An overview on target tracking using multiple models methods

Joana Gomes
Instituto Superior Técnico, Technical University of Lisbon
Lisbon, Portugal

Abstract

Most of the existing algorithms for multiple target tracking are composed of fixed structure multiple models. Observing the recent advances on the algorithms for single target tracking, it should also be expected that the multiple target tracking algorithms evolve from the fixed structure to the variable structure multiple models. The main contribution of this paper is the proposal of a new algorithm capable of tracking multiple targets based on a variable structure multiple model, the likely model-set, and on the joint probabilistic data filter. It is discussed how the variable structure approach can improve the tracking of targets moving close to each other by accentuating their possible different motions.

Keywords: Target Tracking, Multiple Model Estimation, Data Association

1 Introduction

The problem of target tracking dates back as far as the eighteenth century, with the first attempts to determine the orbits of visible planets. More recent work, in the early 1960s, was developed mainly for military purposes, such as ballistic missile defense, battlefield situational awareness and orbital vehicle tracking. Nowadays, an ever-growing number of civilian target tracking applications are emerging, ranging from traditional applications such as air traffic control and building surveillance to emerging applications such as supply chain management and wildlife tracking.

The ubiquitous Kalman Filter (KF) [**?**] is the most widely used filtering technique and is the basis for most of the more complex target tracking estimators. The KF is an optimal single target tracker in the presence of initial state and measurements uncertainties, if there is not target motion uncertainty nor measurement origin uncertainty (terms defined in [**?**]).

The Multiple Model (MM) estimation methods are the main stream approach in single target tracking (STT) under target motion uncertainty and in the absence of measurement origin uncertainty. Basically, these methods resolve the target motion uncertainty by using multiple models at a time for a maneuvering target, being able to explain a larger number of target motions at each time.

The Multiple Target Tracking (MTT) problem extends the single target tracking scenario to a situation where the number of targets may not be known and varies with time, leading to the presence of measurement origin uncertainty in addition to the target motion uncertainty. Besides the need to estimate the targets current state, the identities of the targets may need to be known. The usual method to solve this problem is to assign a single target tracker to each target and use a data association technique to assign the correct measurement to each target.

The remainder of this paper is outline as follows. Some basic concepts necessary to understand the algorithms described are presented in section **??**. In section **??**, the MTT problem is described and divided in two subproblems, commonly studied, the STT problem and the Data Association (DA) problem, discussed in sections **??** and **??**, respectively. In sections **??** and **??**, a common and the new MTT algorithms, respectively, are described and their performance compared in section **??**. In section **??**, the conclusions are draw and the future research purposed.

2 Basic Concepts

In this paper the terms target mode and target model will be used to address two different realities, as introduced [?]. This definition is important for nomenclature purposes on this paper. The target mode refers to the true target behavior or target motion. The target model is a mathematical usually simplified description of the of the target motion with a certain accuracy level. The tracking estimators are based on the target models, which are the known mathematical description of the target motion, and not on the true target modes.

The system mode at a given time k is represented by s_k while the event that model j is in effect at time k is expressed by $m_k^{(j)} \triangleq \{s_k = m^{(j)}\}$. Here, it is considered that the models in target tracking algorithm are a subset of the total mode set $\mathbb{S}_k$, i. e., the models have perfect accuracy but cannot explain all the target modes/motion behaviors.

The sequence of modes, and consequently the sequence of turn rates, is modeled as a Markov chain, which means that at each time step k its value may either jump or stay unchanged with transition probability, for a homogeneous Markov chain, defined by

$$\begin{aligned} P\{s_{k+1}^{(j)}|s_k^{(i)}\} &= P\{\omega_{k+1} = \omega_j | \omega_k = \omega_i\} \\ &= \pi_{ij,k} = \pi_{ij} \;\; \forall i, j, k. \end{aligned} \tag{1}$$

The target model can be casted as a Markov Jump-Linear System (MJLS), i. e., a linear system whose parameters evolve with time according to a finite state Markov chain. The discrete-time general representation of a hybrid system, where the discrete system components have a first order $\{s_k\}$-dependance, can be described by the discrete target model

$$x_{k+1} = F_k(s_k)x_k + G_k(s_k)w_k(s_k), \; w_k \sim \mathcal{N}(0, Q_k). \tag{2}$$

The sensory system has only discrete-time parameters and does not depend on the mode state, thus it will be simply defined by the equation

$$z_k = H_k x_k + v_k, \; v_k \sim \mathcal{N}(0, R_k). \tag{3}$$

In the case of multiple target scenario the sensory gives a vector containing multiple measurements.

3 Multiple Target Tracking

In the MTT problem the number of targets may not be known and can also be variable variable with time. Moreover, the measurements obtained are also not known, since they can be originated from any of the targets. False alarms or measurements originated from clutter are an extra source of complexity in realistic applications. Thus, the MTT problem is composed of the following subproblems:

1. STT: addresses the problem of tracking a single target. In the previous chapter tracking a maneuvering target using multiple models has been shown to be highly effective. As studied earlier the STT estimates the positions of a target based on noisy observations of the target and with some prior regarding the target and sensor characteristics;

2. DA: usually consists in ensuring that the correct measurement is given to each STT tracker so that the trajectories of each target can be accurately estimated.

3.1 Single Target Tracking

In the problem of single target tracking there are two main approaches: 1) Single Model (SM) based tracking and 2) MM based tracking. The first approach is quite simplistic and it is only applicable if the target mode is time invariant and known, which for most of the true life applications is unfeasible due to motion uncertainty. The second approach addresses this issue by using more than one model to describe the target motion.

In the SM based tracking approach there is one single filter based on a unique model. The function of this elemental filter is to reduce the effect of the various disturbances (system and measurement noises) on the state estimate, based on the known target mode and measurements data. The Kalman Filter [?] is one of this algorithms. On the other hand the MM algorithms use a set of models as possible candidates to describe the target at each time step k. This means that at each time instant the MM algorithms run a bank of elemental filters each based on a unique model in the set, and then compute the overall estimate based on all the estimates of each model.

Most existing MM algorithms have a fixed structure, i. e., they have a fixed model set at all times. However these fixed structure MM (FSMM) algorithms have little more room for improvement, since they have been exhaustively studied and improved over for the last decades. In an attempt to break away from the fixed structure a new force has been applied into the development of MM estimation with variable structure (VSMM), i.e, with a variable set of models. This approach intends to overcome the FSMM algorithms especially for problems involving many structural modes.

3.1.1 The Fixed Structure Multiple Models: Interacting Multiple Model algorithm

The Interacting MM (IMM) estimator was originally proposed by Bloom in [**?**]. It is one of the most cost-effective class of estimators for a single maneuvering target. The IMM has been receiving special attention in the last few years, due to its capability of being combined with other algorithms to resolve the multiple target tracking problem.

This algorithm has two fundamental assumptions i) the true mode sequence $\{s_k\}$ is Markov, i. e., the mode evolves accordingly to the memoryless random process described by $P\{s_{k+1}^{(j)}|s_k^{(i)}\} = \pi_{ij,k} \;\; \forall i, j$; ii) the true mode s at any time has a mode space $\mathbb{S}$ that is time invariant and identical to the time-invariant finite model set $\mathbb{M}$ used (i.e., $\mathbb{S}_k = \mathbb{M}, \; \forall k$).

To account the model history or model sequence, the following variable is defined

$$m^{k,l} = \{m^{(i_{1,l})}, \dots, m^{(i_{k,l})}\}, \;\; l = 1, \dots, (N)^k \tag{4}$$

where $i_{k,l}$ is the model index at time k from history l and $1 \leq i_{k,l} \leq N \; \forall k$.

As the size of model sequence increases exponential with time ($(N)^k$), a exponentially increasing number of filters are needed to optimally estimate the state thus making the optimal solution not viable. Suboptimal approaches are commonly and successfully adopted generally consisting in only keeping a limited number of model sequences associated with the largest probabilities, discard the rest, and normalize the mode probability to ensure it sum is equal to unity.

The sequence of events consisting of the true target mode sequence, $s^k = \{s_1, \ldots, s_k\}$, and the correspondent matching models sequence, $m^{(i^k)} = \{m^{(i_1)}, \ldots, m^{(i_k)}\}$, $m^{(i_n)} \in \mathbb{M}$, through time k, is denoted as

$$\begin{aligned}\{s^k = m^{(i^k)}\} &= m^k_{(i^k)} = m^k_{i_1,\ldots,i_k} \\ &= \{m^{k-1}_{(i^{k-1})}, m^{i_k}_k\} \end{aligned} \tag{5}$$

where $m^{k-1}_{(i^{k-1})}$ is a parent sequence and $m^{i_k}_k$ is the last element. For simplicity the notation $m^k_{(i^k)} \in \mathbb{M}$ will be replaced by $i^k = 1, \ldots, N^k$.

Using Baye's formula, the probability of model i is correct given measurement data up to time step k, is given by the recursion

$$\begin{aligned}\mu^{(i)}_k &\triangleq P\{m^{(i)}|z^k\} = P\{m^{(i)}|z_k, z^{k-1}\} \\ &= \frac{p[z_k|z^{k-1}, m^{(i)}]P\{M_i|z^{k-1}\}}{p[z_k|z^{k-1}]} \\ &= \frac{p[z_k|z^{k-1}, m^{(i)}]\mu^{(i)}_{k-1}}{\sum\limits_j p[z_k|z^{k-1}, m^{(j)}]\mu^{(j)}_{k-1}}\end{aligned} \tag{6}$$

where $p[z_k|z^{k-1}, m^{(i)}]$ is the likelihood function of model $m^{(i)}$, which under Gaussian assumption is given by

$$L^{(i)}_k \triangleq p[z_k|m^{(i)}, z^{k-1}] \overset{assume}{=} \mathcal{N}(\tilde{z}^{(i)}_k; 0, S^{(i)}_k) \tag{7}$$

where $\tilde{z}^{(i)}_k$ and $S^{(i)}_k$ are the residual and its covariance from the elemental filter matched to model $m^{(i)}$.

Also of interest, is the definition of the mode transition probability, which given the Markov property, can be written as

$$P\{m^{k-1}_{(i)}, m^{(j)}_k\} = P\{m^i_{k-1}, m^j_k\} \triangleq \pi_{ji} \tag{8}$$

The base state estimator under the same assumptions at time k is given by

$$\begin{aligned}\hat{x}_{k|k} = E[x_k|z_k] &= \sum_{i^k=1}^{(N)^k} E[x_k|m^k_{(i^k)}, z^k]P\{m^k_{(i^k)}|z^k\} \\ &= \sum_{j=1}^{(N)^k} \hat{x}^{(i^k)}_{k|k}\mu^k_{(i^k)}\end{aligned} \tag{9}$$

which has an exponential increasing number of terms reveling the impracticability of the optimal approach.

The IMM simplifies the optimal approach by keeping only N filters, i. e., only N hypotheses, thus

$$\begin{aligned}\hat{x}_{k|k} &= \sum_{j=1}^{N} E[x_k|m^{(j)}_k, z^k]P\{m^{(j)}_k|z^k\} \\ &= \sum_{j=1}^{N} \hat{x}^{(j)}_{k|k}\mu^k_{(j)}\end{aligned} \tag{10}$$

where the posterior mode-sequence probability is defined by

$$\mu^k_{(i^k)} = P\{m^k_{(i^k)}|z^k\} \tag{11}$$

and the posterior mode probability under the fundamental assumptions by

$$\mu^{(j)}_k = P\{m^{(j)}_k|z^k\}. \tag{12}$$

Under the assumptions defined above, the base state estimate is unbiased with covariance given by

$$\begin{aligned}P_{k|k} &= E[(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})'|z^k] \\ &\approx \sum_{j=1}^{N}[E[(x_k - \hat{x}^{(j)}_{k|k})(x_k - \hat{x}^{(j)}_{k|k})'|z^k] + \\ &(\hat{x}_{k|k} - \hat{x}^{(j)}_{k|k})(\hat{x}_{k|k} - \hat{x}^{(j)}_{k|k})']P(m^{(j)}_k|z^k) \\ &= \sum_{j=1}^{N}[P^{(j)}_{k|k} + (\hat{x}_{k|k} - \hat{x}^{(j)}_{k|k})(\hat{x}_{k|k} - \hat{x}^{(j)}_{k|k})']\mu^{(j)}_k.\end{aligned} \tag{13}$$

The simplification from equation (**??**) to equation (**??**) and from the first to the second line in the above equation, reflects the approximation that the past z^{k-1} can be explained entirely by a model-set of size N. Thus at time step $k-1$ there are only N estimates $\hat{x}^{(i)}_{k-1|k-1}$ ($\forall m_i \in \mathbb{M}$) and the associated covariances $P^{(i)}_{k-1|k-1}$ which approximately summarize the past z^{k-1}. This is one of the key features of the IMM algorithm resulting in a low computational complexity and still providing excellent state estimates. The probability of a model m_i being correct is given by **??**. Combining the model probability with the conditioned-model estimates the overall state estimate is given by

$$\hat{x}_{k|k} = \sum_i \hat{x}^{(i)}_{k|k}\mu^{(i)}_k \tag{14}$$

with an overall state covariance given by

$$P_{k|k} = \sum_i P^{(i)}_{k|k}[(\hat{x}_{k|k} - \hat{x}^{(i)}_{k|k})(\hat{x}_{k|k} - \hat{x}^{(i)}_{k|k})']\mu^{(i)}_k. \tag{15}$$

The input of each filter matched to model i consists in a mixture of the estimates $\hat{x}^{(i)}_{k-1|k-1}$ with the mixing probabilities $\mu_{k-1}^{j|i}$, i. e.,

$$\begin{aligned}\bar{x}_{k|k}^{(i)} &= E[x_{k-1}|z^{k-1}, m_k^{(i)}] \\ &= \sum_{j=1}^{N} \hat{x}_{k-1|k-1}^{(j)} P\{m_{k-1}^{(j)}|z^{k-1}, m_k^{(i)}\} \\ &= \sum_{j=1}^{N} \hat{x}_{k-1|k-1}^{(j)} \mu_{k-1}^{j|i} \end{aligned} \quad (16)$$

where the estimate $\hat{x}_{k-1|k-1}^{(i)}$ is computed by the KF based on model m_i and mixing probabilities $\mu_{k-1}^{j|i}$ are given by

$$\mu_{k-1}^{j|i} = P\{m_{k-1}^{(j)}|z^{k-1}, m_k^{(i)}\} = \frac{\pi_{ji}\mu_{k-1}^{(j)}}{\sum_i \pi_{ji}\mu_{k-1}^{(j)}}. \quad (17)$$

The complete description of IMM algorithm cycle is in table **??**. The final combination step (step 4) is not part of the recursive algorithm but a final mixture for output purposes.

3.1.2 The Variable Structure Multiple Models: Likely Model-Set algorithm

The only fundamental assumption valid to the VSMM algorithms is: i) the true mode s is Markov. Thus to guarantee that no track loss occurs a perfect match between the modes and models at all the time should be assured by the model-set adaptation, i. e., $\{s_k \in \mathbb{M}_k\}\ \forall k$. The optimal VSMM estimator, as described in [**?**] and [**?**], is also computationally unfeasible. At each time k, there is a new possible mode set $\mathbb{S}_k$ which is the union of the state dependent mode set at time k, i. e., $\mathbb{S}_k = \{\mathbb{S}_k^{(1)}, \ldots, \mathbb{S}_k^{(n)}\}$ that can be variable sized depending on the previous possible mode set $\mathbb{S}_{k-1}$ and the target's maneuvering restrictions. Thus the set of possible mode sequences through time k, defined as $\mathbb{S}^k = \{\mathbb{S}_1, \ldots, \mathbb{S}_k\}$, grows exponentially. Most of the applicable VSMM algorithms replace the set of possible mode sequences at time k by one, and hopefully the best, model-set sequence $\mathbb{M}_k$.

Defining $\mathbb{E}_n$ as a set of the models in $\mathbb{M}_{k-1}$ that are allowed to switch to the new model $m^{(n)}$, i. e.,

$$\mathbb{E}_n = \{m_l : m_l \in \mathbb{M}_{k-1}, \pi_{ln} \neq 0\}. \quad (18)$$

The initial state of the filter based on m_n can be obtained by

$$\begin{aligned}\bar{x}_{k-1|k-1}^{(n)} &= E[x_k|m_k^{(n)}, \mathbb{M}^{k-1}, z^{k-1}] \\ &= \sum_{m_l \in \mathbb{E}_n} E[x_{k-1}|m_{k-1}^{(l)}, \mathbb{M}^{k-2}, z^{k-1}] \\ &\quad P\{m_{k-1}^{(l)}|m_k^{(n)}, \mathbb{M}^{k-1}\} \\ &= \sum_{m_l \in \mathbb{E}_n} \hat{x}_{k-1|k-1}^{(l)} \mu_{k-1}^{l|n} \end{aligned} \quad (19)$$

where the sequence at time k of sets of the total model-set is $\mathbb{M}^k = \{\mathbb{M}_1, \ldots, \mathbb{M}_k\}$ and

$$\mu_{k-1}^{l|n} = P\{m_{k-1}^{(l)}|m_k^{(n)}, \mathbb{M}^{k-1}\} = \frac{\pi_{ji}\mu_{k-1}^{(j)}}{\sum_{m_j \in \mathbb{E}_n} \pi_{ji}\mu_k^{(j)}}. \quad (20)$$

The assignment of initial model probabilities for new models follows the same logic

$$\begin{aligned}&P\{m_{k-1}^{(n)}|\mathbb{M}^{k-1}, z^{k-1}\} = \\ &\sum_{m_j \in \mathbb{E}_n} P\{m_k^{(n)}|m_{k-1}^{(j)}\} P\{m_{k-1}^{(j)}|\mathbb{M}^{k-1}, z^{k-1}\}. \end{aligned} \quad (21)$$

Analyzing the equations (**??**), (**??**) and (**??**) it is clear the similarity with the IMM algorithm. This leads to the recursive variable structure version of the IMM algorithm called Variable Structure Interacting Multiple Model (VSIMM) presented in table **??**. Note that the model transition probability π_{ij} regards the total model-set and not only the current active model-set.

Also included in the VSMM task of MM estimation given a model-set is the fusion of two MM estimation. This situation arises when after obtaining the MM estimation for a model-set $\mathbb{M}_1$ the model-set adaptation algorithm decides to add to it a new set of models $\mathbb{M}_2$. The efficient solution is to add the new set in the current cycle avoiding repeating the already performed computation, this is achieved by having two distinct MM estimators and a fusion algorithm. Both MM estimators have the same model-set history $\mathbb{M}^{k-1}$ but a different active model sets $\mathbb{M}_1$ and $\mathbb{M}_2$ characterized by

$$\{\hat{x}_{k|k}^{(i)}, P_{k|k}^{(i)}, L_k^{(i)}, \mu_k^{(i)}\}_{m_i \in \mathbb{M}_l} \quad l = \{1, 2\} \quad (22)$$

which can be computed, for instance, by a cycle of the VSIMM $[\mathbb{M}_l, \mathbb{M}_{k-1}]$ algorithm (table **??**).

The optimal MM estimator based on the model set $\mathbb{M}_k = \mathbb{M}_1 \cup \mathbb{M}_2$ and the common model-set history $\mathbb{M}^{k-1}$ is given by

$$\hat{x}_{k|k} = \sum_{m_i \in \mathbb{M}_k} \hat{x}_{k|k}^{(i)} \mu_k^{(i)} \quad (23)$$

$$\begin{aligned}P_{k|k} = \sum_{m_i \in \mathbb{M}_k} & P_{k|k}^{(i)}[(\hat{x}_{k|k} - \hat{x}_{k|k}^{(i)}) \\ & (\hat{x}_{k|k} - \hat{x}_{k|k}^{(i)})']\mu_k^{(i)} \end{aligned} \quad (24)$$

where

$$\begin{aligned}\mu_k^{(i)} &\triangleq P\{m_k^{(i)}|\mathbb{M}^k = \mathbb{M}_1 \cup \mathbb{M}_2, \mathbb{M}^{k-1}, z^k\} \\ &= \frac{\mu_{k|k-1}^{(i)} L_k^{(i)}}{\sum_{m^{(j)} \in \mathbb{M}_k} \mu_{k|k-1}^{(j)} L_k^{(j)}}. \end{aligned} \quad (25)$$

If there are common models in $\mathbb{M}_1$ and $\mathbb{M}_2$, i. e., $\mathbb{M}_1 \cap \mathbb{M}_2 \neq \emptyset$,extra computation is needed to calculate the value of the model probability $\mu_k^{(i)}$.

Table 1: One Cycle of IMM Algorithm.

1. Model-conditioned reinitialization ($\forall m_i \in \mathbb{M}$) :	
Predicted model probability:	$\mu^{(i)}_{k\mid k-1} = \sum_j \pi_{ji}\mu^{(j)}_{k-1}$
Mixing probabilities:	$\mu^{j\mid i}_{k-1} = \pi_{ji}\mu^{(j)}_{k-1}/\mu^{(i)}_{k\mid k-1}$
Mixing estimate:	$\bar{x}^{(i)}_{k-1\mid k-1} = \sum_j \hat{x}^{(j)}_{k-1\mid k-1}\mu^{j\mid i}_{k-1}$
Mixing covariance:	$\bar{P}^{(i)}_{k-1\mid k-1} = \sum_j [P^{(j)}_{k-1\mid k-1} + (\bar{x}^{(i)}_{k-1\mid k-1} - \hat{x}^{(j)}_{k-1\mid k-1})(\bar{x}^{(i)}_{k-1\mid k-1} - \hat{x}^{(j)}_{k-1\mid k-1})']\mu^{j\mid i}_{k-1}$
2. Model-conditioned filtering ($\forall m_i \in \mathbb{M}$) :	
Predicted state:	$\hat{x}^{(i)}_{k\mid k-1} = F^{(i)}_{k-1}\bar{x}^{(i)}_{k-1\mid k-1}$
Predicted covariance:	$P^{(i)}_{k\mid k-1} = F^{(i)}_{k-1}\bar{P}^{(i)}_{k-1\mid k-1}(F^{(i)}_{k-1})' + Q^{(i)}_{k-1}$
Measurement residual:	$\tilde{z}^{(i)}_k = z_k - H^{(i)}_k\hat{x}^{(i)}_{k\mid k-1}$
Residual covariance:	$S^{(i)}_k = H^{(i)}_k P^{(i)}_{k\mid k-1}(H^{(i)}_k)' + R^{(i)}_k$
Filter gain:	$K^{(i)}_k = P^{(i)}_{k\mid k-1}(H^{(i)}_k)'(S^{(i)}_k)^{-1}$
Update state:	$\hat{x}^{(i)}_{k\mid k} = \hat{x}^{(i)}_{k\mid k-1} + K^{(i)}_k\tilde{z}^{(i)}_k$
Update covariance:	$P^{(i)}_{k\mid k} = P^{(i)}_{k\mid k-1} - K^{(i)}_k S^{(i)}_k (K^{(i)}_k)'$
3. Model probability update ($\forall m_i m_i \in \mathbb{M}$) :	
Model likelihood:	$L^{(i)}_k \overset{assume}{=} \mathcal{N}(\tilde{z}^{(i)}_k; 0, S^{(i)}_k)$
Model probability:	$\mu^{(i)}_k = \frac{\mu^{(i)}_{k\mid k-1}L^{(i)}_k}{\sum_j \mu^{(j)}_{k\mid k-1}L^{(j)}_k}$
4. Estimate fusion:	
Overall estimate:	$\hat{x}_{k\mid k} = \sum_i \hat{x}^{(i)}_{k\mid k}\mu^{(i)}_k$
Overall covariance:	$P_{k\mid k} = \sum_i [P^{(i)}_{k\mid k} + (\hat{x}_{k\mid k} - \hat{x}^{(i)}_{k\mid k})(\hat{x}_{k\mid k} - \hat{x}^{(i)}_{k\mid k})']\mu^{(i)}_k$

The Likely Model-Set (LMS) [**?**] algorithm belongs to the family of active model set of the VSMM algorithms. Its basis is to use a subset of the total model set as the active set for any given time. The total model set is finite and can be determined offline, prior to any measurements. The active or working model-set is determined adaptively. On the LMS algorithm this is achieved by implementing a simple rule of: 1) discarding the unlikely models; 2) keeping the significant models; and 3) activating the models adjacent to the principal models.

The adjacency of models is determined by wether one model is allowed to switch to another, if so, the first model is adjacent to the second one. By an appropriated setting of parameters, the number of principal models at any given time can be small and the number of unlikely models as large as needed. This means that the number of active models at time step k can be much smaller than the total model set leading to a computational complexity saving and possibly to an improvement in estimation.

The LMS algorithm can be decomposed in

1. Model-set adaptation: i) Model classification: Identify each model in $\mathbb{M}_{k-1}$ to be unlikely (if its probability is below the threshold t_1) , principal (if its probability exceeds the threshold t_2) or significant (if its probability is between t_1 and t_2); and ii) Model-set adaptation: Obtain $\mathbb{M}_k$ by deleting all the unlikely models and by activating all the models adjacent from any principal models in $\mathbb{M}_{k-1}$.
2. MM estimation given a model-set i) Model-set sequence conditioned estimator: Obtain at least one MM estimative using VSIMM algorithm based on the previous model-set $\mathbb{M}_{k-1}$ and a new model-set. The new and required model-set has all the models in $\mathbb{M}_{k-1}$ minus the unlikely ones. The optional new model-set contains all the models adjacent from the principal models if they were not in the new and required model-set; and ii) Fusion of two MM estimators (if existing): Obtain the optimal estimation by fusing two MM estimators using equations (**??**) and (**??**).

The LMS algorithm described in table **??** has AND logic, i. e., a model is deleted only if its probability is both below the threshold t_1 and not among the K largest. The AND logic guarantees performance and relaxes computation.

3.2 Data Association

As described earlier, to tackle the MTT it is required to solve first the DA problem. Usually the method to solve MTT is to assign a STT to each target and use a DA technique to assign the correct measurement to each target. Numerous techniques have been developed to resolve the measurement origin uncertainty, such as the Nearest Neighbour Standard Filter [**?**], the Joint Probabilistic Data Association Filter (JPDAF) [**?**], described below, and the Multiple Hypothesis Tracking Filter [?].

Table 2: One Cycle of VSIMM $[\mathbb{M}_k, \mathbb{M}_{k-1}]$ Algorithm.

1. Model-conditioned (re)initialization ($\forall m_i \in \mathbb{M}_k$):
   - Predicted model probability: $\mu^{(i)}_{k|k-1} = \sum_{m_j \in \mathbb{M}_{k-1}} \pi_{ji}\mu^{(j)}_{k-1}$
   - Mixing weight: $\mu^{j|i}_{k-1} = \pi_{ji}\mu^{(j)}_{k-1}/\mu^{(i)}_{k|k-1}$
   - Mixing estimate: $\bar{x}^{(i)}_{k-1|k-1} = \sum_{m_j \in \mathbb{M}_{k-1}} \hat{x}^{(j)}_{k-1|k-1}\mu^{j|i}_{k-1}$
   - Mixing covariance: $\bar{P}^{(i)}_{k-1|k-1} = \sum_{m_j \in \mathbb{M}_{k-1}} [P^{(j)}_{k-1|k-1} + (\bar{x}^{(i)}_{k-1|k-1} - \hat{x}^{(j)}_{k-1|k-1})(\bar{x}^{(i)}_{k-1|k-1} - \hat{x}^{(j)}_{k-1|k-1})']\mu^{j|i}_{k-1}$
2. Model-conditioned filtering ($\forall m_i \in \mathbb{M}_k$) :
   - Predicted state: $\hat{x}^{(i)}_{k|k-1} = F^{(i)}_{k-1}\bar{x}^{(i)}_{k-1|k-1}$
   - Predicted covariance: $P^{(i)}_{k|k-1} = F^{(i)}_{k-1}\bar{P}^{(i)}_{k-1|k-1}(F^{(i)}_{k-1})' + Q^{(i)}_{k-1}$
   - Measurement residual: $\tilde{z}^{(i)}_k = z_k - H^{(i)}_k\hat{x}^{(i)}_{k|k-1} - \bar{v}^{(i)}_k$
   - Residual covariance: $S^{(i)}_k = H^{(i)}_k P^{(i)}_{k|k-1}(H^{(i)}_k)' + R^{(i)}_k$
   - Filter gain: $K^{(i)}_k = P^{(i)}_{k|k-1}(H^{(i)}_k)'(S^{(i)}_k)^{-1}$
   - Update state: $\hat{x}^{(i)}_{k|k} = x^{(i)}_{k|k-1} + K^{(i)}_k\tilde{z}^{(i)}_k$
   - Update covariance: $P^{(i)}_{k|k} = P^{(i)}_{k|k-1} - K^{(i)}_k S^{(i)}_k (K^{(i)}_k)'$
3. Model probability update ($\forall m_i \in \mathbb{M}_k$):
   - Model likelihood: $L^{(i)}_k \overset{assume}{=} \mathcal{N}(\tilde{z}^{(i)}_k; 0, S^{(i)}_k)$
   - Model probability: $\mu^{(i)}_k = \frac{\mu^{(i)}_{k|k-1}L^{(i)}_k}{\sum_{m_j \in \mathbb{M}_k}\mu^{(j)}_{k|k-1}L^{(j)}_k}$
4. Estimate fusion:
   - Overall estimate: $\hat{x}_{k|k} = \sum_{m_i \in \mathbb{M}_k}\hat{x}^{(i)}_{k|k}\mu^{(i)}_k$
   - Overall covariance: $P_{k|k} = \sum_{m_i \in \mathbb{M}_k}[P^{(i)}_{k|k} + (\hat{x}_{k|k} - \hat{x}^{(i)}_{k|k})(\hat{x}_{k|k} - \hat{x}^{(i)}_{k|k})']\mu^{(i)}_k$

3.2.1 Joint Probabilistic Data Association Filter

Assume that there are targets $r = \{1, \ldots, T\}$. At each time step k there are total n measurements $z^{(j)}_k$ ($z^{(j)}_k \in \mathbb{Z}_k = \{z^1_k, \ldots, z^{n_k}_k\}$), only $n_k \leq n$ of this measurements are considered valid.

A measurement $z^{(j)}_k$ is validated if and only if at least one target r it lies inside the validation gate $G_k(r)$. The validation gate of a target r is taken to be the same for all models in $\mathbb{M}$ and chosen as the largest of them. The validation gate for each target r is given by

$$G_k(r) = \{y^{(j)}_k = z^{(l_j)}_k : [z^{(l_j)}_k - \hat{z}^{(i_r),(r)}_{k|k-1}]' (S^{(i_r),(r)}_k)^{-1}[z^{(l_j)}_k - \hat{z}^{(i_r),(r)}_{k|k-1}] \leq \gamma\} \quad (26)$$

where i_r is the index of the model in the model set corresponding to the largest residual covariance, i. e.,

$$i_r := \text{argmax}_{i \in \mathbb{M}} \det(S^{(i)}_k). \quad (27)$$

Also, γ is an appropriated threshold, $S^{(i_r),(r)}_k = H^{(i_r)}_k P^{(i_r),(r)}_{k|k-1}(H^{(i_r)}_k)' + R^{(i_r)}_k$ is the largest model-conditioned residual covariance for the target r and $\hat{z}^{(i_r),(r)}_{k|k-1} = H^{(i_r)}_k \hat{x}^{(i_r),(r)}_{k|k-1}$ is the correspondent predicted measurement.

Ideally the value chosen for γ should allow the IMM-JPDAF to work as T isolated IMM algorithms when the targets are far apart. The set of validated measurements is denominated $\mathbb{Y}_k = \{y^1_k, \ldots, y^{n_k}_k\}$ with $n_k \leq n$.

The key to the JPDAF algorithm is the definition of the marginal events θ_{jr} and the evaluation of their the conditional joint probabilities. A marginal association event θ_{jr} is said to be effective at time k when a validated measurement $y^{(j)}_k$ is associated with a target r, i.e., $y^{(j)}_k \in G_k(r)$. A joint association event Θ happens when a set of marginal events holds true simultaneously, i. e.,

$$\Theta = \bigcap_{j=1}^{n_k} \theta_{jr_j} \quad (28)$$

where r_j is the index of the target to which the measurement $y^{(j)}_k$ is associated with.

The validation matrix is given by

$$\Omega = [\omega_{jr}],\ j \in \{1 \ldots n_k\} \text{ and } r \in \{1 \ldots T\} \quad (29)$$

where ω_{jr} is a binary variable indicating whether measurement j lies in the validation gate of target r in event Θ.

Based on the validation matrix Ω, a association event Θ may be represented by the matrix

$$\hat{\Omega} = [\hat{\omega}_{jr}],\ j \in \{1 \ldots n_k\} \text{ and } r \in \{1 \ldots T\} \quad (30)$$

where

$$\hat{\omega}_{jr} = \begin{cases} 1 & \text{if } \theta_{jr} \subset \Theta \\ 0 & \text{otherwise} \end{cases}. \quad (31)$$

A feasible association event is one where each measurement has only one true source, i.e., $\sum_{r=0}^{N} \hat{\omega}_{jr} =$

Table 3: One Cycle of LMS Algorithm.

1. Increase the time counter k by 1. Run the VSIMM $[\mathbb{M}_k, \mathbb{M}_{k-1}]$ cycle.
2. Classify all the models m_i's in $\mathbb{M}_k$ to be principal (i.e., $\mu_k^i > t_2$), unlikely (i.e., $\mu_k^i < t_1$) or significant (i.e., $t_1 \geq \mu_k^i \leq t_1$). Let the set of unlikely models be $\mathbb{M}_u$. If there is neither unlikely nor principal model, output $\hat{x}_{k|k}$, $P_{k|k}$ and $\{\mu_k^{(i)}\}_{m^{(i)} \in \mathbb{M}_k}$, let $\mathbb{M}_{k+1} = \mathbb{M}_k$ and go to step 1.
3. If there is no principal model, then let $\mathbb{M}_a = \emptyset$ and got to step 4. Otherwise, identify the set $\mathbb{M}_a$ of all models adjacent to any principal model. Find the set of new models $\mathbb{M}_n = \mathbb{M}_a \bigcap \bar{\mathbb{M}}_k$ (where $\bar{\mathbb{M}}_k$ is the complement of $\mathbb{M}_k$ and the union set $\mathbb{M}_k := \mathbb{M}_n \bigcup \mathbb{M}_k$.
   Then

   - Run VSIMM $[\mathbb{M}_n, \mathbb{M}_{k-1}]$ cycle, where $\mathbb{M}_n$ is the set of new and only new models.
   - Fusion: Calculate the estimates, error covariances, and mode probabilities for the union set $\mathbb{M}_k$:

$$\mu_k^{(i)} = \frac{\mu_{k|k-1}^{(i)} L_k^{(i)}}{\sum_{m_j \in \mathbb{M}_k} \mu_{k|k-1}^{(j)} L_k^{(j)}}, \forall m_i \in \mathbb{M}_k$$

$$\hat{x}_{k|k} = \sum_{m_i \in \mathbb{M}_k} \hat{x}_{k|k}^{(i)} \mu_k^{(i)}$$

$$P_{k|k} = \sum_{m_i \in \mathbb{M}_k} P_{k|k}^{(i)} [(\hat{x}_{k|k} - \hat{x}_{k|k}^{(i)})(\hat{x}_{k|k} - \hat{x}_{k|k}^{(i)})'] \mu_k^{(i)}$$

   where the estimates $\{\hat{x}_{k|k}^{(i)}\}$, error covariances $\{P_{k|k}^{(i)}\}$, likelihoods $\{L_k^{(i)}\}$, and predicted probabilities $\{\mu_{k|k-1}^{(j)}\}$ were obtained in the above VSIMM $[\mathbb{M}_k, \mathbb{M}_{k-1}]$ and VSIMM $[\mathbb{M}_n, \mathbb{M}_{k-1}]$ cycles.

4. Output $\hat{x}_{k|k}$, $P_{k|k}$ and $\{\mu_k^{(i)}\}_{m^{(i)} \in \mathbb{M}_k}$.
5. If there is no unlikely model, go to step 1; otherwise, identify the discardable set $\mathbb{M}_d = \mathbb{M}_u \bigcap \bar{\mathbb{M}}_a$, that is, the set of unlikely models that are not adjacent from any principal model.
6. Eliminate the models in $\mathbb{M}_d$ from $\mathbb{M}_k$ that have the smallest probability such that $\mathbb{M}_k$ has at least K models, that is, let the likely-model set be $\mathbb{M}_l = \mathbb{M}_k - \mathbb{M}_m$, where $\mathbb{M}_m$ is the set of models in $\mathbb{M}_d$ with smallest probabilities such that $\mathbb{M}_l$ has at least K models.
7. Let $\mathbb{M}_{k+1} = \mathbb{M}_l$ and go to step 1.

$1 \; \forall j$ (where $r = 0$ indicates the measurement is a false alarm, originated from clutter for instances) and where at most one measurement is originate from each target $\delta_r := \sum_{j=0}^{n_k} \hat{\omega}_{jr} \leq 1 \; (\forall r \in \{1, \dots, T\})$,also called target indicator. Other important indicators using the permutation matrix information are the binary measurement association indicator, given by $\tau_j := \sum_{r=1}^{N} \hat{\omega}_{jr}$, with $j = \{1, \dots, n_k\}$ which indicates whether the validated measurement $y_k^{(j)}$ is associated with a target in event Θ; and the number of false alarms (unassociated measurements) given by $\phi = \sum_{j=1}^{n_k} [1 - \tau_j]$.

The marginal association probability is the sum of the probabilities of the joint association events given that measurement j belongs to target r

$$\beta_{jr} = \sum_{\Theta} P\{\Theta | z^k\} \hat{\omega}_{jr}[\Theta], \quad j = 1, \dots, n_k \;\; r = 1, \dots, T \tag{32}$$

where, assuming Gaussian distribution for the residual,

$$P\{\Theta | z^k\} = \frac{1}{c} \frac{\phi!}{V^{\phi}} \prod_{j=1}^{n_k} \{\mathcal{N}(y_k^{(j)}; \hat{z}^{r_j}, S^{r_j})\}^{\tau_j} \prod_{r=1}^{T} (P_D^r)^{\delta_r} (1 - P_D^r)^{1-\delta_r} \tag{33}$$

where c is a normalization factor, V is the volume limited by the validation gate, r_j is the index of the target that measurement j is associated with and P_D^r is the probability of detection of target r.

In table **??**, the full description of the model-conditioned JPDAF algorithm is presented. In order to demonstrate the use of the JPDAF algorithm in solving the DA problem it is incorporated with the filtering process of the IMM or of the VSIMM, which in turn are based on the Kalman filtering process. Besides the DA steps, the state covariance update step has also novelty: its last term is a positive semidifinite matrix which increases the update covariance and the uncertainty of the state estimation due to incorrect measurements.

Table 4: Model-conditioned JPDAF Algorithm.

For all targets r, $(r \in \{1 \dots T\})$:	
Model-conditioned joint probabilistic data association filtering ($\forall m_i \in \mathbb{M}$ and $j = \{1, \dots, n_k\}$) :	
Predicted state:	$\hat{x}_{k\|k-1}^{(i),(r)} = F_{k-1}^{(i)} \bar{x}_{k-1\|k-1}^{(i),(r)}$
Predicted covariance:	$P_{k\|k-1}^{(i),(r)} = F_{k-1}^{(i)} \bar{P}_{k-1\|k-1}^{(i),(r)} (F_{k-1}^{(i)})' + Q_{k-1}^{(i)}$
Measurement validation:	Find $y_k^{(j)} \in G_k(r)$
Association probability:	$\beta_{jr} = \sum_{\Theta} P\{\Theta\|z^k\} \hat{\omega}_{jr}[\Theta]$
Measurement residual:	$\tilde{z}_k^{(ji),(r)} = y_k^{(j)} - H_k^{(i)} \hat{x}_{k\|k-1}^{(i),(r)}$
Weighted measurement residual:	$\tilde{z}_k^{(i),(r)} = \sum_{j=1}^{n_k} \beta_{jr} \tilde{z}_k^{(ji)}(r)$
Residual covariance:	$S_k^{(i),(r)} = H_k^{(i)} P_{k\|k-1}^{(i),(r)} (H_k^{(i)})' + R_k^{(i)}$
Filter gain:	$K_k^{(i),(r)} = P_{k\|k-1}^{(i)}(r) (H_k^{(i)})' (S_k^{(i),(r)})^{-1}$
Update state:	$x_{k\|k}^{(i),(r)} = x_{k\|k-1}^{(i),(r)} + K_k^{(i),(r)} \tilde{z}_k^{(i),(r)}$
Update covariance:	$P_{k\|k}^{(i),(r)} = P_{k\|k-1}^{(i),(r)} - (\sum_{j=1}^{n_k} \beta_{jr}) K_k^{(i),(r)} S_k^{(i),(r)} (K_k^{(i),(r)})' + K_k^{(i),(r)} [\sum_{j=1}^{n_k} \beta_{jr} \tilde{z}_k^{(ji),(r)} (\tilde{z}_k^{(ji),(r)})' - \tilde{z}_k^{(i),(r)} (\tilde{z}_k^{(i),(r)})'] (K_k^{(i),(r)})'$

3.3 Interacting Multiple Model Joint Probabilistic Data Association Filter

The novelty regarding the classic IMM in table **??** appears in the filtering step (step 2) which is replaced by the model-conditioned JPDAF Algorithm in table **??**, where the marginal association probability is introduced in the computation of the weighted measurement residual and on each local estimative off the state covariance. Thus to prevent redundancy the algorithm table description will be omitted.

3.4 Likely Model-Set Joint Probabilistic Data Association Filter

This algorithm was elaborated as we believe that the evolution of the MTT algorithms will necessary pass by the VSMM. The improvement that these algorithms can bring to the MTT algorithms performance are the direct consequence of the improvement on they have already brought to STT problem. Furthermore, by having different model-sets associated with each target, the DA performance can be improved.

The new algorithm combines two previous described approaches in this paper: the LMS algorithm and JPDAF algorithm. Analogously, to the IMM-JPDAF, this algorithm will be named LMS-JPDAF. The first step in the description of this new algorithm is the redefinition of the VSIMM algorithm, into a VSIMM-JPDAF, i. e., an algorithm that combines the JPDAF with the variable structure IMM algorithm. This algorithm appears as an empirical variable structure adaptation of the IMM-JPDAF and is described in table **??**.

In relation to the DA problem, in this algorithm it is solved regarding only the active model-set at each time instant, reducing the computational burden and making it easier to determine which measurement is associated with each target. For each cycle of the LMS-JPDAF, the DA association problem should only be solved once, thus there is the need to define an adapted VSIMM-JPDAF, which will be named VSIMM-JPDAF*. The VSIMM-JPDAF* will be run whenever there are newly activated models and uses the association probability β previously calculated. This algorithm is also described in table **??**.

The LMS-JPDAF algorithm has exactly the same structure as the LMS algorithm described in table **??**, only the steps regarding the VSIMM (or VSIMM $[\mathbb{M}_n, \mathbb{M}_{k-1}]$) algorithm are replace by the VSIMM-JPDAF $[\mathbb{M}_k, \mathbb{M}_{k-1}]$ (or VSIMM-JPDAF* $[\mathbb{M}_n, \mathbb{M}_{k-1}]$) cycle.

4 Performance Evaluation

The target is considered to be described by a constant turn model [**?**] or a constant velocity model [?] depending on its current angular velocity ω. The sensor is a RADAR and is considered to be placed at the origin of the Cartesian axis, i. e., is placed in the coordinates $(x, y) = (0, 0)$, and the sensor coordinate system is polar providing in each measurement the range r and the bearing θ of the target. Following [**?**], it is possible to linearize the sensor measurements in Cartesian coordinates regarding that $\frac{r\sigma_\theta^2}{\sigma_r} < 0.4$ and $\sigma_\theta < 0.4$ rad [**?**], where σ_θ and σ_r are the angle and range uncertainties, respectively.

The mean value of a total of 30 simulations was used to compute the results presented below. The example has a fixed number of targets (two) and no clutter. The two targets cross each others paths once, and for that single time a target-measurement association switch occurs. Target 1 has a fixed angular velocity whereas target 2 has a variable angular velocity.

The mean value of β for each of the algorithms is shown in figure **??**. From this example, its possi-

Table 5: One Cycle of VSIMM-JPDAF and VSIMM-JPDAF* Algorithm.

For all targets r, $(r \in \{1 \dots T\})$:

1. Model-conditioned reinitialization $(\forall m_i \in \mathbb{M}_k)$:
   - Predicted model probability: $\mu_{k|k-1}^{(i),(r)} = \sum_{m_j \in \mathbb{M}_{k-1}} \pi_{ji}^{(r)} \mu_{k-1}^{(j),(r)}$
   - Mixing probabilities: $\mu_{k-1}^{j|i,(r)} = \pi_{ji}^{(r)} \mu_{k-1}^{(j),(r)} / \mu_{k|k-1}^{(i),(r)}$
   - Mixing estimate: $\bar{x}_{k-1|k-1}^{(i),(r)} = \sum_j \hat{x}_{k-1|k-1}^{(j),(r)} \mu_{k-1}^{j|i,(r)}$
   - Mixing covariance: $\bar{P}_{k-1|k-1}^{(i),(r)} = m_j \in \mathbb{M}_{k-1} [P_{k-1|k-1}^{(j),(r)} + (\bar{x}_{k-1|k-1}^{(i),(r)} - \hat{x}_{k-1|k-1}^{(j),(r)})(\bar{x}_{k-1|k-1}^{(i),(r)} - \hat{x}_{k-1|k-1}^{(j),(r)})'] \mu_{k-1}^{j|i,(r)}$
2. Model-conditioned joint probabilistic data association filtering $(\forall m_i \in \mathbb{M}_k$ and $j = \{1, \dots, n_k\})$:
   - Predicted state: $\hat{x}_{k|k-1}^{(i),(r)} = F_{k-1}^{(i)} \bar{x}_{k-1|k-1}^{(i),(r)}$
   - Predicted covariance: $P_{k|k-1}^{(i),(r)} = F_{k-1}^{(i)} \bar{P}_{k-1|k-1}^{(i),(r)} (F_{k-1}^{(i)})' + Q_{k-1}^{(i)}$
   - * Measurement validation: Find $y_k^{(j)} \in G_k(r)$
   - * Association probability: $\beta_{jr} = \sum_{\Theta} P\{\Theta | z^k\} w_{jr}[\Theta]$
   - Measurement residual: $\tilde{z}_k^{(ji),(r)} = y_k^{(j)} - H_k^{(i)} \hat{x}_{k|k-1}^{(i),(r)}$
   - Weighted measurement residual: $\tilde{z}_k^{(i),(r)} = \sum_{j=1}^{n_k} \beta_{jr} \tilde{z}_k^{(ji)}(r)$
   - Residual covariance: $S_k^{(i),(r)} = H_k^{(i)} P_{k|k-1}^{(i),(r)} (H_k^{(i)})' + R_k^{(i)}$
   - Filter gain: $K_k^{(i),(r)} = P_{k|k-1}^{(i)}(r) (H_k^{(i)})' (S_k^{(i),(r)})^{-1}$
   - Update state: $x_{k|k}^{(i),(r)} = x_{k|k-1}^{(i),(r)} + K_k^{(i),(r)} \tilde{z}_k^{(i),(r)}$
   - Update covariance: $P_{k|k}^{(i),(r)} = P_{k|k-1}^{(i),(r)} - (\sum_{j=1}^{n_k} \beta_{jr}) K_k^{(i),(r)} S_k^{(i),(r)} (K_k^{(i),(r)})' + K_k^{(i),(r)} [\sum_{j=1}^{n_k} \beta_{jr} \tilde{z}_k^{(ji),(r)} (\tilde{z}_k^{(ji),(r)})' - \tilde{z}_k^{(i),(r)} (\tilde{z}_k^{(i),(r)})'] (K_k^{(i),(r)})'$
3. Model probability update $(\forall m_i \in \mathbb{M}_k)$:
   - Model likelihood: $L_k^{(i),(r)} \overset{assume}{=} \mathcal{N}(\tilde{z}_k^{(i),(r)}; 0, S_k^{(i),(r)})$
   - Model probability: $\mu_k^{(i),(r)} = \frac{\mu_{k|k-1}^{(i),(r)} L_k^{(i),(r)}}{\sum_j \mu_{k|k-1}^{(j),(r)} L_k^{(j),(r)}}$
4. Estimate fusion:
   - Overall estimate: $\hat{x}_{k|k}^{(r)} = \sum_{m_i \in \mathbb{M}_k} \hat{x}_{k|k}^{(i),(r)} \mu_k^{(i),(r)}$
   - Overall covariance: $P_{k|k}^{(r)} = \sum_{m_i \in \mathbb{M}_k} [P_{k|k}^{(i),(r)} + (\hat{x}_{k|k}^{(r)} - \hat{x}_{k|k}^{(i),(r)})(\hat{x}_{k|k}^{(r)} - \hat{x}_{k|k}^{(i),(r)})'] \mu_k^{(i),(r)}$

* These two steps are omitted on the VSIMM-JPDAF* algorithm.

ble to observe that the LMS-JPDAF provides a softer, though longer, measurement and target association switch. This provides a softer error, but also increases this algorithm's inertia to change.

The mean of the root squared error for the position and velocity of each target for the 30 simulations is shown in figure **??**. Although the overall error is larger for the LMS-JPDAF, specially at the beginning of the simulation, the peak during the targets crossover and data association switch is much smaller, since this is the highest value of the error, the LMS-JPDAF can be useful for applications where the error maximum needs to be smaller.

5 Conclusions and Future Work

The LMS-JPDAF algorithm revealed to be a successful MTT algorithm, but the assumptions made, regarding the false alarms, presence of clutter and the number of detected targets, reduced drastically the measurement origin uncertainty. Nevertheless, the results were quite satisfactory. For the example analyzed the LMS-JPDAF algorithm surpassed the IMM-JPDAF performance under measurement origin uncertainty, although provided a worse STT when the targets were far apart. The LMS-JPDAF algorithm needs further study to make it a competitor of the available MTT algorithms. It would be interesting to analyze possible mathematical designs of the model probability thresholds t_1 and t_2, since the algorithm is quite sensitive to them, and study whether they should be static or adaptive.

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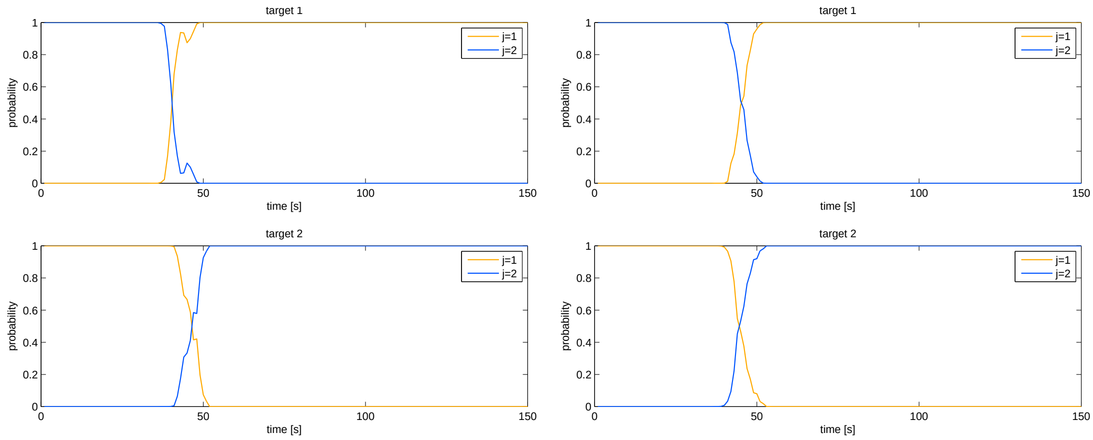
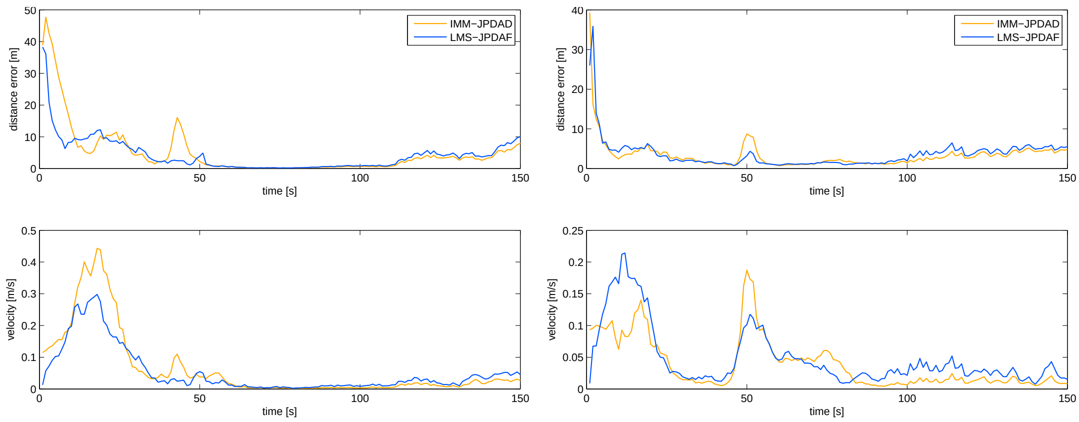


(a) IMM-JPDAF algorithm. (b) LMS-JPDAF algorithm.

Figure 1: IMM-JPDAF and LMS-JPDAF measurement and target association regarding the mean value of β for 30 simulations.



(a) target 1 (b) target 2

Figure 2: IMM-JPDAF and LMS-JPDAF mean of the root squared error for the position and velocity of each target for 30 simulations.

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