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An Innovations Approach to Fault Detection in Sensor Networks

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- >Guidelines
- >Model Based FDI
- >Sensor Validation
- >General Method
- >Linear Dynamic Systems Application
- >Pratical Case Studie
- >Final considerations

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Guidelines

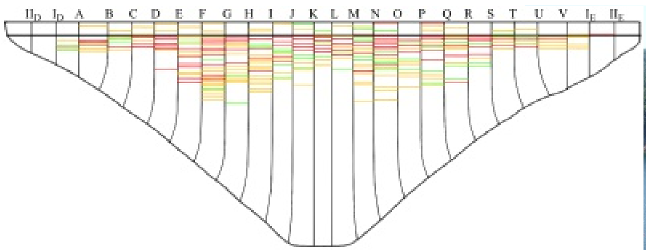
>Structure Health Monitoring (SHM)

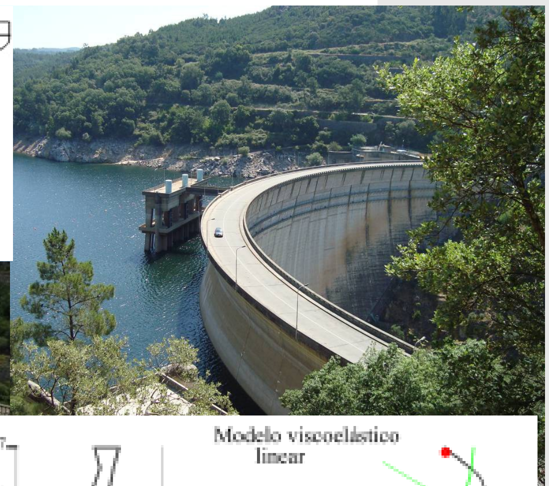
- Set of actions aimed at the detection and diagnosis of abnormal situations during the exploration of major civil engineering works of art in order to keep the security and the reduction of maintenance and inspection costs .

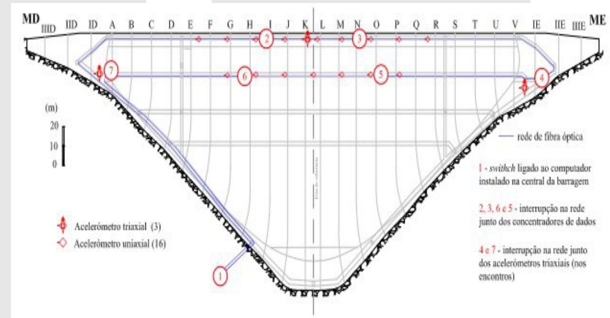
- It is necessary the installation, in the works of art monitored, a large number of sensors according to the established observation plan.

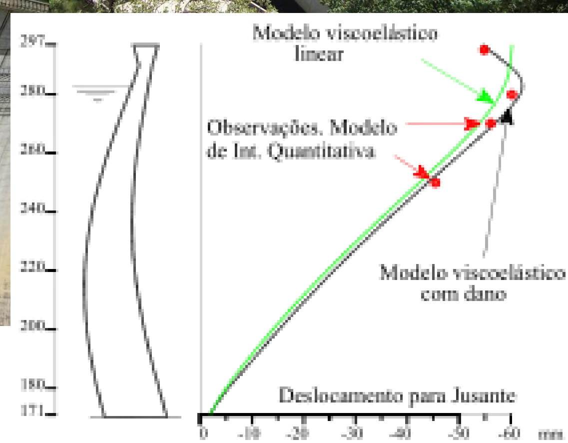
Guidelines

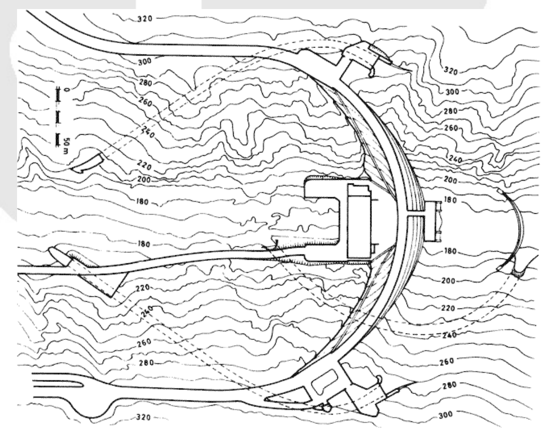
>Structure Health Monitoring (SHM)









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Guidelines

>Structure Health Monitoring (SHM)

- …
- However the sensors, due to its nature and to the harsh environmental conditions that they are subject to, are subject to faults that in the last instance can compromise the quality of the indispensable information for an effective security control.

  - *Note: in a metrological sense, the sensor is a device used in the measuring process that it's directly affected by the measurand and according to a predetermined law generates a related signal related to its value.*
    In the context of this paper it is used a broader sense of the term sensor, comprising all the elements in the measuring chain.

Guidelines

>It is hoped that this study can contribute to the future development of innovative measurement Fault Tolerant Sensor Networks applied to Civil Engineering Structures; witch may have potential application in the Structure Health Monitoring systems.

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Model Based FDI

> Fault

"In the context of an industrial application, a fault is perceived as a non-permitted deviation of a characteristic property that leads to failure of the system or manufacturing facility to fulfill the purpose for which it was designed "

[Isermann, 1997]

Model Based FDI

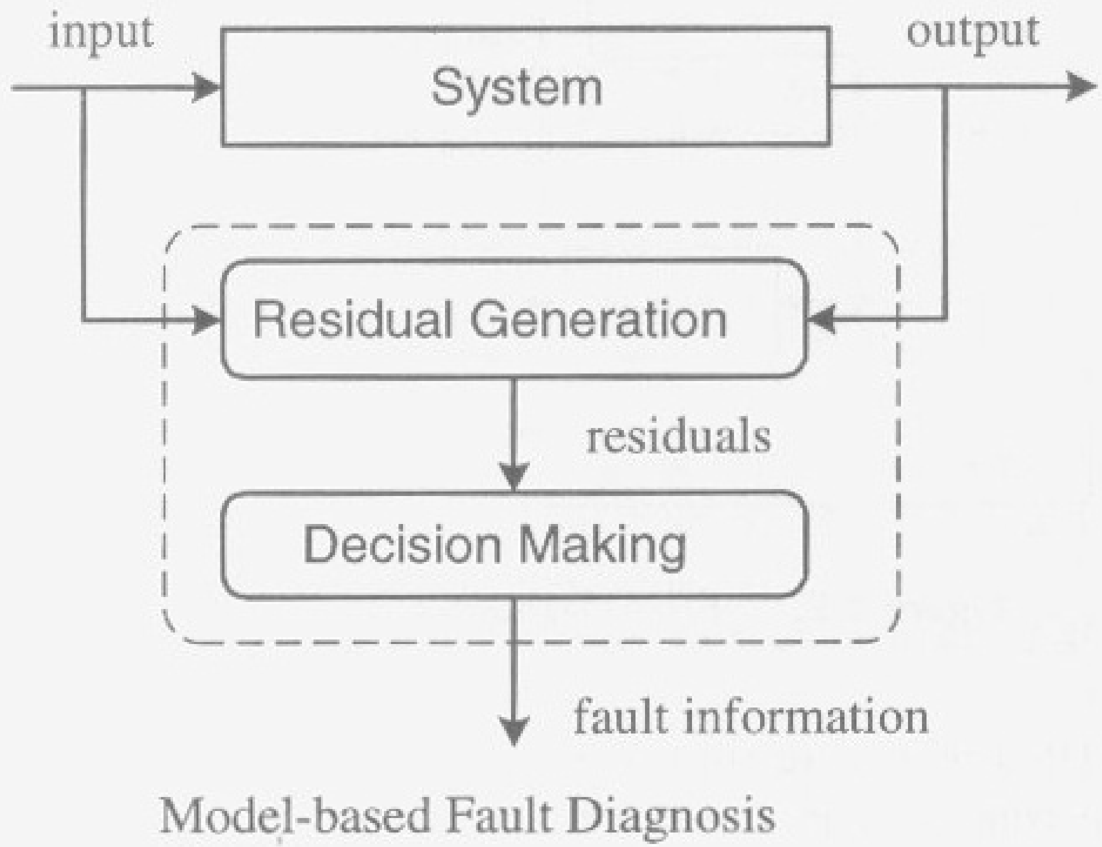

Figure 2.1. Conceptual structure of model-based fault diagnosis

Model Based FDI

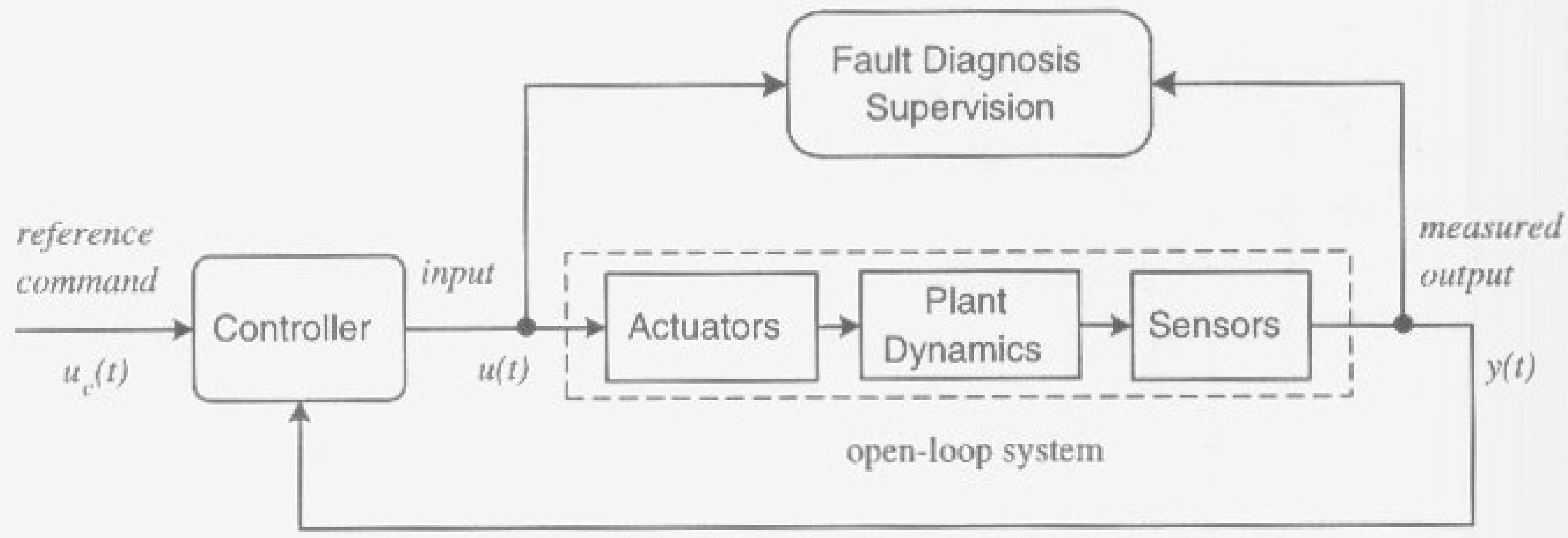

Figure 2.2. Fault diagnosis and control loop

Model Based FDI

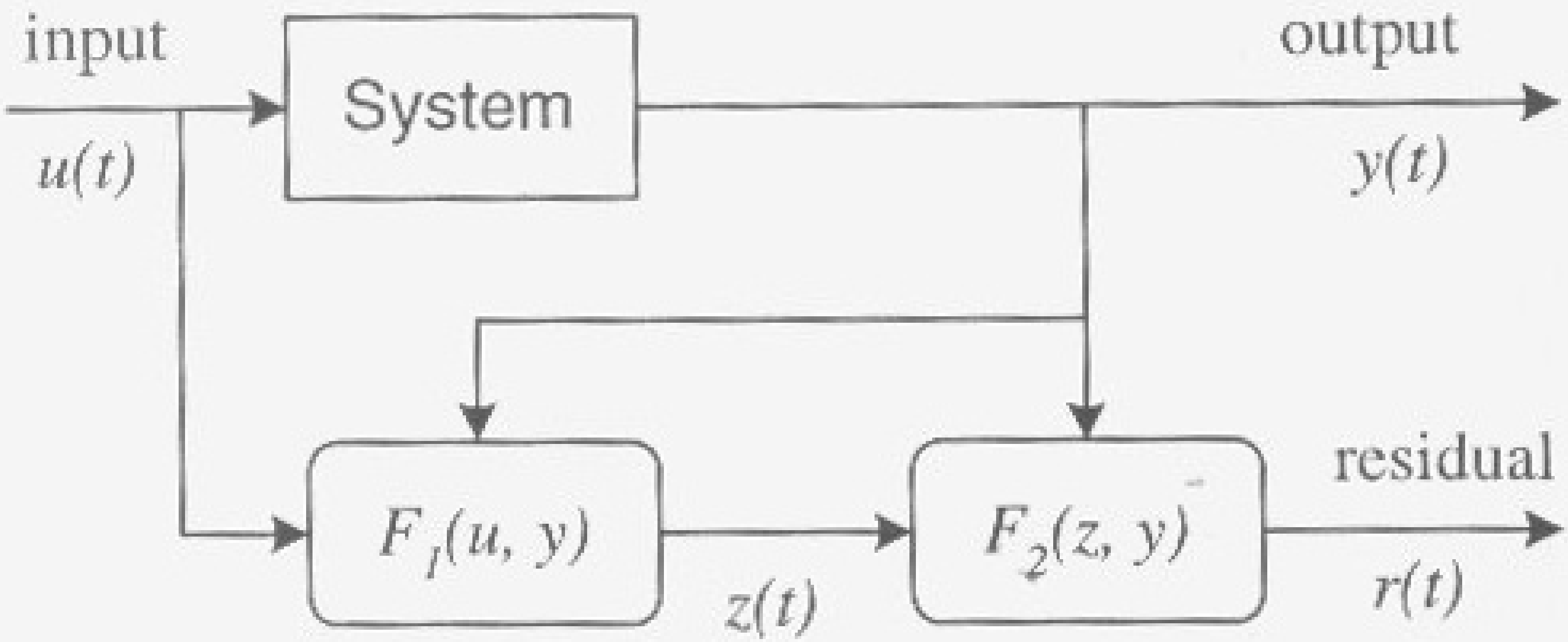

Figure 2.8. Redundant signal structure in residual generation

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Sensor Validation

> Sensor Validation

- *"The fault detection and diagnosis (FDI) when applied to sensors is usually designated as sensor validation "* [Fortuna *et al.*, 2007]

- *"Almost all the techniques of fault detection and diagnosis (FDI) described in the literature can be applied to sensor validation "* [Fortuna *et al.*, 2007].

Sensor Validation

>Sensor Validation

- However, although less frequently, other name is also common in the literature regarding this topic, and is referred as signal validation [Jabłoński, Barszcz & Bielecka, 2011; Fantoni *et al.*, 2003]

Sensor Validation

>As in FDI at the process level, in the validation of sensors, one can compare the results of measurements of the system sensors with mathematical models that describe the static and dynamic relationships between the measured, supported in the techniques fault detection based on models, and the possibility, in the event of a fault to provide an estimate (during a finite time window) of the missing measurements.

Sensor Validation

>However, using this approach, care must be taken to distinguish errors in the sensors from faults in the process or control system.

>A failure in the process may result in abnormal readings from the sensors, the developed algorithms may report a faulty sensor.

>The use of a higher layer in the diagnostic system must consider this situation

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  - Generation of an residual signal or the innovation process
  - Statistics of the residual signal under normal conditions
  - Outlier Detection via limit value checking
  - Sensor fault detection via hypothesis testing
  - Isolation of the sensor fault

General Method - Development of the Model

>A mathematical model is developed for the system based on physical information and statistical data.

>This model can be static or dynamic, linear or non-linear, continuous or discrete and deterministic or stochastic.

>The input and the output variables of the system are clearly defined and all the relevant parameters are identified.

General Method - Development of the Model

> The model describes the behavior of the system under normal operating conditions.

>It also specifies the statistics of the measurement noise in the output variables

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General Method – Residual Generation

> The residual signal or the innovation process is defined as the difference between the actual system output and the expected output based on the model and the previous output data.

> The latter is generated directly by the model if the system is deterministic or by a statistical filter if the system is stochastic, i.e. subject to random inputs and variations.

> It is called the innovation process since it represents the new information brought by the latest observation.

General Method – Residual Generation

>Under normal conditions, the error signal is "small" and corresponds to random fluctuations in the output since all the systematic trends are eliminated by the model.

>However, under faulty conditions, the error signal is "large" and contains systematic trends because the model no longer represents the physical system adequately.

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Statistics of the residual signal under normal conditions

> In deterministic systems, the random fluctuations in the residual signal are due to measurement noise in the output variables. Their statistics are obtained as part of the system description in the modelling step.

> In stochastic systems, the statistics of the error signal are obtained from the filter which is used to predict the output of the system.

Statistics of the residual signal under normal conditions

>For linear dynamic systems with Gaussian random inputs, a Kalman filter generates both the residual signal and its statistics.

- It is known that under normal conditions, the residual signal or the innovation process is a zero mean Gaussian white noise process

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Outlier Detection via limit value checking

- Given the statics of the residual signal under normal conditions, a univariate statistical approach to limit sensing can be used to determine the thresholds for each generated residual
- These thresholds define the boundary limits, and a violation of these limits would indicate a outlier in the actual system output.
- This approach is typically employed using a Shewart control chart

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Sensor fault detection via hypothesis testing

> The problem of sensor fault detection is easily formulated as a problem in Hypothesis Testing

- by regarding the normal operation of the system as the null hypothesis.
- The actual residual signal from the system is tested against this hypothesis at a certain level of significance.
- For example, if the system is described by a set of linear differential equations and a Kalman filter is used to generate the innovation process, the null hypothesis consists of testing the innovation process for normality, zero mean, whiteness and a given covariance

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Isolation of the sensor fault

>If a fault is detected in the system, the present model for the system may no longer apply.

>In order to diagnose the fault it may be necessary to develop a new model for the system.

- Since a failure in the process may result in abnormal readings from the sensors, the developed algorithms may report a faulty sensor.
- As stated before the use of a higher layer in the diagnostic system must consider this situation.

>These subsequent procedures are beyond the scope of this paper

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Linear Dynamic Systems

> Application

- Systems describable by a set of linear difference equations.
- Continuous-time linear systems can be treated in the same way.
- Static systems can be regarded as special cases of the dynamic systems.
- The approach can also be carried over to nonlinear dynamic systems

Linear Dynamic Systems

State Dynamics:

$$x(t+1)=A(t)x(t)+L(t)\xi(t)$$

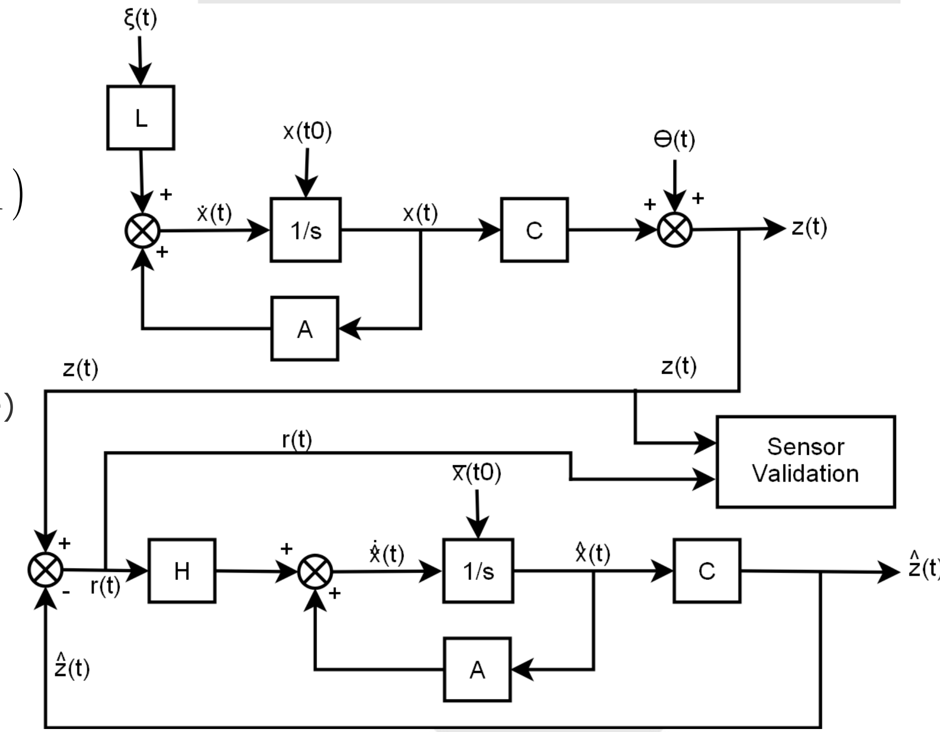
Measurements:

$$z(t+1)=C(t+1)x(t+1)+\theta(t+1)$$

With time index

$$t=0,1,2,\ldots$$

Where



$x(t)$ is a $\mathbb{R}^n$ state vector (stochastic sequence non-white)
$z(t)$ is a $\mathbb{R}^r$ measurement vector
$\xi(t)$ is a $\mathbb{R}^p$ white plant noise
$\theta(t)$ is a $\mathbb{R}^r$ white measurement noise
$A(t)$ is a $\mathbb{R}^{n\times n}$ state-transition matrix
$L(t)$ is a $\mathbb{R}^{n\times p}$ noise distribution matrix

and

$C(t)$ is a $\mathbb{R}^{r\times n}$ output matrix.

Linear Dynamic Systems

Plant noise $\xi(t)$ and Measurement noise $\theta(t)$ are Gaussian discrete white noise, with mean and covariance:

$$E[\xi(t)] = 0, \quad \mathrm{cov}[\xi(t);\xi(\tau)] = \Xi(t)\delta_{t\tau} \qquad (3)$$

$$\Xi(t) = \Xi^T(t) \geq 0 \qquad (4)$$

$$E[\theta(t)] = 0, \quad \mathrm{cov}[\theta(t);\theta(\tau)] = \Theta(t)\delta_{t\tau} \qquad (5)$$

$$\Theta(t) = \Theta^T(t) > 0 \qquad (6)$$

$$\mathrm{cov}[\theta(t);\xi(\tau)] = 0 \qquad (7)$$

Where $\delta_{t\tau}$ denotes the Kronecker delta

$$\delta_{t\tau} = \begin{cases} 1, & t = \tau \\ 0, & t \neq \tau \end{cases}$$

and $E[\cdot]$ denotes the expectation operator , and $\mathrm{cov}[\cdot;\cdot]$ the covariance operator.

The initial state $x(0)$ is also assumed to be random. The distribution of the state variables is Gaussian with mean and covariance:

$$E[x(0)] = \bar{x}(0), \quad \mathrm{cov}[x(0);x(0)] = \Sigma_0 \qquad (8)$$

$$\Sigma_0 = \Sigma_0^T \geq 0 \qquad (9)$$

$$\mathrm{cov}[x(0);\xi(t)] = 0, \quad \mathrm{cov}[x(0);\theta(t)] = 0 \qquad (10)$$

The more general case of correlated plant noise and correlated measurement noise can be reduced to the above case by augmenting the state vector

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> Linear Dynamic Systems Application
  - *Development of the model*
  - *Generation of the innovation sequence*
  - *Statistics of the innovation sequence*
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  - *Sensor fault detection via hypothesis testing*

> Pratical Case Studie

> Final considerations

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> Linear Dynamic Systems Application
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Linear Dynamic Systems - Model

> This consists of identifying matrices

$$A(t),\quad L(t),\Xi(t),C(t),\Theta(t)$$

> and the order n of the system under normal operating conditions.

> It is mostly done by using a combination of physical information and statistical data on the system.

- The various methods for system identification and model validation are useful at this stage

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LDS - *Generation of the innovation sequence*

$$r(t+1) = z(t+1) - \hat{z}(t+1\,|\,t)$$

where $\hat{z}(t+1\,|\,t)$ denotes the unbiased minimum variance estimate of $z(t+1)$ given the sequence of past measurements up to (t), i.e. based on the set $\{z(1), z(2), \ldots, z(t)\}$

LDS - *Generation of the innovation sequence*

>If it is assumed that all the system parameters and statistics are known exactly, the innovation sequence can be generated by a Kalman filter of the following form

1) Off-line Calculations

Initialization (t=0):

$$\Sigma(0\,|\,0) = \mathrm{cov}\left[x(0); x(0)\right]$$

Predict Cycle:

$$\Sigma(t+1\,|\,t) = A(t)\Sigma(t\,|\,t)A^T(t) + L(t)\Xi(t)L^T(t)$$

LDS - *Generation of the innovation sequence*

Update Cycle:

$$\Sigma(t+1|t+1) = \Sigma(t+1|t) - \Sigma(t+1|t)C^T(t+1)\cdot$$

$$\cdot\left[C(t+1)\Sigma(t+1|t)C^T(t+1) + \Theta(t+1)\right]^{-1}\cdot$$

$$\cdot\, C(t+1)\Sigma(t+1|t)$$

1) *Filter Gain Matrix*

$$H(t+1) = \Sigma(t+1|t+1)C^T(t+1)\Theta^{-1}(t+1)$$

where

$H(t+1)$ is $\Re^{n\times r}$ Kalman gain matrix

LDS - *Generation of the innovation sequence*

1) On-Line Calculations

Initialization (t=0):

$$\hat{x}(0 \mid 0) = \mathrm{E}[x(0)]$$

Predict Cycle:

$$\hat{x}(t+1 \mid t) = A(t)\hat{x}(t \mid t)$$

Update Cycle:

$$r(t+1) = z(t+1) - C(t+1)\hat{x}(t+1 \mid t)$$

$$\hat{x}(t+1 \mid t+1) = \hat{x}(t+1 \mid t) + H(t+1)r(t+1)$$

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LDS - *Statistics of the innovation sequence*

It is well known that the innovation sequence $r(t+1)$ is a zero mean Gaussian

$$\mathrm{E}[r(t+1)] = 0$$

white noise sequence with covariance

$$\begin{aligned} S(t+1) &\equiv \operatorname{cov}[r(t+1); r(t+1)] \\ &= C(t+1)\Sigma(t+1 \mid t)C^T(t+1) + \Theta(t+1) \end{aligned}$$

$$\operatorname{cov}[r(t); r(\tau)] = S(t)\delta_{t\tau}$$

LDS - *Statistics of the innovation sequence*

For Testing purposes, it is more convenient to consider the Standardized Innovation Sequence:

$$\eta(t+1) = \left(C(t+1)\Sigma(t+1|\,t)C^T(t+1) + \Theta(t+1)\right)^{-\frac{1}{2}} r(t+1)$$

where $(\cdot)^{-\frac{1}{2}}$ denotes the square root of the inverse of a matrix. Then

$$\operatorname{cov}[\eta(t);\eta(\tau)] = \mathrm{E}\left[\eta(t)\eta^T(\tau)\right] = I\delta_{t\tau}$$

where I denotes the identity matrix.

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LDS - *Outlier Detection via limit value checking*

>Given the statics of the Standardized Innovation Sequence, generated by the Kalman Filter, the upper and lower thresholds on the Shewart Chart can be set.

>Therefore at the 0.27 per cent significance level, the measurement $z_i(t+1)$ is classified as outlier whenever

$$\left|\eta_i(t+1)\right| > 3$$

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LDS - *Sensor fault detection via hypothesis testing*

> Different kinds of faults can develop in the system. Some of these are:
>
> - bias errors in instruments,
> - noisy instruments,
> - change in system parameters,
> - change in level of input noise,
> - change in the structure of the system, etc.

> All these faults make the standardized innovation $\eta(t+1)$, depart from their zero mean, unit variance and whiteness properties.

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    - *Tests of whiteness*
    - *Tests of mean*
    - *Tests of covariance*

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LDS - *Sensor fault detection via hypothesis testing*

>*Normality Tests*

- Several methods can be applied for detecting departures from normality. In frequencist statistics statistical hypothesis testing, data are tested against the null hypothesis that it is normally distributed.

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LDS - *Sensor fault detection via hypothesis testing*

>*Normality Tests*

- *Anderson-Darling test*
  - *The A^2 Empirical distribution function statistics may be used with small sample sizes $5 \leq n \leq 25$. Very large sample sizes may reject the assumption of normality with only slight imperfection. The computation differs based on what is known about the distribution.*
    - Case 1: The mean and the variance are both known.
    - Case 2: Both the mean and the variance are unknown

LDS - *Sensor fault detection via hypothesis testing*

> *Normality Tests*

- *Anderson-Darling test*
  - *Although the parameters are known, the second case is considered because [Stephens,1974] claimes that the test becomes better when the parameters are computed from the data, even if they are known.*
  - *In Case 2 the parameters can be estimated as*

$$\hat{\bar{r}} = \frac{1}{N}\sum_{i=1}^{N} r_i \qquad \hat{c}_{r_0} = \frac{1}{N-1}\sum_{i=1}^{N}\left(r_i - \hat{\bar{r}}_i\right)\left(r_i - \hat{\bar{r}}_i\right)^T$$

LDS - *Sensor fault detection via hypothesis testing*

>*Normality Tests*

- *Anderson-Darling test*
  - *The values of the innovation sequence are then standardized according to the estimated parameters*

$$\hat{\eta}(t+1) = \hat{c}_{r_0}^{-\frac{1}{2}}\left(r(t+1) - \hat{\bar{r}}\right)$$

  - *The standardized Innovation Sequence* $\eta(t+1)$ *in case 1 or* $\hat{\eta}(t+1)$ *in case 2 is then sorted from low to high.*

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LDS - *Sensor fault detection via hypothesis testing*

> *Normality Tests*

- *Anderson-Darling test*

A^2 is then calculated by

$$A^2 = -N - \frac{1}{N}\sum_{i=1}^{N}\left[(2\mathrm{i}-1)\cdot\ln\left(\Phi\left(\eta_i\right)\right) + \left(\left(2\mathrm{N}-i\right)+1\right)\cdot\ln\left(1-\Phi\left(\eta_i\right)\right)\right]$$

Where $\Phi(\square)$ is the standard normal cumulative distribution function.

LDS - *Sensor fault detection via hypothesis testing*

>*Normality Tests*

- *Anderson-Darling test*

A modified statistic is calculated for Case 2:

$$A^{*2} = A^2\left(1 + \frac{4}{N} - \frac{25}{N^2}\right)$$

LDS - *Sensor fault detection via hypothesis testing*

> *Normality Tests*

- *Anderson-Darling test*

At the 5 per cent significance level, the null hypothesys is rejected whenever

$$A^2 > 2.492$$

$$A^{*2} > 0.787$$

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LDS - *Sensor fault detection via hypothesis testing*

>*Normality Tests*

- *Cramér–von Mises criterion*

  - As in the Anderson-Darling test the two cases of known and unknown parameters are considered.

  - The test also follows the same procedure for the standardization and then performes the calculations on the sorted standardized Innovation Sequence.

LDS - *Sensor fault detection via hypothesis testing*

> *Normality Tests*

- *Cramér–von Mises criterion*

W^2 is then calculated by

$$W^2 = \frac{1}{12N} + \sum_{i=1}^{N} \left(\Phi\left(\eta_i\right) - \frac{2i-1}{2N} \right)^2$$

LDS - *Sensor fault detection via hypothesis testing*

>*Normality Tests*

- *Cramér–von Mises criterion*

A modified statistic is then calculated for Case 1

$$W_1^{*2} = \left(W^2 - \frac{0.4}{N} + \frac{0.6}{N^2} \right)\left(1.0 + \frac{1.0}{N} \right)$$

and Case 2

$$W_2^{*2} = W^2 \left(1 + \frac{0.5}{N} \right)$$

LDS - *Sensor fault detection via hypothesis testing*

> *Normality Tests*
- *Cramér–von Mises criterion*

Then at the 5 per cent significance level, the null hypothesis is rejected whenever

$$W_1^{*2} > 0.461$$

$$W_2^{*2} > 0.126$$

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LDS - *Sensor fault detection via hypothesis testing*

>*Normality Tests*

- Tests of whiteness
  - *The most important property of the innovation sequence is whiteness or independence at different time instants. Most of the tests of independence are based on the autocorrelation function of a stationary process for lag $k = 1, 2, \ldots$ as follows:*

$$c_k = \mathrm{E}\left[\left(\eta_i - \overline{\eta}\right)\left(\eta_{i-k} - \overline{\eta}\right)^T\right]$$

  - c_k *is often estimated as*

$$\hat{c}_k = \frac{1}{N}\sum_{i=k}^{N}\left(\eta_i - \hat{\overline{\eta}}\right)\left(\eta_{i-k} - \hat{\overline{\eta}}\right)^T \qquad \hat{\overline{\eta}} = \frac{1}{N}\sum_{i=1}^{N}\eta_i$$

LDS - *Sensor fault detection via hypothesis testing*

>*Normality Tests*

- Tests of whiteness

  - *Under the null hypothesis,* $\hat{c}_k$*, k=1, 2, … are asymptotically independent and normal with zero mean and covariance of I/N.*

  - *Thus they can be regarded as samples from the same normal distribution and must lie in the band* $\pm 1.96/\sqrt{n}$ *more than 95 per cent of the times for the null hypothesis.*

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    - *Tests of covariance*

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LDS - *Sensor fault detection via hypothesis testing*

>*Normality Tests*

- Tests of mean
  - *These tests check whether the observed innovation sequence is zero mean or not.*
  - *The mean of the innovation sequence is estimated by*

$$\hat{\eta} = \frac{1}{N} \sum_{i=1}^{N} \eta_i$$

  - *Under the null hypothesis, has a Gaussian distribution with zero mean and covariance*

$$\mathrm{E}\left[\hat{\eta}\,\hat{\eta}^T\right] = I/N$$

LDS - *Sensor fault detection via hypothesis testing*

> *Normality Tests*

- Tests of mean
  - *Therefore at the 5 per cent significance level, the null hypothesis is rejected whenever*

$$\left|\hat{\eta}\right| > 1.96\, I \big/ \sqrt{N}$$

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    - *Tests of mean*
    - *Tests of covariance*

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LDS - *Sensor fault detection via hypothesis testing*

>*Normality Tests*

- Tests of covariance
  - *The covariance of the innovation sequence is estimated as*

$$\hat{c}_0 = \frac{1}{N}\sum_{i=1}^{N}\left(\eta_i - \hat{\bar{\eta}}\right)\left(\eta_i - \hat{\bar{\eta}}\right)^T$$

  - *Under the Under the null hypothesis,* $\hat{c}_0$ *has a WISHART Distribution. The trace of* $\hat{c}_0$ *has a Chi-Square distribution with (N-l)r degrees of freedom. Thus can be tested for its null hypothesis covariance equal to an identity matrix.*
  - *Both the tests of mean and covariance assume that the innovation sequence is white.*
    - Therefore, it is important to test the innovation sequence for whiteness first, especially using tests which are invariant with respect to the mean and covariance of the distribution.

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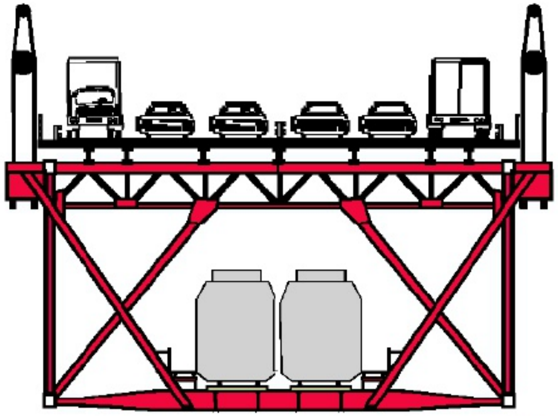
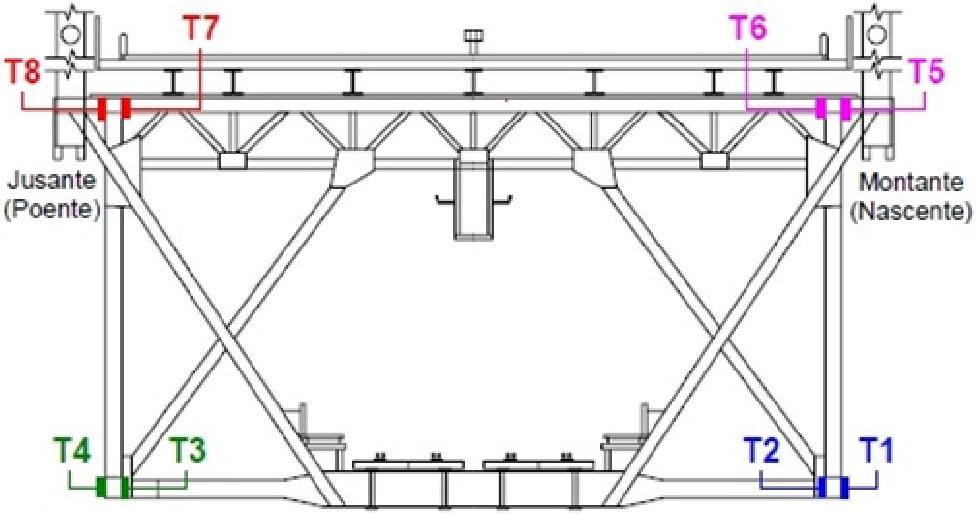
Pratical Case Studie

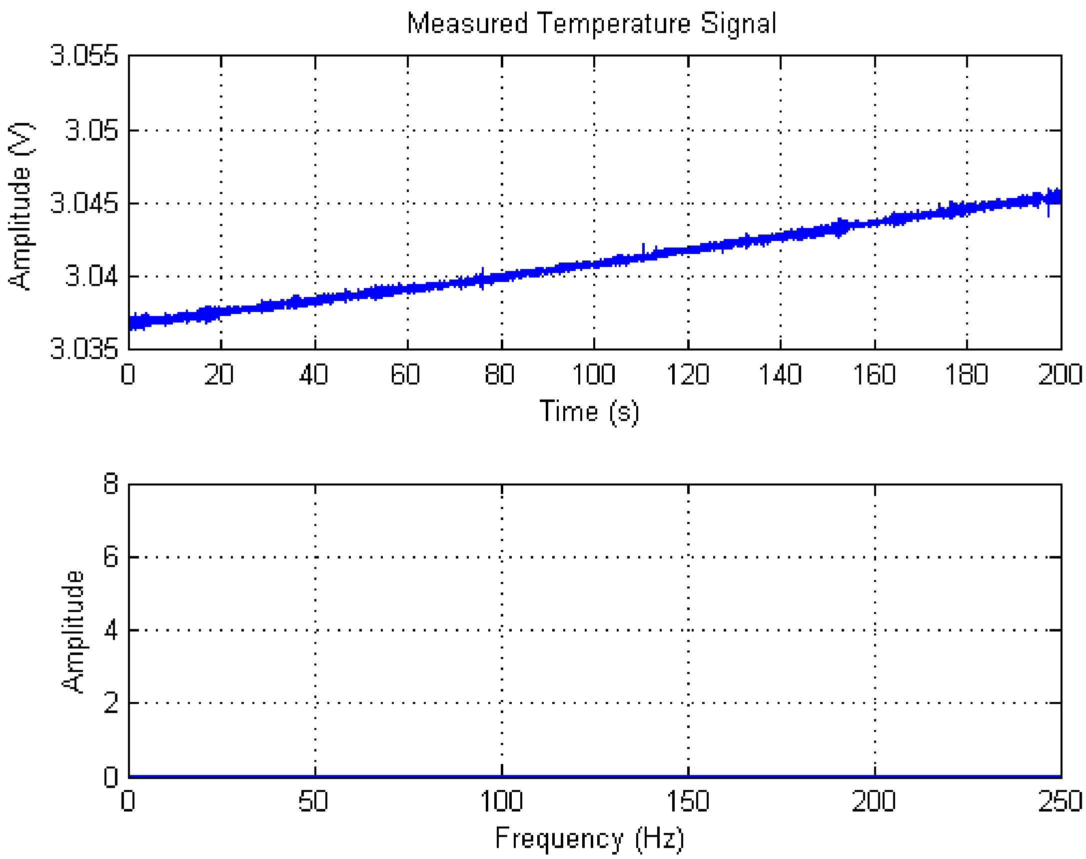
Instrumentação do tabuleiro com os extensómetros e termistores



Secção transversal do tabuleiro da Ponte 25 de Abril



Pratical Case Studie



Pratical Case Studie

The dynamics of the system where described by :

$$\begin{cases} x(t+1) = x(t) + v(t) \\ v(t+1) = v(t) + \xi(t) \end{cases}$$

The measurement model considered was:

$$Z(t+1) = x(t+1) + \theta(t+1)$$

Pratical Case Studie

For the intensities of Plant noise $\xi(t)$ and Measurement noise $\theta(t)$

where considered the following values:

$$\Xi(t) = 8.6703\mathrm{E}-16$$

$$\Theta(t) = 2.729\mathrm{E}-8$$

Pratical Case Studie

For the initialization step of the kalman filter was considered

$$x(0) = \begin{bmatrix} x(0) \\ \Xi \end{bmatrix}, \quad \Sigma_0 = \begin{bmatrix} \Theta & 0 \\ 0 & \Xi \end{bmatrix}$$

The calculated steady state filter gain was

$$H = \begin{bmatrix} 0.0187 \\ 1.7657E-4 \end{bmatrix}$$

Pratical Case Studie

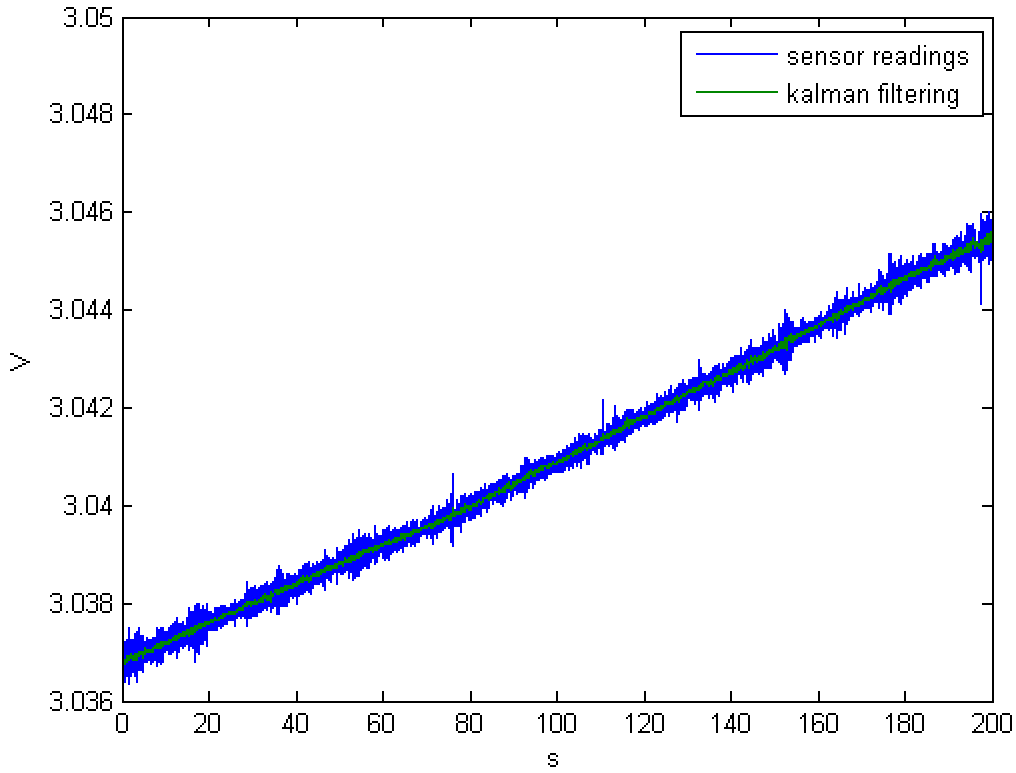
And the covariance matrices :

$$\Sigma_p = 1\text{E-}9 \begin{bmatrix} 0.5202 & 0.0049 \\ 0.0049 & 0.0001 \end{bmatrix}$$

$$\Sigma = 1\text{E-}9 \begin{bmatrix} 0.5104 & 0.0048 \\ 0.0048 & 0.0001 \end{bmatrix}$$

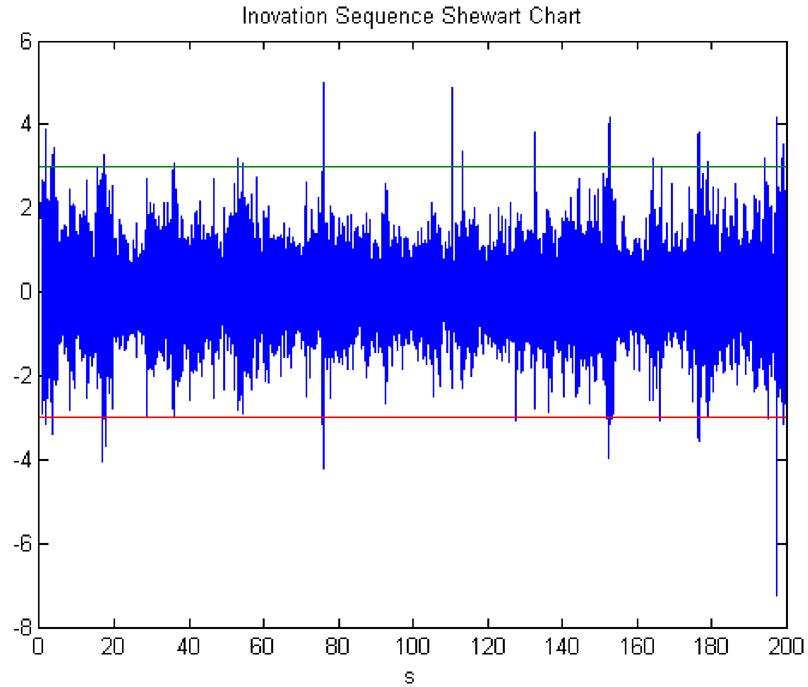
$$S(t+1) = 2.7810\text{E-}8$$

Pratical Case Studie

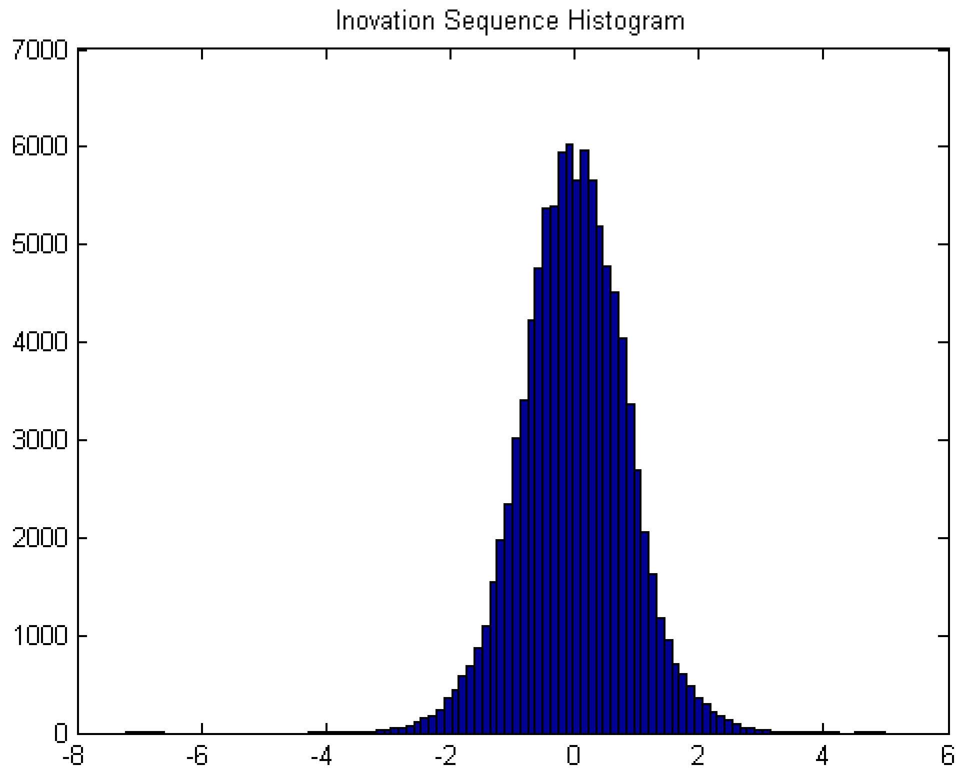


Pratical Case Studie

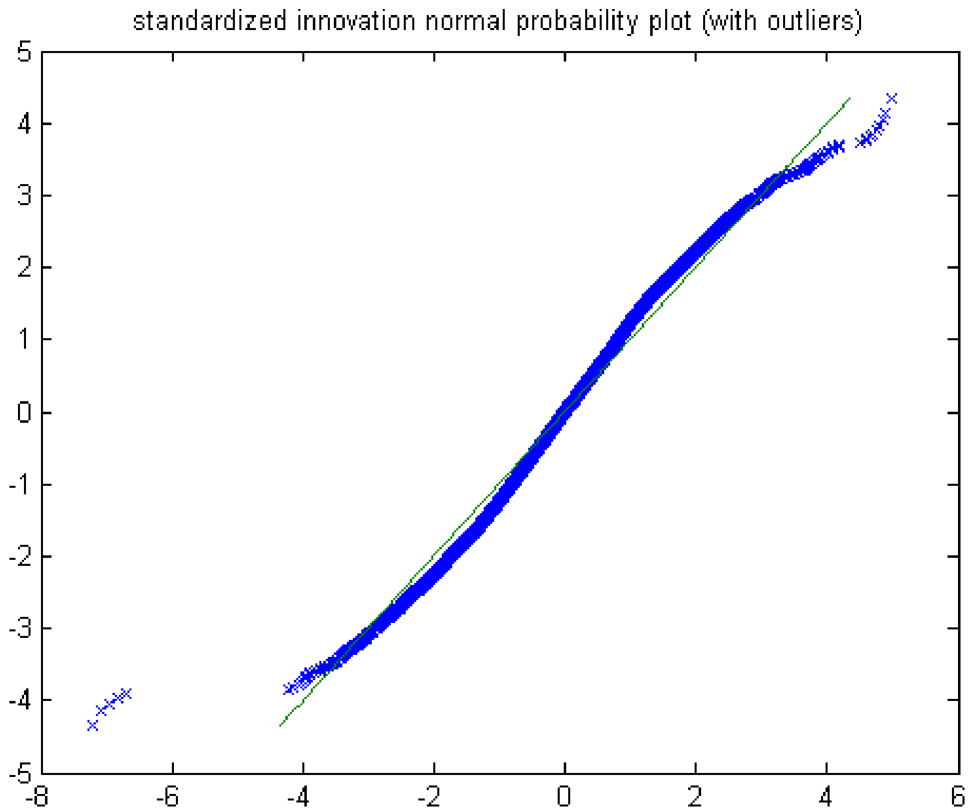
>Standardizing the innovation sequence and applying the defined threshold 225 outliers where detected witch is within the significance level since we have 0.22 per cent outliers (1E5 samples).



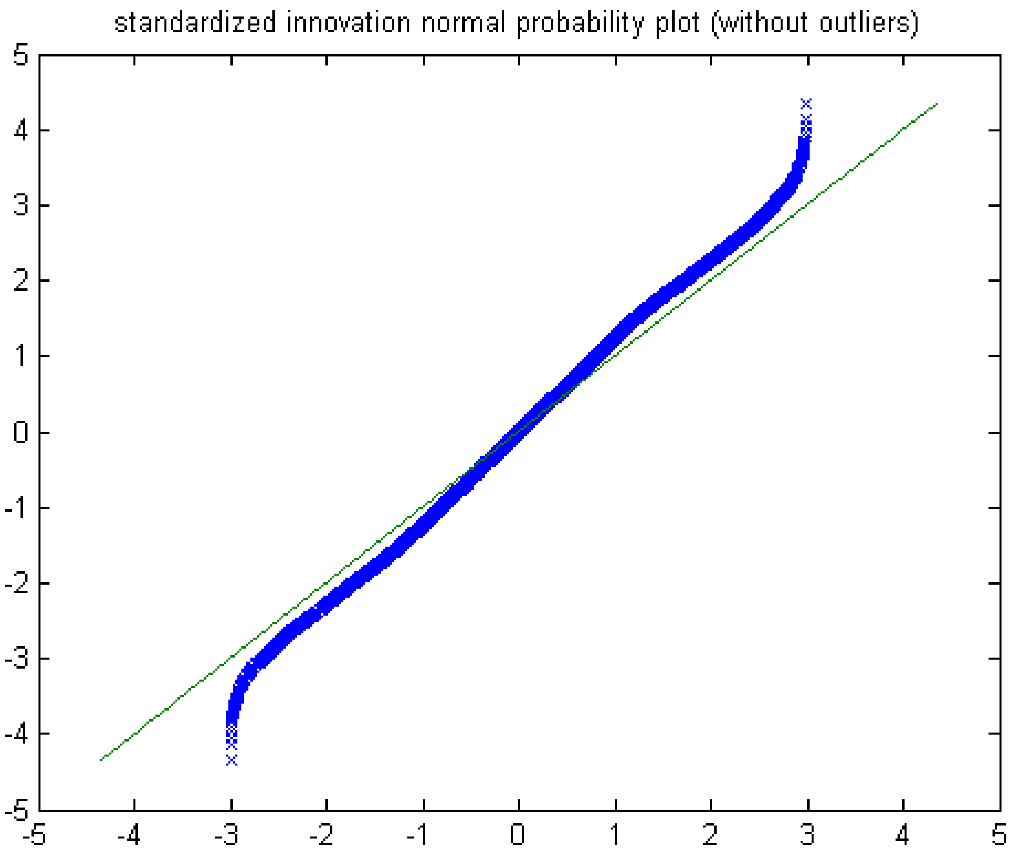
Pratical Case Studie



Pratical Case Studie



Pratical Case Studie



Pratical Case Studie

> Appling the normality tests the null hypotheses is rejected in all of them.

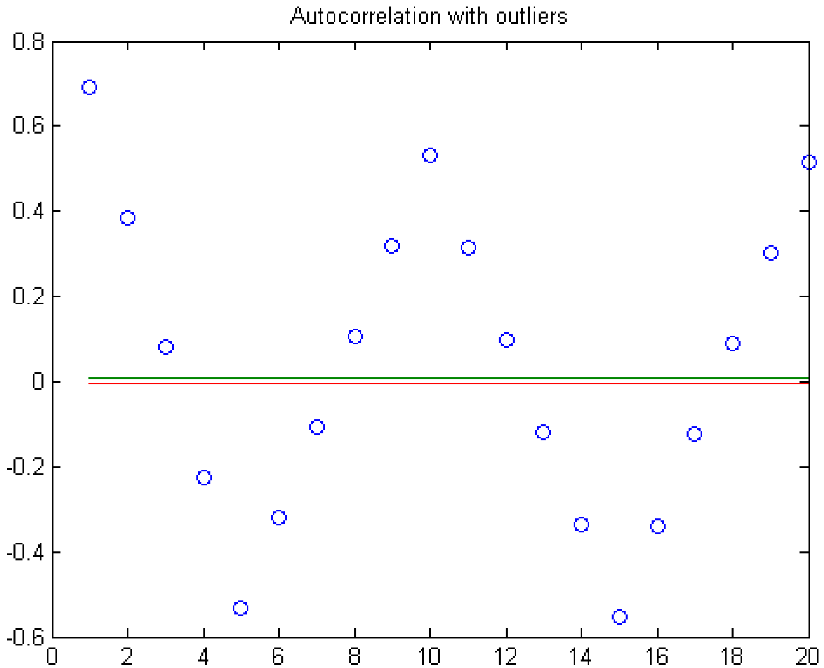
- The best results where obtained for the Cramér–von Mises criterion with the parameters estimated from the data.

NORMALITY TESTS RESULTS

Normality Test	Data with Outliers	Data without Outliers	Normal Distribution	5% Signif.
A^2	3704.9	3738	0.4825	2.492
A^{*2}	112.99	19.106	0.5477	0.787
W_1^{*2}	566.79	570.50	0.0976	0,461
W_2^{*2}	16.575	2.7471	0.0438	0,126

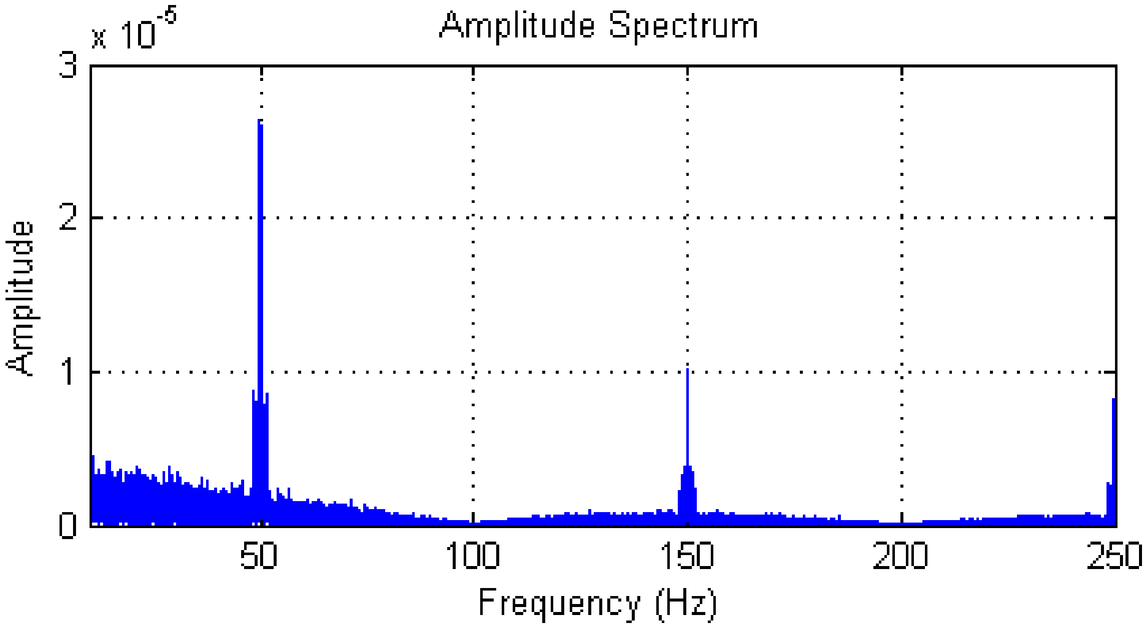
Pratical Case Studie

>Calculating the autocorrelation for the innovation sequence for the whiteness test, the null hypothesis is also rejected



Since both the tests of mean and covariance assume that the innovation sequence is white. Therefore, the test sequence should be stopped.

Pratical Case Studie



>The higher layer in the diagnostic system must consider this situation. The subsequent procedures are beyond the scope of this paper

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Final considerations

>A method for detecting faults in sensor networks based in the innovation sequence of a kalman filter was proposed.

>The proposed method was applied to a real world data measurement with successful results, since it could identify that the tested signal sensor data had a fault resulting from the interference of a 50Hz non linear noise.

>Subsequent work should consider multivariate statistical tests

Thank you for your attention!