Nonlinear Navigation System Design
with Application to Autonomous Vehicles

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1. Introduction
   - Thesis outline
   - Main contributions

2. Kalman Filter Based Navigation Systems
   - INS/GPS aided by selective frequency contents
   - Embedded UAV model for INS aiding
   - Nonlinear complementary Kalman filter

3. Lyapunov Based Navigation System Design
   - Attitude and position nonlinear observers
   - Combination of Lyapunov and density functions for nonlinear observer stability analysis
   - Nonlinear GPS/IMU navigation system

4. Conclusions and Future Work
The proposed navigation systems are derived using two methodologies:
- Kalman filtering techniques;
- Lyapunov and density functions stability theory.
Introduction
Main contributions: Kalman Filtering

Main contributions in the field of navigation systems based on Kalman filtering

(A) Advanced aiding techniques for EKF/INS navigation systems
- modelling frequency contents of vector readings,
- integrating efficiently the dynamic of the vehicle.

(B) Nonlinear complementary Kalman filters
- endowed with stability and performance properties,
- combining stochastic approach and frequency domain design.
Introduction
Main contributions: Nonlinear Observers

- Main contributions in the field of nonlinear observers
  - **(A)** Nonlinear attitude and position observers for inertial systems
    - with diverse aiding sensors.
    - endowed with stability properties for worst-case errors.
  - **(B)** New stability analysis tools
    - based on the combination of density and Lyapunov functions,
    - for almost GAS and almost ISS analysis of nonlinear systems.
    - that eventually yield almost ISS of the proposed observers.
Presentation Outline

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4. Conclusions and Future Work
INS/GPS aided by selective frequency contents
EKF/INS navigation system architecture

- High-accuracy, multi-rate INS computes position, velocity and attitude based on the inertial sensor readings.
- Gravity Selective Frequency Contents are introduced in the EKF for error estimation and compensation.
- Direct-feedback error compensation.
INS/GPS aided by selective frequency contents
Gravity Frequency Contents: Pendular Measurement

\[ B \mathbf{a}_{SF} = B \mathbf{g} + \mathbf{a}_{LA} + \boldsymbol{\omega} \times B \mathbf{v} \]

- Pendular measurements are distorted by accelerations:
  - **Coriolis terms** can be compensated by the INS estimates,
  - **linear accelerations** can be modeled as bandpass signals for terrestrial and oceanic vehicles.

- The **linear acceleration component** is characterized in the frequency domain using the EKF state model.
The pendular measurement is a function of the EKF states.

\[
\delta z_g = B \hat{g} - g_r = -a_{LA} + (\omega) \times R' \delta v + \delta b_a + (B v) \times \delta b_\omega
\]

\[
+ [R' (E g) \times + (\omega) \times (B v) \times R'] \delta \lambda - n_a - (B v) \times n_\omega
\]

The integration of the pendular measurement \( \delta z_g \) in the EKF enhances bias compensation and roll estimation.
Embedded UAV model for INS aiding
Vehicle Model Aiding Techniques

- The **external vehicle dynamics** (VD) aiding is implemented using a standalone simulator.
- The INS and VD error derivation, estimation, and compensation techniques are similar.
• The **embedded vehicle dynamics** (VD) are integrated directly in the EKF architecture.

• The vehicle model equations are computed using the inertial estimates.

• The computational load of VD aiding is reduced and the implementation flexibility is added, while retaining the accuracy enhancements of VD aiding.
The embedded VD aiding technique was validated for a nonlinear dynamic model of the Vario X-Treme R/C Helicopter.

The helicopter dynamics are described using a six degree of freedom rigid body model that includes the effects of the main rotor, Bell-Hiller stabilizing bar, tail rotor, fuselage, horizontal tail plane, and vertical fin.
Embeddred UAV model for INS aiding

Simulation Results

- Bias and velocity estimation results are enhanced.
- The application of the embedded VD aiding to a complex and nonlinear Vario X-Treme helicopter model validates the proposed technique.
Embedded UAV model for INS aiding
Simulation Results

<table>
<thead>
<tr>
<th>Aiding information</th>
<th>GPS Aided</th>
<th>External VD Aided</th>
<th>Embedded VD Aided</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>δp</td>
</tr>
<tr>
<td>GPS</td>
<td>2.42</td>
<td>3.50 × 10^{-1}</td>
<td>1.26 × 10^{-2}</td>
</tr>
<tr>
<td>Vario X-Treme Model</td>
<td>3.07 × 10^{-1}</td>
<td>2.84 × 10^{-2}</td>
<td>1.27 × 10^{-2}</td>
</tr>
<tr>
<td>Vario X-Treme Model (z_v only)</td>
<td>5.91 × 10^{-1}</td>
<td>3.20 × 10^{-2}</td>
<td>1.26 × 10^{-2}</td>
</tr>
</tbody>
</table>

- The embedded VD aiding is computationally more efficient than the classical VD aiding.

- The VD linear velocity aiding yields about the accuracy of the full VD aiding and has a negligible computational cost with respect to GPS aiding.
The navigation system is cascade architecture of an attitude and a position complementary filters.

The proposed system exploits the sensor information over **complementary frequency regions**.

The attitude and position filters are endowed with **stability and performance** properties.

**Multirate architecture** is obtained by a synthesis methodology based on periodic systems.
Nonlinear complementary Kalman filter
Sensor fusion

<table>
<thead>
<tr>
<th></th>
<th>Position Filter</th>
<th>Attitude Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Frequency</td>
<td>GPS</td>
<td>Vector Observations</td>
</tr>
<tr>
<td>High Frequency</td>
<td>Accelerometer Integration</td>
<td>Rate Gyro Integration</td>
</tr>
</tbody>
</table>

- The complementary filters combine sensor information in the frequency domain.
- The feedback gains can be designed using a stochastic characterization of the sensors.
Nonlinear complementary Kalman filter

Attitude complementary filter

- The gains $K_{1\lambda}$, $K_{2\lambda}$ are computed for the LTI system

\[
\begin{bmatrix}
    x_{\lambda k+1} \\
    x_{b k+1}
\end{bmatrix}
= \begin{bmatrix}
    I & -T \bar{I} \\
    0 & I
\end{bmatrix}
\begin{bmatrix}
    x_{\lambda k} \\
    x_{b k}
\end{bmatrix}
+ \begin{bmatrix}
    -T \bar{I} & 0 \\
    0 & I
\end{bmatrix}
\begin{bmatrix}
    w_{\omega_r k} \\
    w_{b k}
\end{bmatrix},
\]

\[
y_{\lambda k} = \begin{bmatrix}
    I & 0
\end{bmatrix}
\begin{bmatrix}
    x_{\lambda k} \\
    x_{b k}
\end{bmatrix}
+ v_{\lambda k}.
\]

- The attitude complementary filter is uniformly asymptotically stable for bounded roll ($\theta<\pi$).

- The complementary filter is the steady-state Kalman filter for the attitude kinematics, assuming constant roll and pitch.

- Performance degradation for time-varying roll and pitch is small.
Nonlinear complementary Kalman filter
Position complementary filter

The gains $K_{1p}$, $K_{2p}$ are computed for the LTI system

$$
\begin{bmatrix}
x_{p\,k+1} \\
x_{v\,k+1}
\end{bmatrix} =
\begin{bmatrix}
I & T I \\
0 & I
\end{bmatrix}
\begin{bmatrix}
x_{p\,k} \\
x_{v\,k}
\end{bmatrix} +
\begin{bmatrix}
I & -T^2 \\
0 & -T
\end{bmatrix}
\begin{bmatrix}
w_{p\,k} \\
w_{v\,k}
\end{bmatrix},
$$

$$
y_{x\,k} =
\begin{bmatrix}
I & 0
\end{bmatrix}
\begin{bmatrix}
x_{p\,k} \\
x_{v\,k}
\end{bmatrix} + v_{p\,k}.
$$

The position complementary filter is uniformly asymptotically stable and is identified with the steady-state Kalman filter for the position kinematics with isotropic accelerometer noise.
Nonlinear complementary Kalman filter
Experimental results: DELFIMx
Nonlinear complementary Kalman filter
Experimental results: Time domain

![Diagram of Trajectory Estimation](image1)

![Diagram of Body Velocity Estimation](image2)

![Diagram of Yaw Estimation](image3)

![Diagram of Pitch and Roll Estimation](image4)
Nonlinear complementary Kalman filter
Experimental results: Frequency domain

<table>
<thead>
<tr>
<th>AIDING SENSOR</th>
<th>INERTIAL SENSOR INTEGRATION</th>
<th>FILTER ESTIMATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Pendulum)</td>
<td>(Rate Gyro)</td>
<td>$\hat{\theta}$</td>
</tr>
<tr>
<td>(GPS)</td>
<td>(Accelerometer)</td>
<td>$\hat{p}_x$</td>
</tr>
</tbody>
</table>

Experimental results: Frequency domain
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4. Conclusions and Future Work
Problem formulation: landmark readings

Objective (Landmark based observer)
Estimate attitude ($\hat{R}$) and position ($\hat{p}$) of a rigid body using landmark observations ($q_r$) and non-ideal angular and linear velocity measurements ($\omega_r$ and $v_r$, respectively).

Rigid Body Kinematics
\[
\dot{R} = R(\omega) \times, \quad \dot{B}p = Bv - (\omega) \times Bp
\]

Velocity Measurements
\[
\omega_r = \omega + b_\omega, \quad v_r = Bv + b_v
\]

Landmark Measurements
\[
q_{ri} = \mathcal{R}^Lx_i - Bp
\]
Problem formulation: vector observations

Objective (Vector based attitude observer)
Estimate attitude ($\hat{R}$) of a rigid body using vector observations ($h_r$) and non-ideal angular velocity measurements ($\omega_r$).

Rigid Body Kinematics

$$\dot{R} = R(\omega) \times$$

Velocity Measurements

$$\omega_r = \omega + b_\omega$$

Body Frame Vectors (measured)

$$H_r = \begin{bmatrix} h_{r1} & \ldots & h_{rn} \end{bmatrix}$$

$$= \begin{bmatrix} R h_1 & \ldots & R h_n \end{bmatrix} = R' H.$$

Reference Vectors (known)

$$H = \begin{bmatrix} L h_1 & \ldots & L h_n \end{bmatrix},$$

$$\{L\} \rightarrow \text{local frame}$$

$$\{B\} \rightarrow \text{body frame}$$
Observer Design Methodology

- **Define** a Lyapunov function $V(x)$ based on the observation error of the landmarks.
- **Derive** a feedback law such that $\dot{V}(x) \leq 0$.
- **Analyze** the stability properties of the closed loop system using suitable Lyapunov based tools.

Design Issues

- **Topological obstacles** to global stability on manifolds, namely $SE(3)$. Relaxation to **almost global stability** and **almost ISS** frameworks is adopted.
- The feedback law must be a **function of the sensor readings**. The true position and orientation are unknown.
- Velocity readings can be distorted by **bias** and/or **noise**.
Lyapunov Based Navigation System Design

Observer design: Landmark based observer

• The **synthesis Lyapunov Function** is a linear combination of landmark readings and bias estimation errors

\[ V_b = \gamma_\varphi \sum_{i=1}^{n-1}\|B\hat{u}_i - B\hat{u}_i\|^2 + \gamma_p\|B\hat{u}_n - B\hat{u}_n\|^2 + \gamma_{b_\omega}\tilde{b}_\omega \tilde{b}_\omega + \gamma_{b_v}\tilde{b}_v \tilde{b}_v \]

where

\[ B\hat{u}_{j=1..(n-1)} = \sum_{i=1}^{n-1} a_{ij}(q_{i+1} - q_i), \quad B\hat{u}_n = -\frac{1}{n}\sum_{i=1}^{n} q_i. \]

• The obtained **observer kinematics** are given by

\[ \hat{\mathbf{R}} = \hat{\mathbf{R}}(\hat{\omega}) \times, \quad B\hat{p} = B\hat{v} - (\hat{\omega}) \times B\hat{p}, \]

\[ \hat{b}_\omega = \gamma_b(\gamma_\varphi s_\omega - \gamma_p (B\hat{p}) \times s_v), \quad \hat{b}_v = \frac{\gamma_p}{\gamma_b} s_v, \]

where

\[ \hat{\omega} = \omega_r - \hat{b}_\omega - k_{\omega}s_\omega, \quad B\hat{v} = v_r - \hat{b}_v + \left( (\omega_r - \hat{b}_\omega) \times - k_v I \right) s_v + k_{\omega}(B\hat{p}) \times s_\omega, \]

\[ s_\omega = \sum_{i=1}^{n} (\hat{\mathbf{R}}'XD_XA_Xe_i) \times (QD_XA_Xe_i), \quad s_v = B\hat{p} + \frac{1}{n}\sum_{i=1}^{n} q_i, \]

that are a function of the sensor readings (**output feedback**).
Lyapunov Based Navigation System Design

Observer design: Vector based observer

- The **synthesis Lyapunov Function** is a linear combination of vector readings and bias estimation error

\[ V_b = \sum_{i=1}^{n-1} \| B \hat{u}_i - B u_i \|^2 + \gamma_{b\omega} \hat{b}_\omega \hat{b}_\omega \]

where \( B u_j = \sum_{i=1}^{n} a_{ij} h_i \).

- The obtained **observer kinematics** are given by

\[ \dot{\hat{R}} = \hat{\omega} \times, \quad \dot{\hat{b}}_\omega = k_{b\omega} s_\omega, \]

where

\[ \hat{\omega} = \hat{\omega} \omega' H A_H A'_H H'_r \left( \omega_r - \hat{b}_\omega \right) - k_\omega s_\omega, \quad s_\omega = \sum_{i=1}^{n} (\hat{\omega} H A_H e_i) \times (H_r A_H e_i), \]

that are a function of the sensor readings (**output feedback**).
Lyapunov Based Navigation System Design

Observer results

**Observer Properties**

- **Almost GAS with exponential convergence**
  Almost all initial conditions converge to the origin for ideal velocity measurements.

- **Dynamic bias estimation with exponential convergence for worst-case initial conditions**
  Almost all initial conditions converge to the origin for the case of biased velocity readings.

- **Output feedback formulation**
  The derived feedback law is an explicit function of the sensor measurements.

- **Sensor setup characterization**
  Necessary and sufficient landmark/vector geometry for pose estimation is derived.
Combination of Lyapunov and density functions
Almost ISS analysis

Analysis Method
Obtain almost ISS by combining local ISS with weakly almost ISS.

**Step 1 (Local ISS)**
Find a region $\gamma_1(\|u\|_{\infty}) < |x(t)| < r$ where $\dot{V} < 0$, yielding
$$\forall u \forall |x(t)| < r \limsup_{t \to \infty} |x(t)| \leq \gamma_1(\|u\|_{\infty}).$$

**Step 2 (Weakly almost ISS)**
Find a density function $\rho$ such that $\text{div}(\rho f) > 0$ for $|x(t)| > \gamma_2(\|u\|_{\infty})$, which implies
$$\forall u \forall a.a. x(t_0) \liminf_{t \to \infty} |x(t)| \leq \gamma_2(\|u\|_{\infty}).$$

**Proposition (Almost ISS)**
If the system is locally ISS and weakly almost ISS, then the system is almost ISS, with
$$\forall u \forall a.a. x(t_0) \limsup_{t \to \infty} |x(t)| \leq \gamma_1(\|u\|_{\infty}).$$
Combination of Lyapunov and density functions
Almost GAS analysis

Analysis Method
Given the largest invariant set $M$ in $\{x : \dot{V}(x) = 0\}$, a density function $\rho$ can show that $M \setminus \{0\}$ is unstable, yielding aGAS of $\{0\}$.

The stability of an equilibrium point can be excluded using a density function analysis.

Theorem 1
Suppose there exists a density function $\rho : \mathbb{R}^n \rightarrow \mathbb{R}^+$ that is $C^1$ and integrable in a neighborhood $U$ of $x_u \in \mathbb{R}^n$. If $\text{div}(\rho f) > 0$ in $U$ then the global inset of $\mathcal{X}_U$ has zero measure.

Almost GAS of the origin is obtained by combining Theorem 1 with LaSalle's invariance principle.

Proposition
Let the $M$ be the largest invariant set in $\{x : \dot{V}(x) = 0\}$ and assume that $M$ is a countable union of isolated points. If there is a density function $\rho$ that satisfies the conditions of Theorem 1 for all $x_u \in M \setminus \{0\}$, then the origin is almost GAS.
Combination of Lyapunov and density functions
Application to Nonlinear Observers

- If rate gyro noise is considered, the proposed stability analysis tools show that the nonlinear attitude observer is almost ISS with respect to \( \dot{\mathbf{R}} = \mathbf{I} \), that is \( \forall u \forall a.a. \mathcal{R}(t_0) \)

\[
\limsup_{t \to \infty} \|\mathbf{I} - \mathcal{R}(t)\|^2 \leq \gamma_1(\|u\|_\infty).
\]

- Almost GAS of a nonlinear observer with biased velocity readings is obtained, namely almost all the trajectories of the system

\[
\dot{\theta} = -\sin(\theta) + b, \quad \dot{b} = -\sin(\theta)
\]

converge to the set

\[
\{(\theta, b) = (2\pi k, 0), k \in \mathbb{Z}\}.
\]
Nonlinear GPS/IMU navigation system
Problem formulation

**Objective**
Estimate attitude ($\hat{\mathbf{R}}$) and position ($\hat{\mathbf{p}}$) of a rigid body using pseudorange observations ($\rho_{ij}$) and non-ideal angular velocity and acceleration measurements ($\mathbf{\omega}_r$ and $\mathbf{a}_r$, respectively).

**Rigid Body Kinematics**
\[
\dot{\mathbf{R}} = \mathbf{R} (\mathbf{\omega}) x, \quad \dot{\mathbf{p}} = \mathbf{v}, \quad \dot{\mathbf{v}} = \mathbf{a}
\]

**Inertial Measurements**
\[
\mathbf{\omega}_r = \mathbf{\omega} + \mathbf{b}_\mathbf{\omega} + \mathbf{n}_\mathbf{\omega},
\]
\[
\mathbf{a}_r = \mathbf{R}^T (\mathbf{a} - \mathbf{g}) + \mathbf{n}_\mathbf{a}
\]

**Pseudorange Measurements**
\[
\rho_{ij} = \|\mathbf{p}_j - \mathbf{p}_{Si}\| + b_c
\]
(satellite $i$, receiver $j$)
The navigation system is described by a cascade of an attitude observer and a position observer.

Three GPS receivers onboard the vehicle are necessary for attitude estimation. In alternative, vector readings can be adopted.
Lyapunov Based Navigation System Design
GPS/INS observer results

GPS/INS Observer Properties

- **Almost GAS with exponential convergence**
  Almost all initial conditions converge to the origin for ideal inertial measurements.

- **Dynamic bias estimation with exponential convergence for worst-case initial conditions**
  Almost all initial conditions converge to the origin for the case of biased velocity readings.

- **Almost ISS with inertial sensor noise**
  The estimation errors converge to a known neighborhood of the origin.

- **Output feedback formulation**
  The derived feedback law is an explicit function of the sensor measurements.
Lyapunov Based Navigation System Design

GPS/INS simulation results
Conclusions and Future Work

**Performance driven design**
- High-accuracy navigation systems;
- Kalman filter based architectures;
- Integration of advanced aiding techniques;
- Accounts for a variety of sensor non-idealities and disturbances.

**Stability driven design**
- Almost global stabilization;
- Guarantees robustness to some sensor non-idealities;
- Accounting for extra non-idealities implies system redesign.
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