, for each snapshot of the respective gait data set, the pose of the horseback (as rigid body) relative to the reference pose is determined in a least squares sense. This amounts to find a rigid body transfor-
mation matrix $\left(\mathbf{R}_{l}, \mathbf{t}_{l}\right) \in S E(3)$, i.e. an 'optimal' rotation $\mathbf{R}_{l}$ and translation $\mathbf{t}_{l}$ mapping the marker coordinates $\left\{\mathbf{p}_{1, l}, \ldots, \mathbf{p}_{n, l}\right\}$ of frame $l$ to the marker coordinates $\left\{\mathbf{r}_{1}, \ldots, \mathbf{r}_{n}\right\}$ of the reference pose, so that (with weights $w_{i}$ )

$$
\begin{equation*}
\left(\mathbf{R}_{l}, \mathbf{t}_{l}\right)=\underset{\left(\mathbf{R}_{l}, \mathbf{t}_{l}\right) \in S E(3)}{\arg \min } \sum_{i=1}^{n} w_{i}\left\|\left(\mathbf{R}_{l} \mathbf{p}_{i, l}+\mathbf{t}_{l}\right)-\mathbf{r}_{i}\right\|^{2} \tag{1}
\end{equation*}
$$

A closed form solution of Eq. (1), also known as weighted orthogonal Procrustes problem [2], is described in [3, 4]. With the weighted centroids $\overline{\mathbf{p}}_{l}$ and $\overline{\mathbf{r}}$, the solution is given as

$$
\begin{align*}
\mathbf{S}_{l} & =\mathbf{U}_{l} \boldsymbol{\Sigma}_{l} \mathbf{V}_{l}^{\top}  \tag{2}\\
\mathbf{R}_{l} & =\mathbf{V}_{l}\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \operatorname{det}\left(\mathbf{V}_{l} \mathbf{U}_{l}^{\top}\right)
\end{array}\right] \mathbf{U}_{l}^{\top}  \tag{3}\\
\mathbf{t}_{l} & =\overline{\mathbf{r}}-\mathbf{R}_{l} \overline{\mathbf{p}}_{l} \tag{4}
\end{align*}
$$

where $\mathbf{S}_{l}:=\mathbf{X}_{l} \mathbf{W} \mathbf{Y}^{\top}$ is the covariance matrix of the centered vectors $\mathbf{x}_{i, l}=\mathbf{p}_{i, l}-\overline{\mathbf{p}}_{l}$ and $\mathbf{y}_{i}=\mathbf{r}_{i}-\overline{\mathbf{r}}$. Via singular value decomposition of $\mathbf{S}_{l}$, as in Eq. (2), the optimal rotation and translation are found with Eq. (3) and Eq. (4). With the sequence of poses $\left\{\left(\mathbf{R}_{1}, \mathbf{t}_{1}\right), \ldots,\left(\mathbf{R}_{N}, \mathbf{t}_{N}\right)\right\}$ a raw trajectory of the horseback motion can be extracted from the movement data, following above method. The translation of the horseback is described as the movement in $x, y$ and $z$-direction, the orientation is described using Cardan angles following the rotation sequence $\mathrm{X}-\mathrm{Y}^{\prime}-\mathrm{Z} \mathrm{Z}^{\prime}$. In order to generate smooth, cyclic motion trajectories executable by robotic systems the position and orientation components are approximated by Fourier series. Figure 2 shows the components of the approximated trajectory of one movement cycle of the walking motion.
The presented approach provides an easy and robust method to synthesize horseback motion and to generate smooth, cyclic and realistic trajectories for robotic hippotherapy. Based on the trajectories generated with this approach, future research will comprise the design and simulation of a robotic hippotherapy system.


Fig. 2: Generated trajectory for one single movement cycle of the walking motion, (a) position in $x$ (dashed blue line), $y$ (dot-dashed red line) and $z$ (solid yellow line) direction and (b) orientation described as Cardan angles $\alpha$ (dashed blue line), $\beta$ (dot-dashed red line) and $\gamma$ (solid yellow line).

## References

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