Advanced Control Systems
DETECTION, ESTIMATION, AND FILTERING

Sensor-based Globally Asymptotically Stable Range-Only Simultaneous Localization and Mapping

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Presentation Outline

Introduction

Outline

1. Introduction
   - Mission Scenario
   - Range-Only Simultaneous Localization and Mapping
   - Proposed Solution

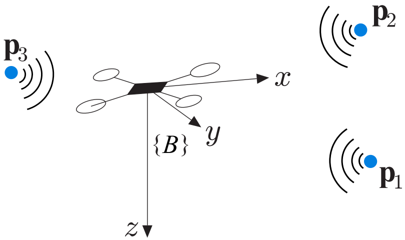
Introduction

Mission Scenario

Autonomous vehicle missions with no absolute positioning available.

Sensor Suite

- Angular rates: IMU
- Linear velocity: visual odometry
- Distance to beacons: acoustic/radio receiver



Introduction

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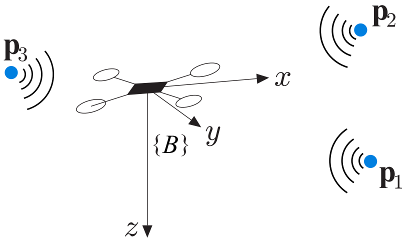


Problem Statement

Design a navigation filter in the space of sensors using the available sensor suite.

Introduction

Range-Only Simultaneous Localization and Mapping

Definition

- The problem of navigating a vehicle in an unknown environment, by building a map of unknown landmarks by measuring distances and using this map to deduce its location, without the need for a priori knowledge of location.

RO-SLAM details

- No need for data association algorithms as the ranging signals are tagged.
- **Problem**: landmark initialization

Introduction

Proposed Solution

- **Landmark initialization** is one of the main issues.
- Some works use trilateration techniques on the first instants that a landmark is observed to obtain a first estimate to insert in the filter.
- Our solution solves this problem by introducing **global** convergence and stability results rooted in source localization

P. Batista, C. Silvestre, and P. Oliveira.
Single range aided navigation and source localization: Observability and filter design.
In *Systems & Control Letters*, vol. 60, no. 8, pp. 665–673, 2011.

Sensor-based RO-SLAM Filter

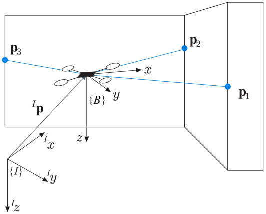
Outline

2. Sensor-based RO-SLAM Filter
   - Overview
   - Nonlinear System Dynamics
   - Observability Analysis
   - Filter Design

Sensor-based RO-SLAM Filter

Overview

- Coordinate frames
  - $\{I\}$ – Inertial reference frame
  - $\{B\}$ – Body-fixed frame
- Body-fixed frame defined by
  - ${}^I\mathbf{p} \in \mathbb{R}^3$ – Vehicle position
  - $\mathbf{R} \in SO(3)$ – Rotation matrix from $\{B\}$ to $\{I\}$
- Landmarks
  - ${}^I\mathbf{p}_i \in \mathbb{R}^3$ – Landmark position in $\{I\}$
  - $\mathbf{p}_i \in \mathbb{R}^3$ – Landmark position in $\{B\}$



Nonlinear System Dynamics
Definitions

Landmark sets

$\mathcal{L}_O$ Set of visible landmarks

$\mathcal{L}_U$ Set of non-visible landmarks

$\mathcal{L}$ Set of all landmarks: $\mathcal{L} = \mathcal{L}_O \cup \mathcal{L}_U$

Measured quantities

$\|\mathbf{p}_i\|$ Ranges to sensor-based landmarks

$\mathbf{v}$ Linear velocity of the body-fixed frame

$\boldsymbol{\omega}$ Angular velocity

Nonlinear System Dynamics

Nonlinear System

- A nonlinear system is designed:

$$\begin{cases} \dot{\mathbf{p}}_i(t) = -\mathbf{S}\left[\boldsymbol{\omega}(t)\right]\mathbf{p}_i(t) - \mathbf{v}(t) \\ \dot{\mathbf{v}}(t) = \mathbf{0} \\ r_i(t) = \|\mathbf{p}_i(t)\| \\ \mathbf{y}_v(t) = \mathbf{v}(t) \end{cases} \tag{NLS}$$

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- Problem: the output is **nonlinear**!

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- Problem: the output is **nonlinear**!
- Proposed solution: **augment** the state to include the nonlinear **observation**:

$$\begin{cases} \mathbf{x}_{L_i}(t) := \mathbf{p}_i(t) \\ \mathbf{x}_V(t) := \mathbf{v}(t) \\ x_{R_i}(t) := \|\mathbf{x}_{L_i}(t)\| \end{cases}$$

Nonlinear System Dynamics

Augmented Nonlinear System

- The augmented system dynamics are

$$\begin{cases} \dot{\mathbf{x}}_{L_i}(t) = -\mathbf{S}\left[\boldsymbol{\omega}(t)\right]\mathbf{x}_{L_i}(t) - \mathbf{x}_V(t), \; i \in \mathcal{L} \\ \dot{\mathbf{x}}_V(t) = \mathbf{0} \\ \dot{x}_{R_j}(t) = -\mathbf{y}_v^T(t)\, y_{R_j}^{-1}(t)\, \mathbf{x}_{L_j}(t), \; j \in \mathcal{L}_O \\ \dot{x}_{R_k}(t) = -\mathbf{y}_v^T(t)\, x_{R_k}^{-1}(t)\, \mathbf{x}_{L_k}(t), \; k \in \mathcal{L}_U \\ \mathbf{y}(t) = \begin{bmatrix} \mathbf{y}_v^T(t) & y_{R_1}(t) & \cdots & y_{R_{N_O}}(t) \end{bmatrix}^T \end{cases}$$

Nonlinear System Dynamics

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- There is nothing in the dynamics imposing the **state constraint** $x_{R_i}(t) := \|\mathbf{x}_{L_i}(t)\|$.

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- There is nothing in the dynamics imposing the **state constraint** $x_{R_i}(t) := \|\mathbf{x}_{L_i}(t)\|$.
- The **state dependences** on the visible ranges state and the velocity state are **replaced** by the corresponding **output**.

Observability Analysis

Outline

Nominal nonlinear system → Augmented nonlinear system → LTV-like nonlinear system → Transformed LTV system → Observability analysis → Globally asymptoticaly stable observer

Discard non-visible landmarks and ranges

$$\begin{cases} \dot{\mathbf{z}}(t) = \mathbf{A}(t)\mathbf{z}(t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{z}(t) \end{cases} \qquad \text{(LTV)}$$

Lyapunov transformation:
$\boldsymbol{\chi}(t) = \operatorname{diag}\left(\mathbf{R}(t), \mathbf{I}_3, 1\right) \mathbf{z}(t)$

$$\begin{cases} \dot{\boldsymbol{\chi}}(t) = \boldsymbol{\mathcal{A}}(t)\boldsymbol{\chi}(t) \\ \mathbf{y}(t) = \mathbf{C}\boldsymbol{\chi}(t) \end{cases} \qquad \text{(TLTV)}$$

Observability Analysis

Observability of the LTV system

Theorem 1

Consider the (LTV) *system and let* $\mathcal{T} := [t_0, t_f]$*. If there exist three instants* $\{t_1, t_2, t_3\} \in \mathcal{T}$ *such that the linear velocity of the vehicle expressed in the inertial frame is linearly independent in those instants, i.e.,* $\det\left(\begin{bmatrix} {}^I\mathbf{v}(t_1) & {}^I\mathbf{v}(t_2) & {}^I\mathbf{v}(t_3) \end{bmatrix}\right) \neq 0$*, and all the ranges are within* $[R_m, R_M]$*, both positive, then the system is observable in the sense that, given the system output* $\{\mathbf{y}(t),\ t \in \mathcal{T}\}$*, the initial condition* $\mathbf{z}(t_0)$ *is uniquely defined.*

Observability Analysis

Observability of the LTV system (cont.)

Sketch of the proof.

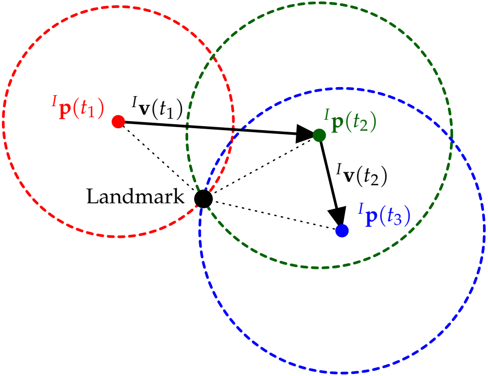
- System (LTV) is observable if the observability Gramian associated with $(\mathbf{A}(t), \mathbf{C})$ is invertible;
- Lyapunov transformation maintains observability properties;
- Proof by contraposition: assume the non-observability of the system (TLTV) which implies the non-invertibility of the observability Gramian $\mathcal{W}(t_0, t_f)$ that cannot hold if the conditions of the theorem apply.
- $\mathcal{W}(t_0, t_f)$ is invertible in $\mathcal{T}$, yielding the system (LTV) observable.

$\square$

Observability Analysis

Observability of the LTV system (cont.)

- First condition (simplified 2-D version)



Observability Analysis

Observability of the LTV system (cont.)

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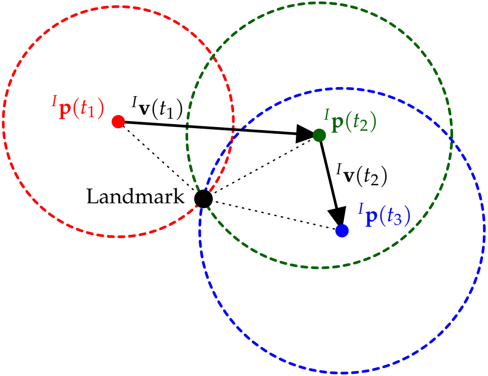


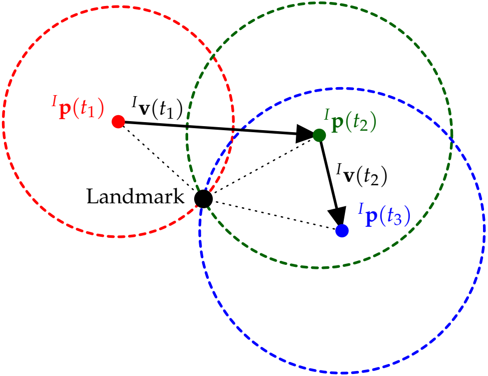
- Second condition

$$\dot{x}_{R_j}(t) = -\frac{\mathbf{y}_v^T(t)}{y_{R_j}(t)}\mathbf{x}_{L_j}(t)$$

Observability Analysis

Observability of the LTV system (cont.)

- First condition (simplified 2-D version)



- Second condition

$$\dot{x}_{R_j}(t) = -\frac{\mathbf{y}_v^T(t)}{\boxed{y_{R_j}(t)}}\mathbf{x}_{L_j}(t)$$

Observability Analysis

Equivalence of the LTV and the Nonlinear Systems

Theorem 2

Consider the LTV system (LTV) *and the original nonlinear system* (NLS). *If the conditions of Theorem 1 hold, then*

(i) *the state of the original nonlinear system and that of the LTV system are the same and uniquely defined, provided that the invisible landmarks are discarded. Furthermore the constraints expressed by* (AS) *become naturally imposed by the dynamics; and*

(ii) *a state observer with uniformly globally asymptotically stable error dynamics for the LTV system is also a state observer for the underlying nonlinear system, retaining the same error dynamics properties.*

Observability Analysis

Equivalence of the LTV and the Nonlinear Systems

Theorem 3

The pair $(\mathbf{A}(t), \mathbf{C})$ *is uniformly completely observable if there exist* $\delta > 0$ *and* $\alpha^* > 0$ *such that, for all* $t \geq t_0$*, it is possible to choose a set of instants* $\{t_1, t_2, t_3\} \in \mathcal{T}_\delta$*, with* $\mathcal{T}_\delta := [t, t+\delta]$*, for which the linear velocity of the vehicle in the inertial frame respects*

$$\left|\det\left(\left[{}^I\mathbf{v}(t_1) \quad {}^I\mathbf{v}(t_2) \quad {}^I\mathbf{v}(t_3)\right]\right)\right| > \alpha^*.$$

Sketch of the proof.

The proof follows steps similar to the proof of Theorem 1, but considering uniform bounds for all $t \geq t_0$ and intervals $[t, t+\delta]$. $\square$

Observability Analysis

Convergence and stability

- Following the approach of:

  P. Batista, C. Silvestre, and P. Oliveira.
  Single Range Aided Navigation and Source Localization: observability and filter design.
  Systems & Control Letters, 60(8):665–673, August 2011.

- Since the pair $(\mathbf{A}(t), \mathbf{C})$ is **uniformly completely observable**, a Kalman filter for system (LTV) has **GAS** error dynamics.

  B. Anderson.
  Stability properties of Kalman-Bucy filters.
  Journal of the Franklin Institute, 291(2):137-144, 1971.

- The state of the nonlinear system (NLS) is the same as that of sytem (LTV).
- The **observer** for (LTV) is **also a GAS observer** for (NLS).

Sensor-based RO-SLAM Filter

Filter Design

Discrete dynamics

- Forward Euler discretization
- Rotation of a landmark from instant k to $k+1$ done using $\mathbf{R}_{k+1}^T \mathbf{R}_k = \exp\left(-\mathbf{S}\left[\boldsymbol{\omega}_k\right] T_s\right)$

Predict Step

- Non-visible landmarks and ranges are propagated in **open loop**.
- Standard discrete LTV Kalman filter prediction equations.

Update Step

- Standard LTV Kalman filter update equations

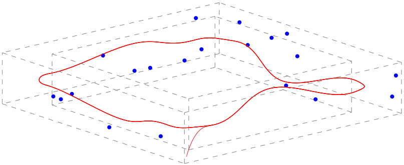
SIMULATION RESULTS
OUTLINE

3 Simulation Results
- Simulation Parameters
- Resulting map
- Landmark Estimation Results

Simulation Results

Simulation Parameters

Environment



- $16\text{m}\times16\text{m}\times3\text{m}$ map with 20 random landmarks

Measurement noise

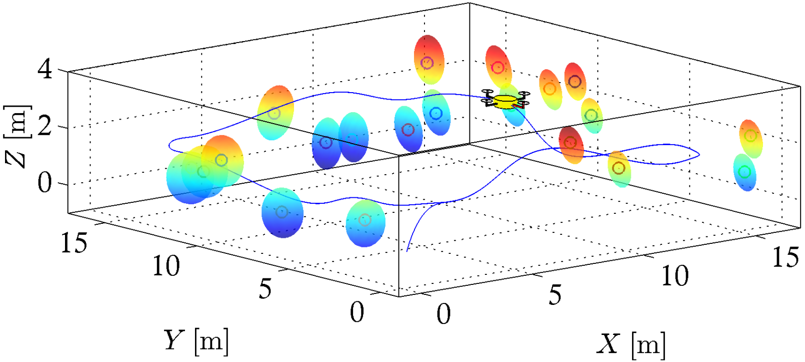
- $\sigma_\omega = 0.05\ ^\circ/\text{s}$
- $\sigma_v = 0.03\ \text{m/s}$
- $\sigma_r = 0.03\ \text{m}$

Algorithm Parameters

- State covariance $\mathbf{Q} = T_s \operatorname{diag}\left(10^{-3}\mathbf{I}_{3N}, 10^{-2}\mathbf{I}_3, 10^{-5}\mathbf{I}_N\right)$
- Observation covariance $\mathbf{R} = 10^{-3} \operatorname{diag}\left(\mathbf{I}_3, \mathbf{I}_{N_O}\right)$

Simulation Results

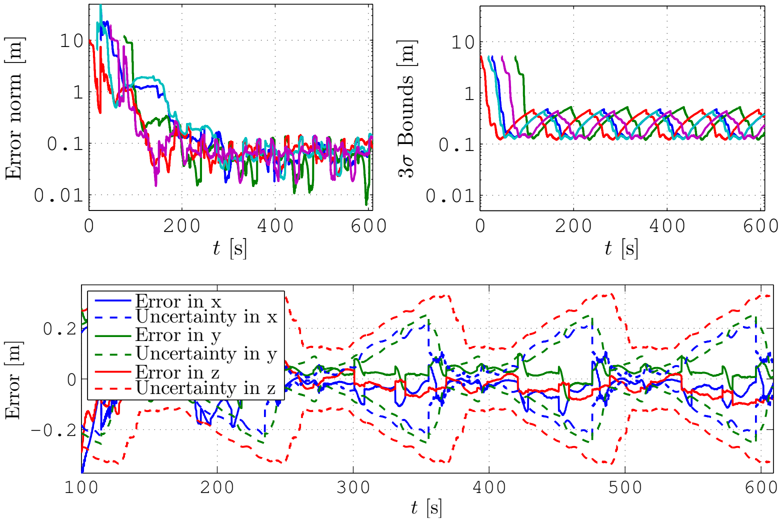
Resulting map



- The trajectory designed to respect observability conditions and ensure sufficient excitation in all directions.

Simulation Results

Landmark Estimation Results



Experimental Results

Outline

Experimental Results

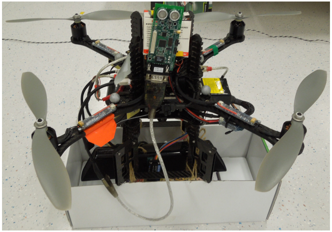
Setup

Instrumented *AscTec Pelican* quadrotor

- *Crossbow Cricket* acoustic/radio receiver
- *Microsoft Kinect* facing down for visual odometry
- *Microstrain 3DM-GX3-25*

Environment

- *Crossbow Cricket* constellation of 7 beacons
- *VICON* motion capture system for ground truth



Experimental Results

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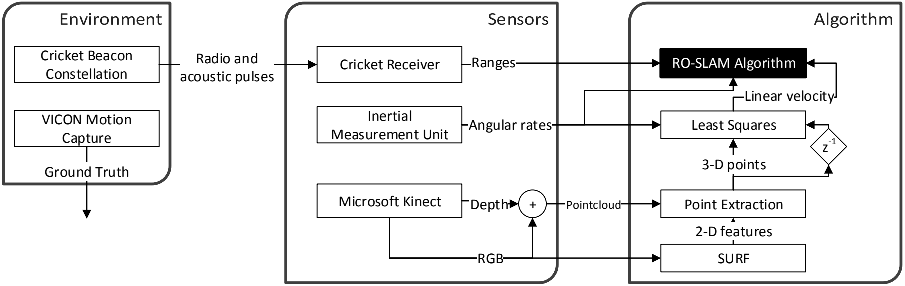
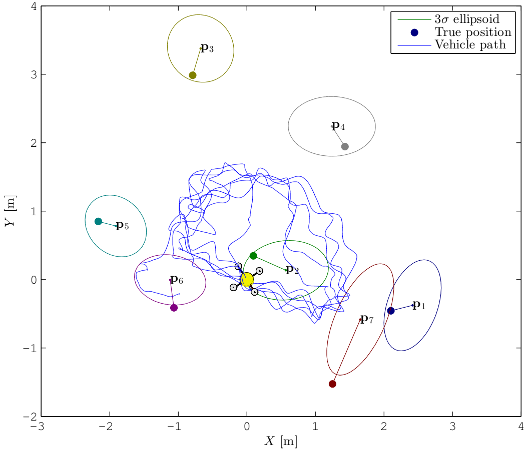


H. Bay, A. Ess, T. Tuytelaars, and L. Van Gool
Speeded-Up Robust Features (SURF)
Computer Vision and Image Understanding, 110(3):346-359, 2008.

J. Neira and J.D. Tardós
Data assoc. in stoch. mapp. using the joint comp. test
IEEE Transactions on Robotics and Automation, 17(6):890-897, 2001.

Experimental Results

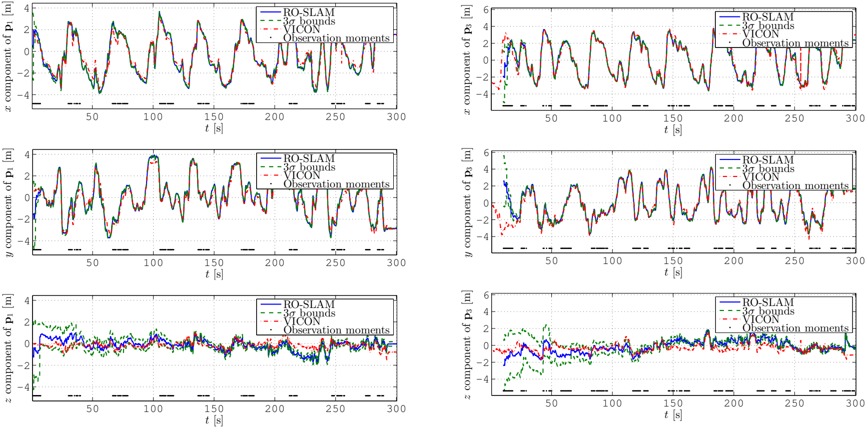
Resulting map



- Trajectory was not sufficiently rich in the vertical direction.
- The velocity measurements from the visual odometry are noisy, which can make the direction appear observable.

Experimental Results

Ground Truth Data



Conclusions & Further Work

Conclusions

- Novel RO-SLAM algorithm was designed, analysed and validated through simulated and experimental tests.
- Formal proof of stability and convergence of the sensor-based filter for a nonlinear system was obtained through state augmentation.
- Theoretical observability results establish a constructive basis for trajectory design.

Recent and further work

- Necessary conditions for observability of the original nonlinear system have been found.
- Extension of the algorithm to make use of the full capabilities of a sensor network (sensor-to-sensor ranging) is an interesting point of research.

THE END.

- Thank you.