Onboard 3D Position Estimation of a Single Quadcopter Vehicle

By

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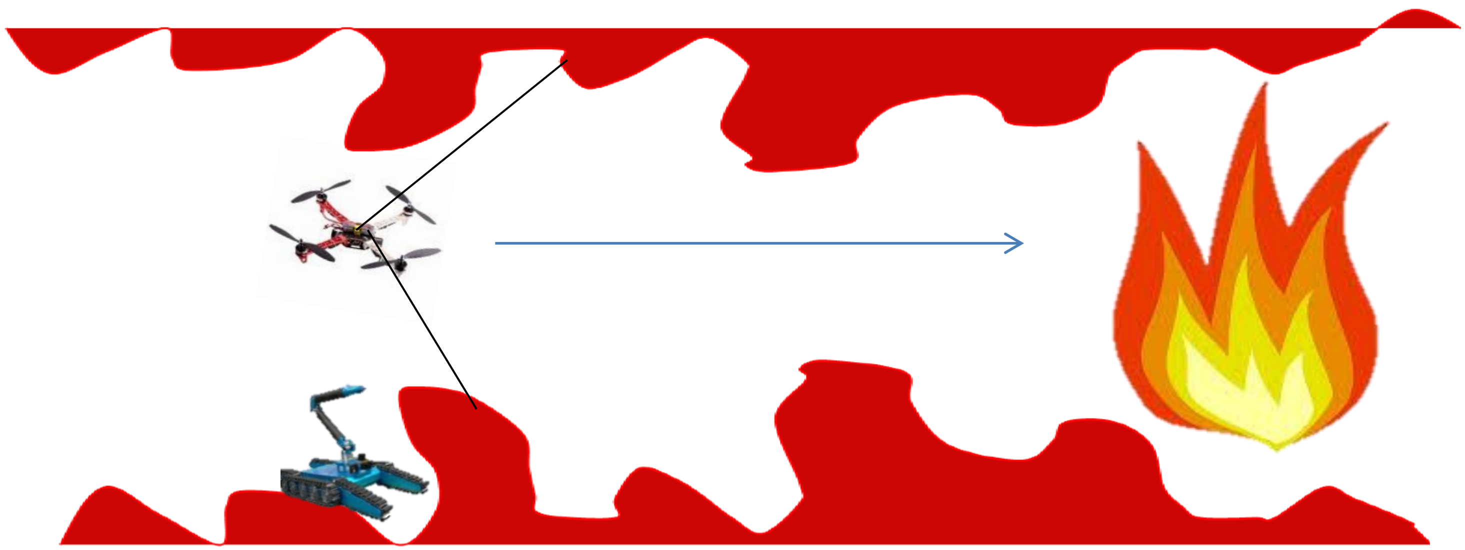
Outline

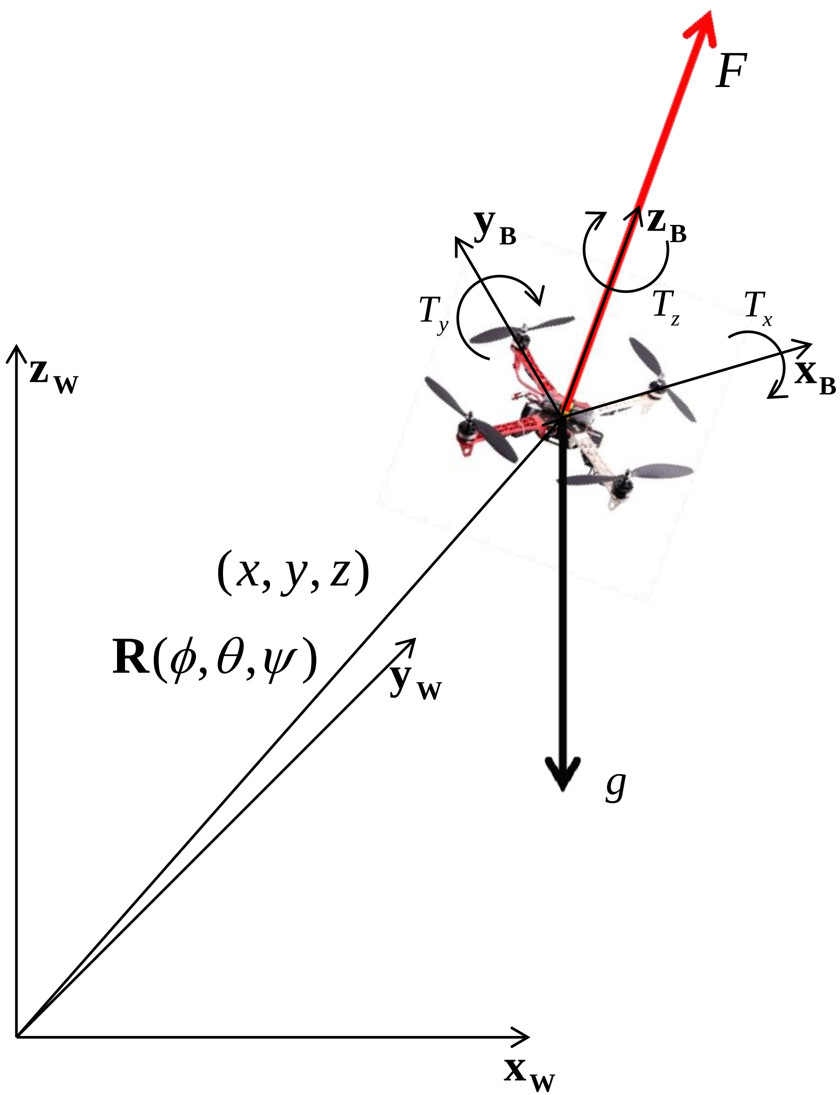
- Motivation
- Quadcopter dynamics and sensors
- Challenges / Objectives
- Proposed localization system
- Results

Why UAVs?

No GPS



Quadcopter Dynamics



$$\begin{bmatrix} \ddot{x}(t) \\ \ddot{y}(t) \\ \ddot{x}(t) \end{bmatrix} = F(t)\mathbf{R}(\phi, \theta, \psi)\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - g\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Attitude known assumed known

Estimated using auto-pilot programs on-board
The quadcopter:

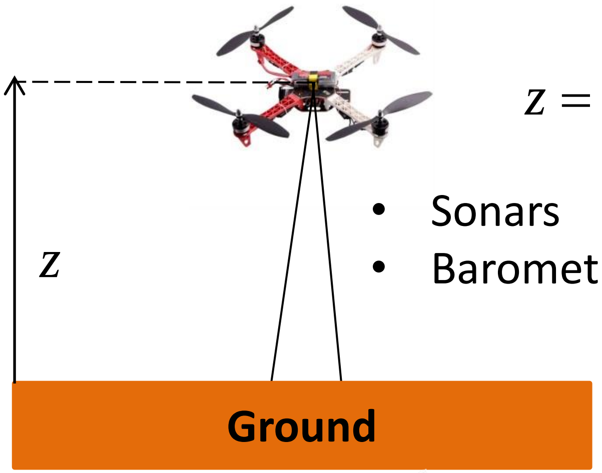
Dynamics:

$$\begin{bmatrix} \ddot{\phi}(t) \\ \ddot{\theta}(t) \\ \ddot{\psi}(t) \end{bmatrix} = f\left(\begin{bmatrix} T_x(t) \\ T_y(t) \\ T_z(t) \end{bmatrix}\right)$$

Observations:

Onboard IMU
Sensor

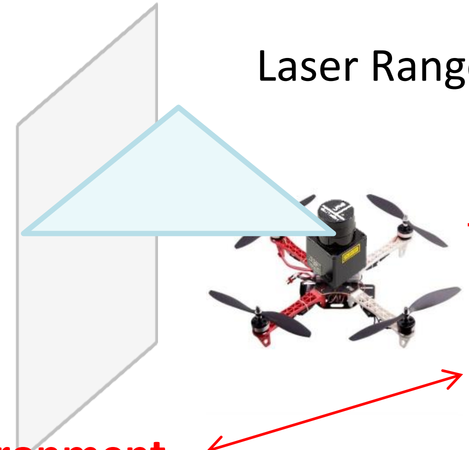
Sensors for Quadcopters



$$z = \begin{bmatrix} 0 & 0 & 1 & \mathbf{0}_3 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{v} \end{bmatrix}$$

- Sonars
- Barometric sensors

Laser Range Finder



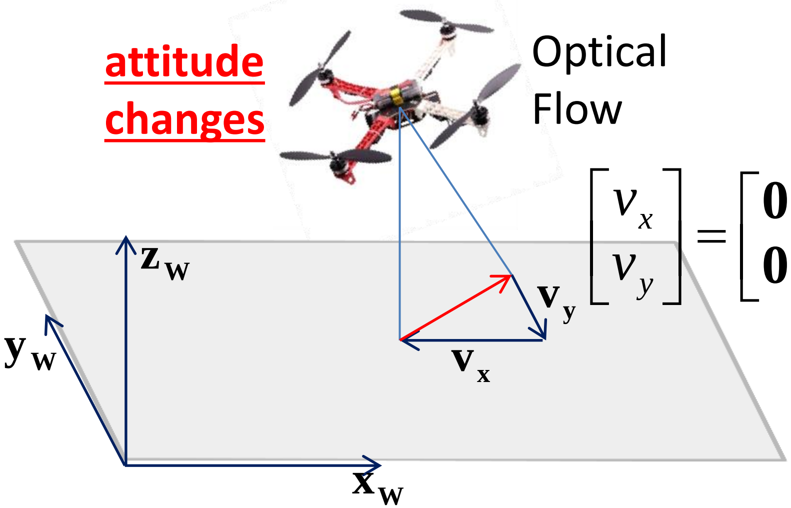
heavy

attitude changes

environment unpredictability

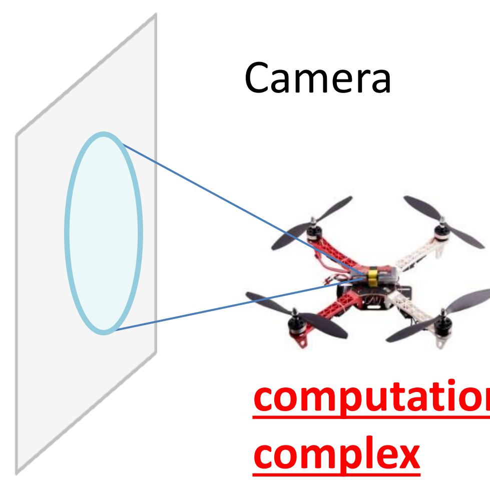
Optical Flow

attitude changes



$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \mathbf{0}_3 & 1 & 0 & 0 \\ \mathbf{0}_3 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{v} \end{bmatrix}$$

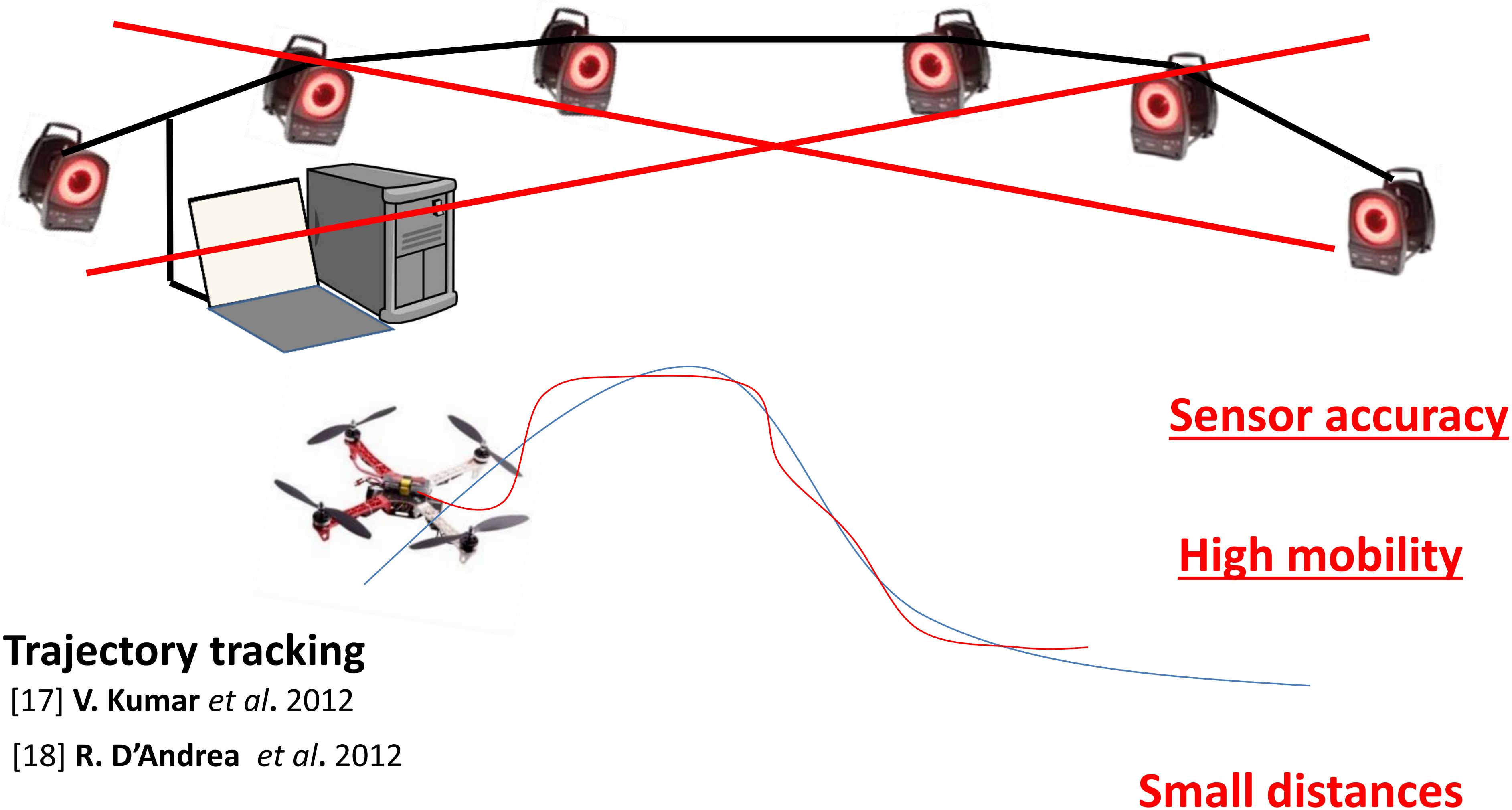
Camera



computationaly complex

[21] **V. Kumar** *et. al*. Robotics Automation Magazine, 2012

Challenge



Trajectory tracking

[17] **V. Kumar** *et al*. 2012

[18] **R. D'Andrea** *et al*. 2012

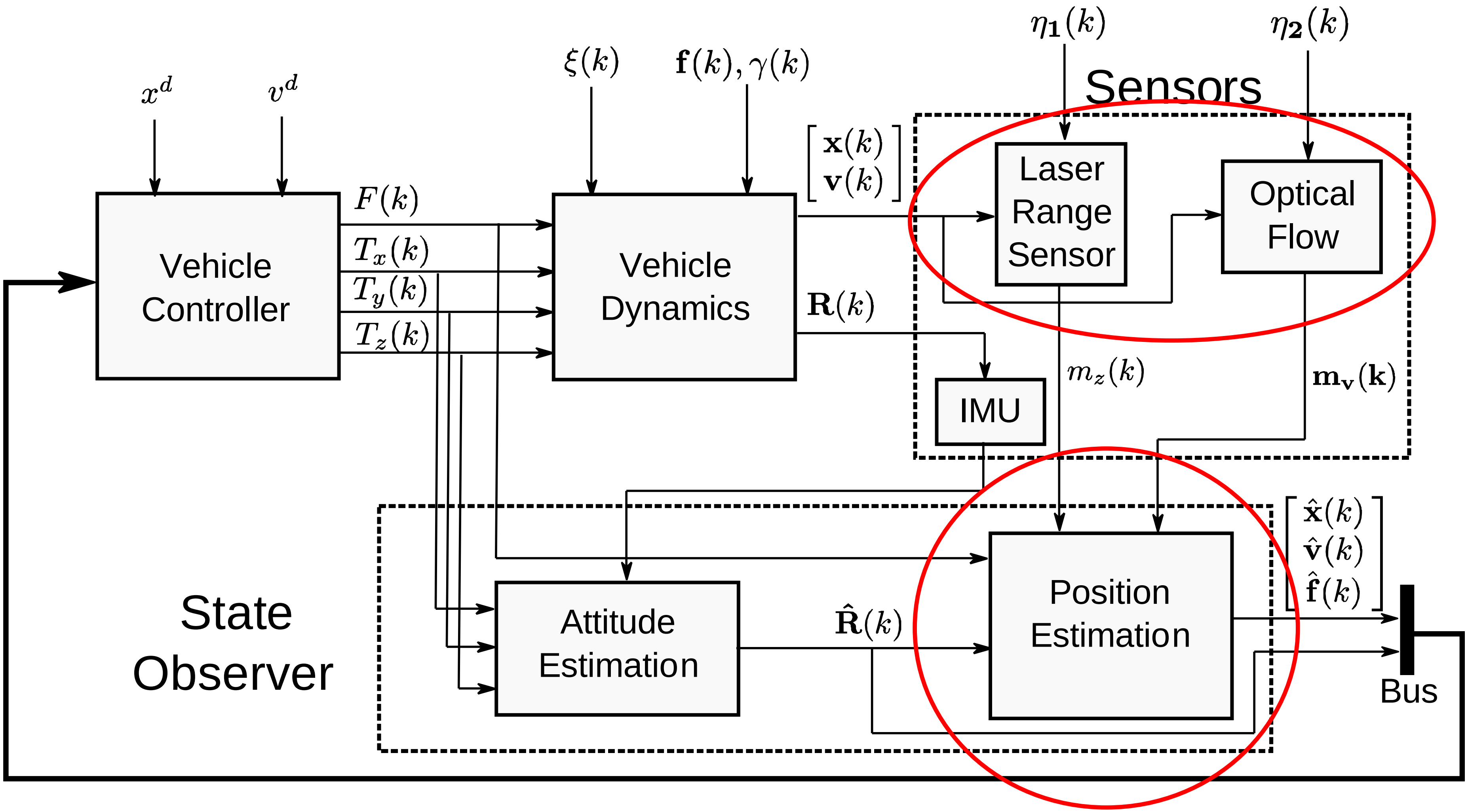
Path Following

[19] **A. Pascoal** *et al*. 2012

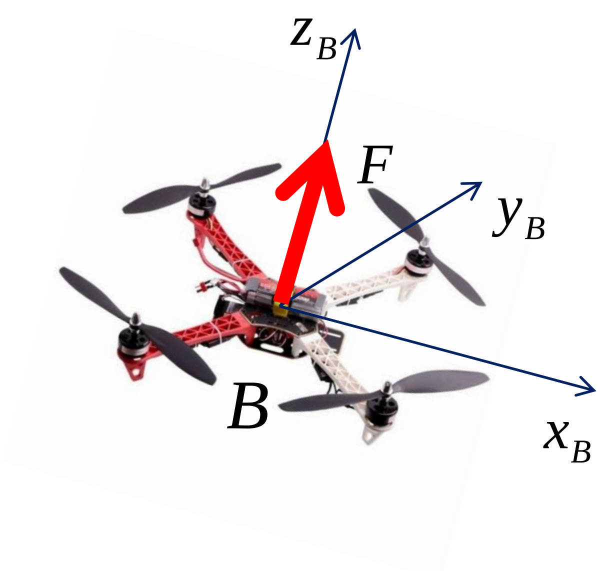
Objective

- **Propose an autonomous localization method deployable onboar the quadcopter vehicle:**
  - Onboard Sensor Setup
  - Estimation Method

Localization System



Motion Model



Inertial Navigation Model:

$$\boxed{\mathbf{x}(k+1)} = \mathbf{x}(k) + \mathbf{v}(k)\Delta t + 0.5\Delta t^2 \mathbf{a}(k+1)$$

$$\boxed{\mathbf{v}(k+1)} = \mathbf{v}(k) + \Delta t \mathbf{a}(k+1)$$

$$\mathbf{a}(k+1) = F(k+1)\mathbf{R}(k)\mathbf{z_W} - g\mathbf{z_W} \boxed{+\mathbf{f}(k) + \boldsymbol{\xi}(k+1)}$$

Unmodeled aerodynamics

Model identification uncertainty

$$\boxed{\mathbf{f}(k+1)} = \mathbf{f}(k) + \boldsymbol{\gamma}(k+1)$$

[20] **R. D'Andrea** *et. al*. 2012.

[13] **V. Kumar** *et. al*. 2010

Motion Model

Model

$$\begin{bmatrix} \hat{\mathbf{x}}(k+1) \\ \hat{\mathbf{v}}(k+1) \\ \hat{\mathbf{f}}(k+1) \end{bmatrix} = \mathbf{A}\begin{bmatrix} \hat{\mathbf{x}}(k) \\ \hat{\mathbf{v}}(k) \\ \hat{\mathbf{f}}(k) \end{bmatrix} + \mathbf{B}(F(k+1)\hat{\mathbf{R}}(k)\mathbf{z}_{\mathrm{w}} - g\mathbf{z}_{\mathrm{w}}) + \mathbf{W}\begin{bmatrix} \xi(k) \\ \gamma(k) \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{I}_{3x3} & dt\mathbf{I}_{3x3} & 0.5dt^2\mathbf{I}_{3x3} \\ \mathbf{0}_{3x3} & \mathbf{I}_{3x3} & dt\mathbf{I}_{3x3} \\ \mathbf{0}_{3x3} & \mathbf{0}_{3x3} & \mathbf{I}_{3x3} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0.5dt^2\mathbf{I}_{3x3} \\ dt\mathbf{I}_{3x3} \\ \mathbf{0}_{3x3} \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} 0.5dt^2\mathbf{I}_{3x3} & \mathbf{0}_{3x3} \\ dt\mathbf{I}_{3x3} & \mathbf{0}_{3x3} \\ \mathbf{0}_{3x3} & \mathbf{I}_{3x3} \end{bmatrix}$$

Prediction

$$\begin{bmatrix} \hat{\mathbf{x}}(k+1|k) \\ \hat{\mathbf{v}}(k+1|k) \\ \hat{\mathbf{f}}(k+1|k) \end{bmatrix} = \mathbf{A}\begin{bmatrix} \hat{\mathbf{x}}(k|k) \\ \hat{\mathbf{v}}(k|k) \\ \hat{\mathbf{f}}(k|k) \end{bmatrix} + \mathbf{B}(F(k+1)\hat{\mathbf{R}}(k)\mathbf{z}_{\mathrm{w}} - g\mathbf{z}_{\mathrm{w}})$$

$$\mathbf{P}(k+1|k) = \mathbf{A}\mathbf{P}(k|k)\mathbf{A}^T + \mathbf{W}\begin{bmatrix} \mathbf{Q}_\xi & 0 \\ 0 & \mathbf{Q}\gamma \end{bmatrix}\mathbf{W}^{\mathrm{T}}$$

Observation Model

Model

Height Sensor

$$m_z(k) = \begin{bmatrix} 0 & 0 & 1 & \mathbf{0}_{1x3} & \mathbf{0}_{1x3} \end{bmatrix} \begin{bmatrix} x(k) \\ y(k) \\ z(k) \\ \mathbf{v}(k) \\ \mathbf{f}(k) \end{bmatrix} + \eta_1$$

Optical Flow

$$\mathbf{m}_{\mathbf{v}}(k) = \begin{bmatrix} \mathbf{0}_{1x3} & 1 & 0 & 0 & \mathbf{0}_{1x3} \\ \mathbf{0}_{1x3} & 0 & 1 & 0 & \mathbf{0}_{1x3} \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ v_x(k) \\ v_y(k) \\ v_z(k) \\ \mathbf{f}(k) \end{bmatrix} + \boldsymbol{\eta}_2$$

Update

$$\begin{bmatrix} \hat{\mathbf{x}}(k+1|k+1) \\ \hat{\mathbf{v}}(k+1|k+1) \\ \hat{\mathbf{f}}(k+1|k+1) \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{x}}(k+1|k) \\ \hat{\mathbf{v}}(k+1|k) \\ \hat{\mathbf{f}}(k+1|k) \end{bmatrix} + \mathbf{K}\left(\mathbf{m} - \mathbf{C} \begin{bmatrix} \hat{\mathbf{x}}(k+1|k) \\ \hat{\mathbf{v}}(k+1|k) \\ \hat{\mathbf{f}}(k+1|k) \end{bmatrix} \right)$$

$$\mathbf{P}(k+1|k+1) = (\mathbf{I}_{9x9} - \mathbf{KC})\mathbf{P}(k+1|k)$$

$$\mathbf{K} = \mathbf{P}(k+1|k)\mathbf{C}^{\mathrm{T}}(\mathbf{CP}(k+1|k)\mathbf{C}^{\mathrm{T}} + \mathbf{R}_{\boldsymbol{\eta}})^{-1}$$

Observation Model

$$\mathbf{O} = \left[\mathbf{C}^T \ \ (\mathbf{CA})^T \ \ (\mathbf{CA}^2)^T \ \ \cdots \ \ (\mathbf{CA}^{n-1})^T \ \right]^T$$

$$\mathbf{O} = \begin{bmatrix}
0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.0000 & 0 & 0 & 0.0500 & 0 & 0 & 0.0125 \\
0 & 0 & 0 & 1.0000 & 0 & 0 & 0.0500 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0.0500 & 0 \\
0 & 0 & 1.0000 & 0 & 0 & 0.1000 & 0 & 0 & 0.0275 \\
0 & 0 & 0 & 1.0000 & 0 & 0 & 0.1000 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0.1000 & 0 \\
0 & 0 & 1.0000 & 0 & 0 & 0.1500 & 0 & 0 & 0.0450 \\
0 & 0 & 0 & 1.0000 & 0 & 0 & 0.1500 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0.1500 & 0 \\
0 & 0 & 1.0000 & 0 & 0 & 0.2000 & 0 & 0 & 0.0650 \\
0 & 0 & 0 & 1.0000 & 0 & 0 & 0.2000 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0.2000 & 0 \\
0 & 0 & 1.0000 & 0 & 0 & 0.2500 & 0 & 0 & 0.0875 \\
0 & 0 & 0 & 1.0000 & 0 & 0 & 0.2500 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0.2500 & 0 \\
0 & 0 & 1.0000 & 0 & 0 & 0.3000 & 0 & 0 & 0.1125 \\
0 & 0 & 0 & 1.0000 & 0 & 0 & 0.3000 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0.3000 & 0 \\
0 & 0 & 1.0000 & 0 & 0 & 0.3500 & 0 & 0 & 0.1400 \\
0 & 0 & 0 & 1.0000 & 0 & 0 & 0.3500 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0.3500 & 0 \\
0 & 0 & 1.0000 & 0 & 0 & 0.4000 & 0 & 0 & 0.1700 \\
0 & 0 & 0 & 1.0000 & 0 & 0 & 0.4000 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0.4000 & 0
\end{bmatrix}$$

$$rank(\mathbf{O}) = 7 < 9$$

Results – Stochastic Properties

Prediction:

Attitude measurements:

On-board auto-pilot

Thrust Control:

Rate: 20Hz

Precision:

$$\mathbf{R}_{\xi} = (0.08\,\mathbf{I}_{3x3}\; m/s^2)^2$$
$$\mathbf{R}_{\gamma} = (0.1\,\mathbf{I}_{3x3}\; m/s^2)^2$$

Observations:

Height sensor:

Rate: 10Hz

Precision: $\mathbf{R}_{\eta 1} = (0.01\; m)^2$

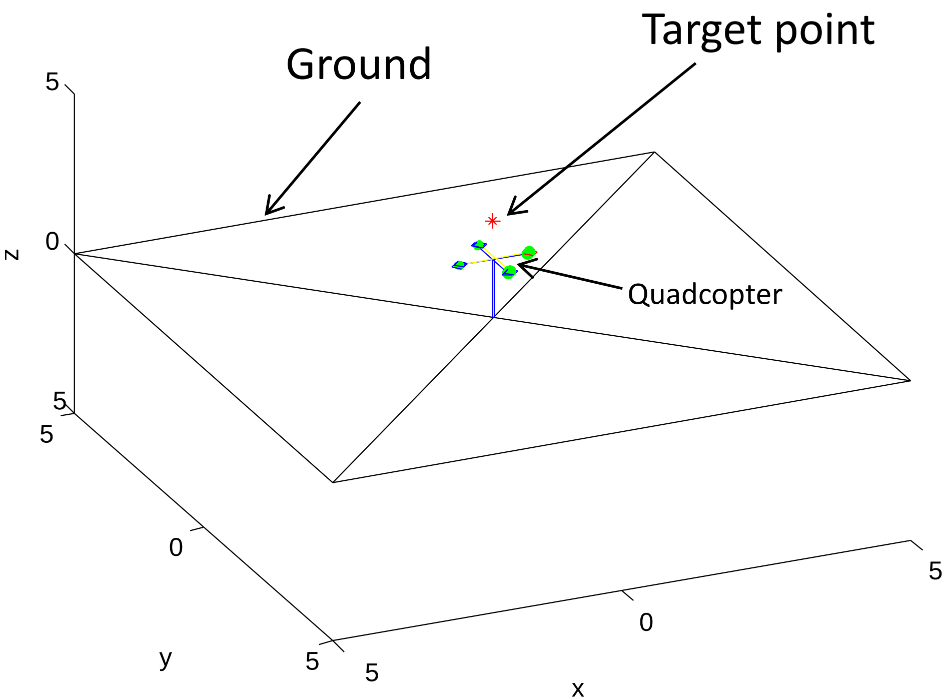
Horizontal velocity sensor:

Rate: 100Hz

Precision: $\mathbf{R}_{\eta 2} = (0.1\,\mathbf{I}_{2x2}\; m/s^2)^2$

Results – Simulator

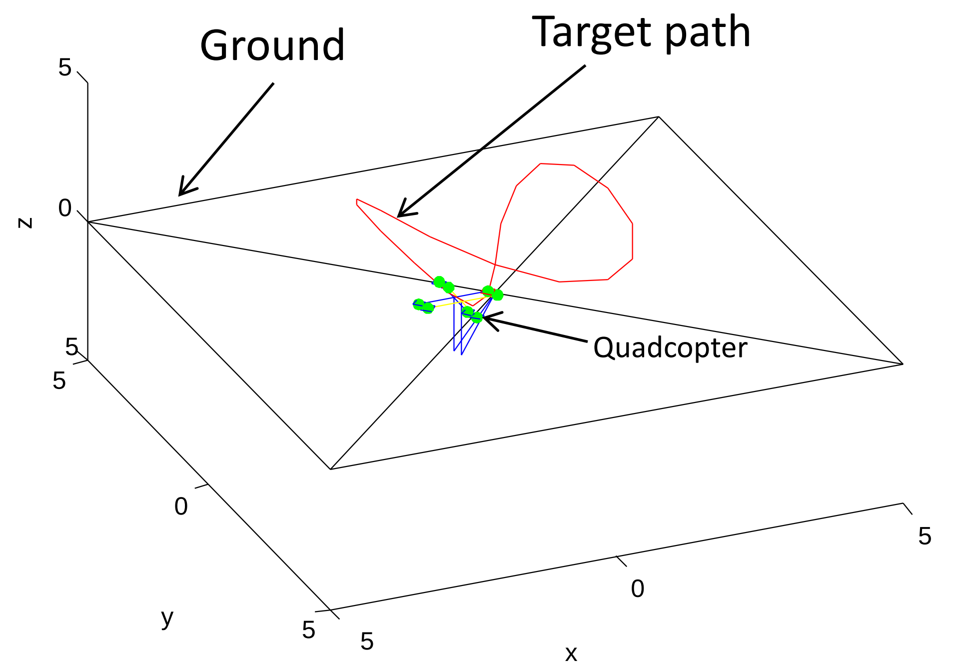
Position Control



Target point:

$$\mathbf{x} = \begin{bmatrix} 0 & 0 & 3 \end{bmatrix}^T$$

Path Following



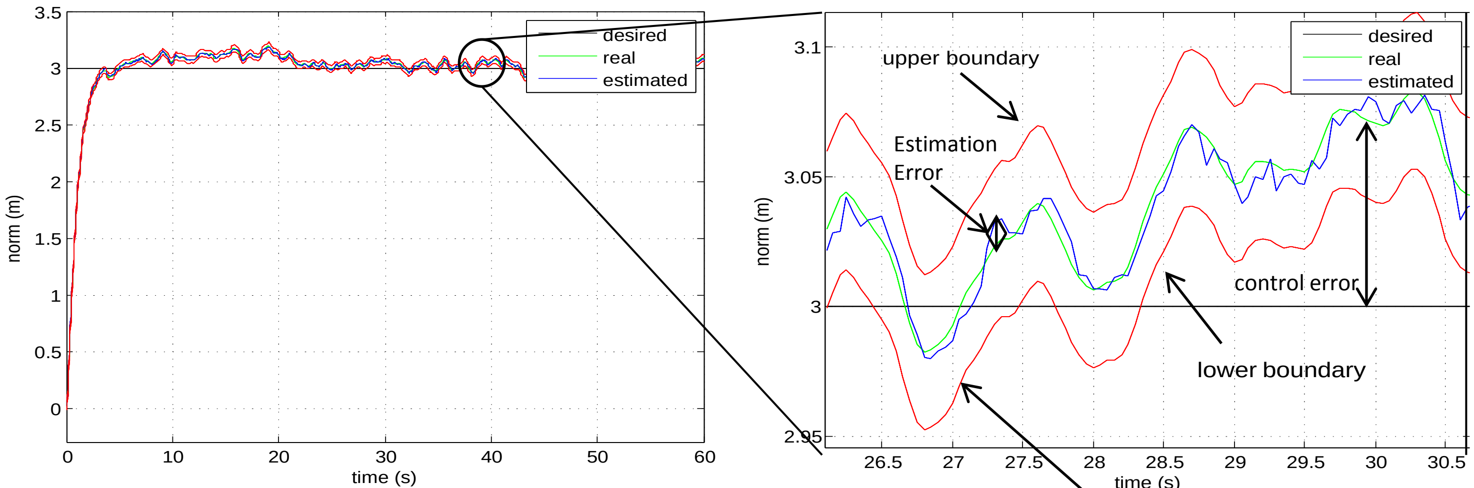
Target path:

$$\mathbf{x} = \begin{bmatrix} 2\cos(\theta) & 2\sin(2\theta) & 2+\cos(2\theta) \end{bmatrix}^T$$

$$\theta = 0 : \frac{\pi}{10} : 2\pi$$

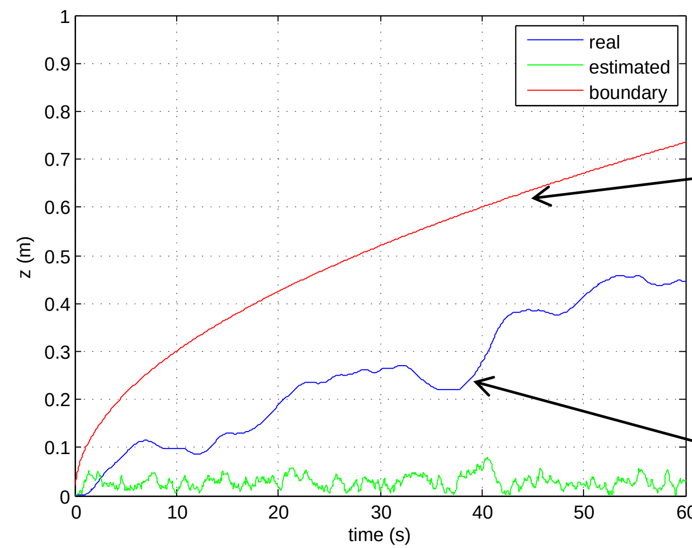
Results – Position Control

Height



Computed from Heigh sensor noise

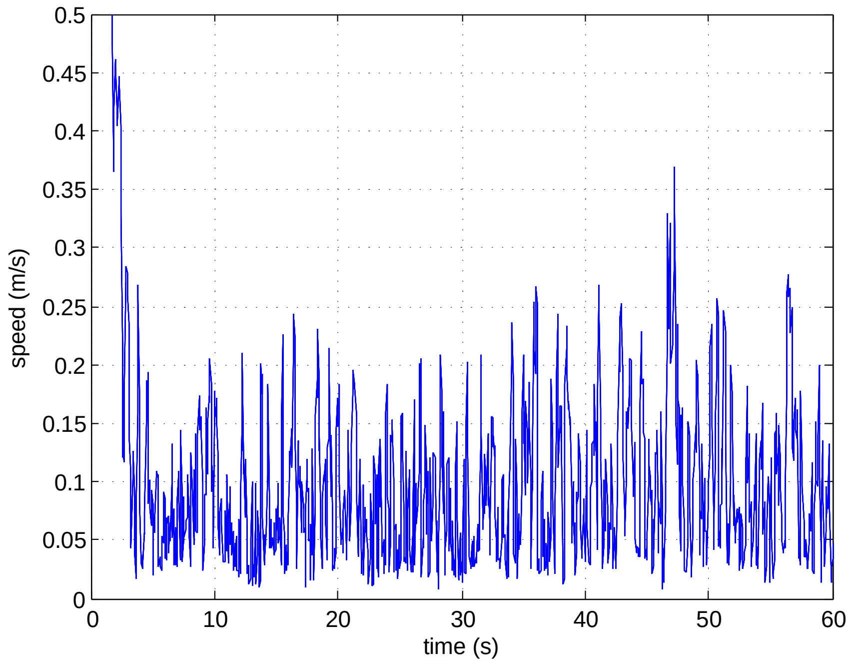
Horizontal Position Norm



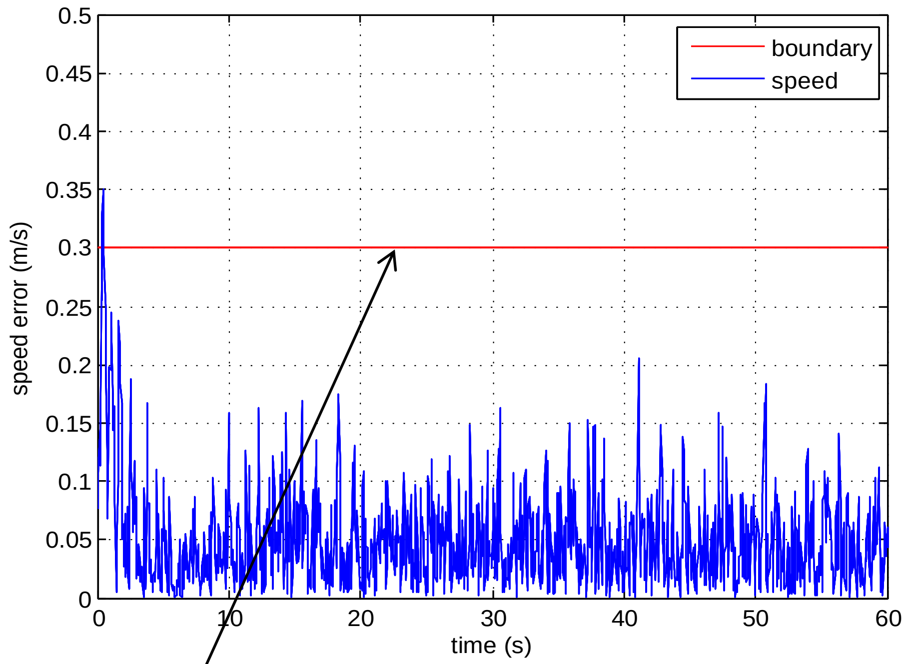
Computed from Optical flow noise

Observed drift

Results – Position Control



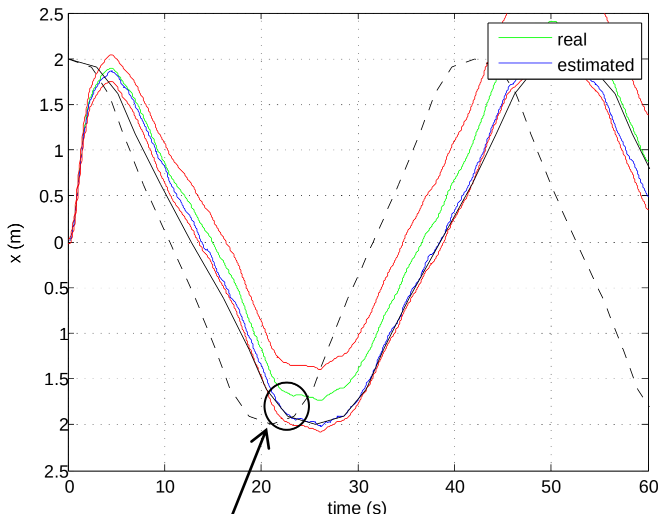
Real robot speed



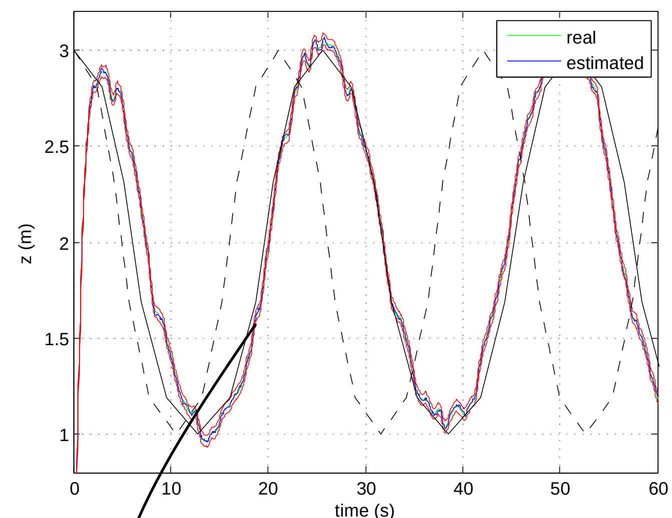
Difference between real and Estimated speeds

Computed from Optical flow noise

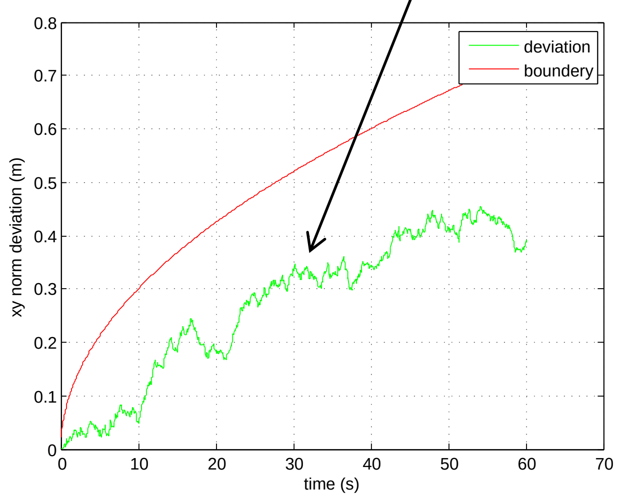
Results – Path Following



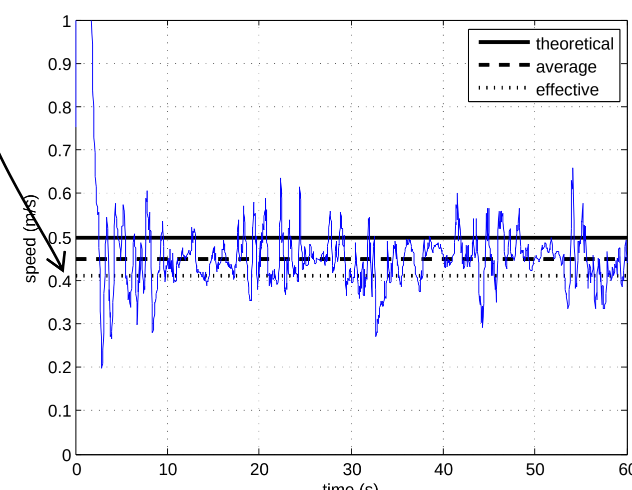
x coordinate



z coordinate

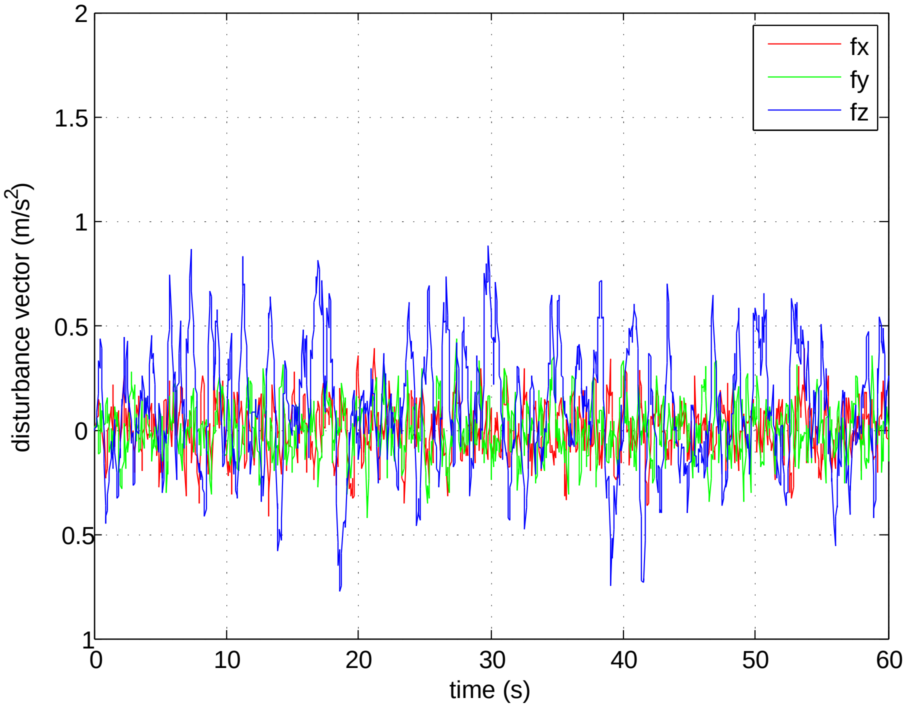


Difference between real and Estimated horizontal position

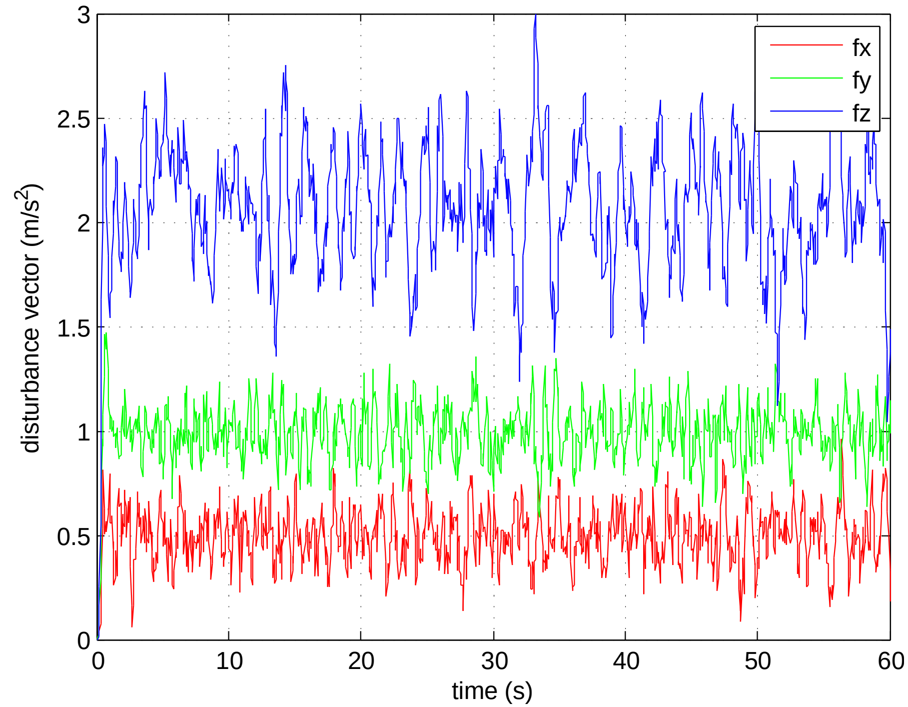


Real robot speed

Results – Disturbances



No disturbances



Emulated wind gust

$$\mathbf{a} = \begin{bmatrix} 0.5 & 1 & 2 \end{bmatrix}^T$$

Conclusions

- **The onboard sensor setup and localization method was successfully implemented.**
- **The used models are simple, but flexibile with respect to model imperfections.**
- **Additional compensation parameters can be be used to obtain better motion prediction under environment and model uncertainty.**

Future Work

- Extra capabilities on the onboard sensor setup, to eliminate observability problems.

- Consider attitude dynamics and uncertanity to improve the estimation method.

Thank You!