Nonlinear Marine Animals Tracking System from Multiple USBL/INS Units

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Abstract— **This thesis presents a new sensor fusion technique for tracking of underwater targets, with application to marine animal study, from multiple Ultra Short Baseline (USBL) receiver arrays. The proposed strategy is based on a marked target and relies on acoustic signal Direction of Arrival (DoA) information provided by the arrays and array relative positioning information. Two methods of obtaining the target position are devised based on the available spatial information. Least Squares (LS) and Kalman estimation techniques are applied in filtering approaches designed according to Recursive Least Squares (RLS), Kalman, and Extended Kalman Filter (EKF) methods, which yield increase position estimate accuracy and add velocity and acceleration estimation. The performance of the obtained solutions is evaluated and compared using simulation.**

I. INTRODUCTION

A. Motivation

The importance of water in the existence of life has been one of the main driving forces of evolutionary processes, and its predominance across the Earth's surface has shaped human progress since the beginning of history. Some of the most important human settlements have been located in the vicinity of large bodies of water, be it rivers, lakes or oceans. Beyond the essential function of sustaining life, these bodies of water have found themselves deeply rooted in mankind's growth, progress, survival and culture for their great potential as sources of food and mineral wealth, their functions as ways of communication and transportation, their importance in energy generation and even for their leisure value.

Most recently, some of the gravest concerns regarding this seemingly immense environment are the depletion of marine food reserves due to overfishing and the effect of human activities in coastal waters in the health of marine flora and fauna. In order to study and understand the precise mechanisms that are at work in such situations, multidisciplinary teams of researchers involving marine biologists, scientists and engineers have been constituted into groups around the world. Additionally, this sort of study adds undeniable value in scientific knowledge from which expected and unexpected advances may arise as a result of observing the effects of pollution, fishing, transportation, oil drilling and many other activities on the health, behavior and migratory patterns of various marine species of interest.

This work was supported by project FCT PEst-OE/EEI/LA0009/2013 and by project FCT MAST/AM - PTFC/EEA - CRO/111107/2009

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Although a long running field of study, the tools used in this area of research have not developed much in recent years due to a number of issues including development and upgrade costs, backward compatibility considerations, maintenance concerns and others. The present work aims to explore advanced tracking techniques to be used in low cost tools with the intent of helping researchers by obtaining more accurate movement data on targeted individuals of marine animal populations.

B. State of the Art

Most current marine animal tracking systems in use worldwide work with electronic tags which can be broadly categorized as archival tags, transmission tags, or acoustic tags.

Archival tags work by collecting data such as time, water pressure, animal and water temperatures, and even satellite position. These are attached internally or externally to an animal and must be recovered in order to access the collected information. This may be accomplished through the recapture of the tagged animal or by pop-up mechanisms, which consists of the tag detaching itself from the tracked individual and floating to the surface.

Transmission tags gather similar information to archival tags. These, however, do not require recovery of the implanted hardware to recover the information gathered. By limiting the hardware to externally implanted tags, the data gathered can be remotely downloaded by researchers when the animals surface via satellite up-link or, if the tracked individual regularly visit coastal waters, via mobile communication networks.

The first two categories of tags incur in high deployment costs, the former due to the cost of having two missions, one for deployment and one for recovery, and the latter due to the higher complexity of the hardware and, especially in the case of the satellite up-link tags, the cost of the download bandwidth.

A less costly option are the acoustic category of tags. These can also be implanted internal or externally and transmit at semi-regular intervals acoustic pulses which may contain encoded identification, temperature and pressure data. The emitted pulses are then detected and decoded by a receiver if in range. The majority of acoustic tracking material is by, or compatible with, Canadian company VEMCO© receivers and transmitters.

Concerning the acoustic tags, these consist of pinger-type implantable devices of varying size that upon activation emit omni-directional semi-periodic acoustic pulses in which can be encoded information including identification, water pressure, and temperature data. Due to the level of complexity and specialization of these emitters, since there is already a wide market base using the systems, and that both emitter detection and its associated identification information decryption is available, these will be viewed as the standard and the work will be developed considering their limitations.

As for the case of detectors, commercially available solutions are restricted to fixed or mobile presence detectors and manually operated directional detectors. The latter make use of directional hydrophones for which cost increases with precision.

A typical scenario of tracking animals with these systems requires implantation of the acoustic pinger in the intended target and the deployment of presence detectors in specified locations. These detectors use a single omni-directional hydrophone and can only mark the presence of the intended target in its effective range, logging it with any information carried by the signals. Mostly deployed in buoys, the information provided by such detectors is limited and implies several deployments with overlapping ranges in order to extract minimal and error prone trajectory information. This information has the added drawback of not being available in real-time, depending on data collection and cross-referencing.

In order to gather real-time precision information on animals directions relative to the receiver, directional hydrophones are used. However, due to the high costs, fragility, and the need for mechanical scanning in order to track a target, these are handled manually by researchers.

Parallelly, position tracking systems have been studied for underwater applications and most used systems use arrays of omni-directional hydrophones or transducers in order to extract signal DoA from differences in Time of Arrival (ToA) between pairs of hydrophones using their spacial diversity. These arrays are usually divided into three major categories: Long Baseline (LBL), Short Baseline (SBL) and USBL. The LBL arrays are fixed arrays that entail distances between transducers of hundreds of meters making these very expensive with a very complex calibration process and a high deployment time. The SBL arrays, are typically hull mounted arrays with distances in the tens of meters and that require constant monitoring of these distances due to the natural deforming of the structures that house the transducers. The USBL arrays are an evolution of the SBL systems with distances in the order of tens of centimeters that can be factory calibrated due to their smaller size and lesser deformation susceptibility. Furthermore, the latter, due to their reduced size, are flexible in mounting and deployment.

Although USBL systems are commercially available, this work will be based on the assumption that an in-house solution being developed in parallel, the MAST-AM tool, will be used. This allows for full control over the reception hardware and detection algorithms.

C. Notation

Throughout the paper, a bold symbol stands for a multi-dimensional variable, the symbol $\mathbf{0}$ denotes a matrix of zeros and $\mathbf{I}$ an identity matrix, both of appropriate dimensions.

II. PROBLEM STATEMENT

The object of this work consists in determining the position of a moving target in an underwater environment through the use of acoustic signals. The target is equipped with an acoustic pinger type marker which produces a signal in which an identification number may be encoded. In order to determine the position of this target, two receivers are available. A manned underwater tool used for aiding a diver in the identification, tracking and observation of the target, and a surface transponder, used for precise target positioning and diver localization in an inertial frame. Both the tool and the transponder are equipped with hydrophone arrays in an inverted-USBL configuration, which are used to obtain a Direction of Arrival (DoA) from a received signal. The assumed mission scenario is depicted in figure 1.

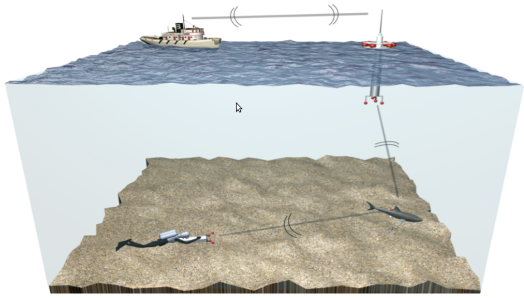

Fig. 1: Graphical representation of the mission scenario [1]

Based on the work in [2], it is possible for a vehicle equipped with as USBL array to obtain its relative position to a similarly equipped transponder which is assumed as stationary in the inertial frame. This is accomplished through the transmission of an acoustic signal by the vehicle which is detected by the USBL array present on the transponder. This operation and its reverse, emission by the transponder and reception by the vehicle array, allows each receptor to determine the other's direction in their respective body-frames, resorting to the plane-wave approximations to the received signal and from the differences in Time of Arrival between pairs of elements in the hydrophone arrays, obtain the DoA of said signal and relative direction of its source.

Furthermore, the distance between both objects can be determined by the emission of an interrogating signal from one and an adequate response from the other. By measuring the time between the interrogation and the reception of the response it is possible to obtain the round-trip time (t_{rt}), assuming that the response involves a fixed and known delay between the reception of the interrogations and the emission of the response. With the already available DoA of the response signal available, this measurement of time allows for the full precise positioning of the transponder in the vehicle's body-frame, and vice versa, assuming a constant known speed of sound in the medium.

At this point, the focus of this work is the third element, the moving target. Since the target is tagged with a pinger type marker, the range cannot be measured in the same way as the vehicle-transponder range. Thus an indirect form of range measurement must be developed. This will be the subject of the following chapter.

A. Framework

B. System Dynamics

III. LOCALIZATION FILTER DESIGN

A. Geometric Solutions

In Fig. 2, the mission scenario is depicted with the available positioning elements and measurements. In this problem all quantities are indicated as represented in the body-frame of the vehicle and as such, the vehicle's position is always the origin of the frame. From this we define the target position vector $\mathbf{p}_t$ and the transponder position vector $\mathbf{p}_b$ as the vectors that give the respective positions in the frame of reference. Additionally, we define $\mathbf{p}_{tb}$ as the vector that, in the vehicle's body-frame, represents the position of the target relative to the position of the transponder. For these vectors, their direction cosines are defined as $\mathbf{d}_{p_t}$, $\mathbf{d}_{p_b}$, and $\mathbf{d}_{p_{tb}}$ respectively for $\mathbf{p}_t$, $\mathbf{p}_b$, and $\mathbf{p}_{tb}$.

It is possible from the figure to identify a simple triangular geometry for the problem. Firstly, this triangle is in a three dimensional space and any three non collinear points form a plane in such a frame. Thus, assuming the situation of non collinearity of the three elements of the problem holds, then their three positions in the body frame of the vehicle can be used as the plane defining points. With this in mind, we can observe that any measurements taken between any two elements of the problem are from two points in a same plane and thus these measurements are projected in that plane. Therefore, the three dimensional problem in the three dimensional space may be represented, without loss of information, as a problem in a plane within the three dimensional space and may be solved accordingly.

Based on the considerations above, the problem can be viewed as solving a triangle of which some elements are directly measurable by the vehicle, namely the direction cosines $\mathbf{d}_{p_t}$, $\mathbf{d}_{p_b}$, and the vehicle-transponder distance $\|\mathbf{p}_b\|$. Additionally, the director cosine $\mathbf{d}_{p_{tb}}$ is directly measurable by the transponder and may be made available to the vehicle. At this point, there are enough elements to allow for the complete and unambiguous determination of the target distance and, consequently, its position in the vehicle's body-frame.

According to the figure, let

$$\cos\alpha = \mathbf{d}_{p_t} \cdot \mathbf{d}_{p_b} \tag{1}$$

and

$$\cos\beta = \mathbf{d}_{p_{tb}} \cdot (-\mathbf{d}_{p_b}) \ , \tag{2}$$

And, according to the internal angles of a triangle, let also

$$\gamma = \pi - \alpha - \beta \ . \tag{3}$$

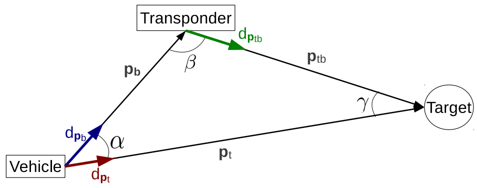

Fig. 2: Graphical presentation of the problem situation

There are now enough available elements of the triangle geometry to allow the determination of its remaining elements using the Law of Sines

$$\frac{\|\mathbf{p}_t\|}{\sin\beta} = \frac{\|\mathbf{p}_b\|}{\sin\gamma} \ , \tag{4}$$

Which can be rearranged in order to isolate the target range as

$$\|\mathbf{p}_t\| = \frac{\sin\beta}{\sin\gamma}\|\mathbf{p}_b\| \ . \tag{5}$$

The determination of the complete target position is completed by multiplying the computed target range, $\|\mathbf{p}_t\|$, by the measured target position direction cosine, $\mathbf{d}_{p_t}$.

An alternative to the Law of Sines approach may be devised, using relations between the distances $\|\mathbf{p}_t\|$ and $\|\mathbf{p}_{tb}\|$, based on the physical characteristics of the acoustic signal used to track the target.

Due to the nature of the problem, the emitted signal from the target is transmitted simultaneously to both the transponder and the vehicle from the target. Thus, each length traveled by the signal to each of the receivers can be expressed as a product of the signal velocity, v_s, by a time, τ, which it takes to reach that receiver. Assuming a known and constant v_s throughout the length of travel and due to the fact that the signal is emitted at the same point in time for both $\mathbf{p}_t$ and $\mathbf{p}_{tb}$ paths, their difference in length can be expressed through a difference of travel time, $\Delta\tau$. This reinterpretation of the problem, presented in Fig. 3, has the reference travel time τ coupled to the $\|\mathbf{p}_{tb}\|$ interval, as before, the $\mathbf{p}_b$ vector is available though direct measurements and τ_0 is the travel time between the vehicle and the transponder. At this point, the problem distances can be rewritten accordingly.

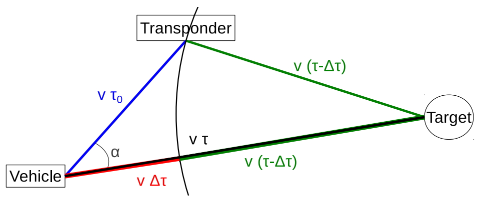

Fig. 3: Graphical reinterpretation of the problem

Let the target range be given as

$$\|\mathbf{p}_t\| = v_s \ \tau \ , \tag{6}$$

the transponder range as

$$\|\mathbf{p}_b\| = v_s \ \tau_0 \ , \tag{7}$$

and the distance between the transponder and the target as

$$\|\mathbf{p}_{tb}\| = v_s \; (\tau - \Delta\tau) \;, \tag{8}$$

in which τ is the travel time from the target to the vehicle, τ_0 is the travel time between the vehicle and the transponder and $\Delta\tau$ is the difference in signal travel time between the emitter-vehicle and the emitter-transponder paths.

By applying the distributive property to (8) and substituting (6) into it gives, after rearranging,

$$\Delta\tau = \frac{\|\mathbf{p}_t\| - \|\mathbf{p}_{tb}\|}{v_s} \;. \tag{9}$$

Using this redefinition of the problem variables it is possible to use a Law of Cosines approach to relate the new quantities. Applying the Law of Cosines to the triangle in Fig. 3 gives

$$\|\mathbf{p}_{tb}\|^2 = \|\mathbf{p}_t\|^2 + \|\mathbf{p}_b\|^2 - 2\|\mathbf{p}_t\|\|\mathbf{p}_b\| \cos\alpha \tag{10}$$

and replacing (6), (7), and (8) in (10) finds

$$[v_s \; (\tau - \Delta\tau)]^2 = (v_s \; \tau)^2 + (v_s \; \tau_0)^2 - 2(v_s \; \tau)(v_s \; \tau_0)\cos\alpha \;. \tag{11}$$

Both sides of equation (11) can be divided by v_s^2 resulting in

$$(\tau - \Delta\tau)^2 = \tau^2 + \tau_0^2 - 2 \; \tau \; \tau_0 \; \cos\alpha \;, \tag{12}$$

which rewritten for τ gives

$$\tau = \frac{\tau_0^2 - \Delta\tau^2}{2(\tau_0\cos\alpha - \Delta\tau)} \;. \tag{13}$$

By transmission of a target detected signal from the transponder to the vehicle, the time difference $\Delta\tau$ can be directly obtained if a constant and known delay is added by the transponder, by subtracting from the time difference between the reception of the target signal and the transponder target detection signal, the transponder-vehicle travel time and the known transponder delay. With this information, the target distance signal travel time, τ, can be computed.

With this determination of τ, inserting it in (6), gives the calculated target range. In order to determine a position from this value it is a matter of multiplying this scalar value with the direction cosine $\mathbf{d}_{p_t}$, already available through direct measurement.

After devising the methods of finding the target position, it is of interest to study in which situations these methods degrade their computations and become unusable. Firstly let us remember the Law of Sines approach (5)

$$\|\mathbf{p}_t\| = \frac{\sin\beta}{\sin\gamma}\|\mathbf{p}_b\| \;.$$

This equation can not be applied when the denominator approaches zero, resulting in

$$\sin\gamma = 0 \Leftrightarrow \gamma = k\pi, k \in \mathbb{Z} \tag{14}$$

which, for the considered situation geometrical constraints, may take only two physically acceptable values

$$\gamma = 0 \vee \gamma = \pi \tag{15}$$

Translating these values into the problem geometry, it means that the proposed method breaks down precisely as it approaches the limits of the assumption validity, namely the non-collinearity hypothesis. Additionally, there is one further situation in which the method fails, which is for a triangle in which the $\|\mathbf{p}_t\|$ dimension is much larger than $\|\mathbf{p}_b\|$ and forces γ to approach zero.

Let us now observe the expression for the delay obtained with (13) of the Law of Cosines approach

$$\tau = \frac{\tau_0^2 - \Delta\tau^2}{2(\Delta\tau + \tau_0\cos\alpha)} \;.$$

This solution is invalid if at any time the denominator of the equation becomes zero

$$\begin{aligned} 2(\tau_0\cos\alpha - \Delta\tau) = 0 \\ \tau_0\cos\alpha = \Delta\tau \;. \end{aligned} \tag{16}$$

Multiplying both sides of (16) by v_s and substituting (7) and (9) results in

$$\|\mathbf{p}_b\|\cos\alpha = \|\mathbf{p}_t\| - \|\mathbf{p}_{tb}\| \;. \tag{17}$$

By rearranging (10) into

$$\cos\alpha = \frac{\|\mathbf{p}_{tb}\|^2 - \|\mathbf{p}_t\|^2 - \|\mathbf{p}_b\|^2}{-2\|\mathbf{p}_t\|\|\mathbf{p}_b\|} \tag{18}$$

and substituting into (17) gives

$$\begin{aligned} \|\mathbf{p}_b\|\frac{\|\mathbf{p}_{tb}\|^2 - \|\mathbf{p}_t\|^2 - \|\mathbf{p}_b\|^2}{-2\|\mathbf{p}_t\|\|\mathbf{p}_b\|} &= \|\mathbf{p}_t\| - \|\mathbf{p}_{tb}\| \\ \|\mathbf{p}_b\|^2 &= \|\mathbf{p}_t\|^2 - 2\|\mathbf{p}_t\|\|\mathbf{p}_{tb}\| + \|\mathbf{p}_{tb}\|^2 \\ \|\mathbf{p}_b\| &= \|\mathbf{p}_t\| - \|\mathbf{p}_{tb}\| \;. \end{aligned} \tag{19}$$

The final result of (19) represents, for a triangle, the same situation as for the Law of Sines of point colinearity that violates the modelled assumptions.

B. *Least Squares Estimator*

$$\mathbf{p}_t(t) = \mathbf{p}_{t_0} + \mathbf{v}_{t_0}t + \frac{1}{2}\mathbf{a}_{t_0}t^2 \tag{20}$$

and

$$\mathbf{v}_t(t) = \mathbf{v}_{t_0} + \mathbf{a}_{t_0}t \;, \tag{21}$$

where $\mathbf{p}_t(t)$ and $\mathbf{v}_t(t)$ are the target position, and velocity at time t, respectively, and $\mathbf{p}_{t_0}$, $\mathbf{v}_{t_0}$ and $\mathbf{a}_{t_0}$ are constants representing the initial target position, velocity and acceleration, respectively.

With the chosen model, the position of the target at any time, is obtainable with knowledge of the three initial conditions of the motion ($\mathbf{p}_{t_0}$, $\mathbf{v}_{t_0}$ and $\mathbf{a}_{t_0}$) and the present time by means of (20). Since it has been established that it is possible to obtain measurements of the target position in some instants of known t, this problem may be solved through the fitting of (20) to the data points measured in order to determine the problem constants.

For a least squares solution to be possible, any k^{th} element of a measurement vector $\mathbf{y}$, $\mathbf{y}_k$, must be a linear combination of the elements of a constant parameter vector $\mathbf{x}$ to be estimated with the addition of some measurement noise v. It may be expressed in vector form as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + v_k \ . \tag{22}$$

By minimizing a cost function

$$\begin{aligned} J = \boldsymbol{\epsilon}^T\boldsymbol{\epsilon} &= (\mathbf{y} - \mathbf{H}\hat{\mathbf{x}})^T (\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}) \\ &= \mathbf{y}^T\mathbf{y} - \hat{\mathbf{x}}^T\mathbf{H}^T\mathbf{y} - \mathbf{y}^T\mathbf{H}\hat{\mathbf{x}} + \hat{\mathbf{x}}^T\mathbf{H}^T\mathbf{H}\hat{\mathbf{x}} \ , \end{aligned} \tag{23}$$

in which

$$\boldsymbol{\epsilon} = \mathbf{y} - \mathbf{H}\hat{\mathbf{x}} \tag{24}$$

is called the measurement residual and is defined as the difference between the measurements $\mathbf{y}$and the model's prediction for such measurements, the best estimate of $\mathbf{x}$ can be shown to be given by

$$\hat{\mathbf{x}} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{y} \ . \tag{25}$$

However, using this approach requires that a record be maintained of every measurement taken in the $\mathbf{y}$ vector as well as requiring an ever expanding $\mathbf{H}$ matrix. This is computationally expensive and due to the unpredictable number of measurements is inadvisable. In order to circumvent this issue a recursive estimation method is of interest and thus the RLS method is used. This approach provides the least squares estimate recursively with each new available measurement based on the previous estimate and an estimation-error covariance estimate. A different cost function is also chosen to be minimized. This new function, defined in (27) as the sum of the estimate-error variances at each time step, leads to the addition of a dependence on the variance of the noisy measurements though a measurement-error covariance matrix to be later defined.

Taking (22), a linear recursive estimator can be written in the form

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k-1} + \mathbf{K}_k(\mathbf{y}_k - \mathbf{H}_k\hat{\mathbf{x}}_{k-1}) \ , \tag{26}$$

where $\hat{\mathbf{x}}_{k-1}$ is the estimate after measurement $\mathbf{y}_{k-1}$ and $\hat{\mathbf{x}}_k$ is the estimate after measurement $\mathbf{y}_k$. The $\mathbf{K}_k$ matrix is the estimator gain, and is obtained by minimizing the new cost function

$$J_k = \sum_i^n E\left[\theta_{ik}\theta_{ik}\right] \tag{27}$$

in which θ_{nk} is the estimation error for the n^{th} parameter at the k^{th} step

$$\theta_{nk} = \mathbf{x}_{nk} - \hat{\mathbf{x}}_{nk}, \tag{28}$$

By defining an estimation-error covariance matrix $\mathbf{P}_k$ as an $n \times n$ diagonal matrix with

$$\mathbf{P}_k = E\left(\boldsymbol{\theta}_k\boldsymbol{\theta}_k^T\right) \tag{29}$$

solving for the gain matrix finds

$$\mathbf{K}_k = \mathbf{P}_{k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k-1}\mathbf{H}_k^T + \mathbf{R}_k)^{-1} \tag{30}$$

where $\mathbf{R}_k$ is the measurement-error covariance matrix

$$\mathbf{R}_k = E\left[(\mathbf{y}_k - \hat{\mathbf{y}}_k)(\mathbf{y}_k - \hat{\mathbf{y}}_k)^T\right] \tag{31}$$

and the estimation-error covariance matrix for the present step is found to be defined recursively as

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k-1}(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)^T + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^T \ . \tag{32}$$

A first approach to the problem of defining a measurement model is to use the results from the algebraic solution as measurements, which are the measured positions of the target and are directly related to the problem constants by (20). With this observation model, there is a further degree of freedom in the choice of assumed value of the constant acceleration, $\mathbf{a}_0$, which can be assumed to be zero, for a constant velocity motion approximation, or any value as the most general case. Both cases will be evaluated and thus for the time of the k^{th} measurement, at time t_k, the zero acceleration case transforms (20) into

$$\mathbf{p}_t(t_k) = \mathbf{p}_{t_0} + \mathbf{v}_{t_0}t_k \ , \tag{33}$$

and the problem reduces to the determination of only two constants. Making the measurement

$$\mathbf{y}_k = \tilde{\mathbf{p}}(t_k) = \begin{bmatrix} \tilde{p}_x(t_k) & \tilde{p}_y(t_k) & \tilde{p}_z(t_k) \end{bmatrix}^T \tag{34}$$

and the parameter vector

$$\begin{aligned} \mathbf{x} &= \begin{bmatrix} \mathbf{p}_{t_0} & \mathbf{v}_{t_0} \end{bmatrix}^T \\ &= \begin{bmatrix} p_{t_0x} & p_{t_0y} & p_{t_0z} & v_{t_0x} & v_{t_0y} & v_{t_0z} \end{bmatrix}^T \ , \end{aligned} \tag{35}$$

the matrix $\mathbf{H}_k$ that satisfies (22) is

$$\mathbf{H}_k = \begin{bmatrix} \mathbf{I}_3 & t_k\mathbf{I}_3 \end{bmatrix} \ . \tag{36}$$

This solution will be referred to as the Constant Velocity Recursive Least Squares (RLS-V).

For the constant, non-zero acceleration case (20) maintains it's form and the measurements remain given by (34). As for the parameter vector, it is given by

$$\mathbf{x} = \begin{bmatrix} \mathbf{p}_{t_0} & \mathbf{v}_{t_0} & \mathbf{a}_{t_0} \end{bmatrix}^T \tag{37}$$

and the matrix $\mathbf{H}_k$ to satisfy (22) now becomes

$$\mathbf{H}_k = \begin{bmatrix} \mathbf{I}_3 & t_k\mathbf{I}_3 & \frac{1}{2}t_k^2\mathbf{I}_3 \end{bmatrix} . \tag{38}$$

This solution will be referred to as the Constant Acceleration Recursive Least Squares (RLS-A).

C. Kalman Filter

Following the success in the design and simulation of the RLS algorithm, it is desirable to produce an alternative solution that better copes with the situations in which the previous models have difficulties, namely the situation of variable acceleration, and thus a Kalman filter based approach was developed.

Given a problem presented in the state-space form, a Kalman filter allows the estimation of the system states based on direct or indirect measurements of such states, fitted to some assumed model for the system dynamics.

Let (39) represent any generic dynamic system in continuous time

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t) + w(t) \tag{39}$$

where $\mathbf{x}(t)$ is the system state, $\mathbf{u}(t)$ is the system input and $w(t)$ is a continuous-time white noise process. Assuming it is possible to direct or indirectly observe the system state, these observations are related to the state by (40)

$$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + v(t) \ , \tag{40}$$

where $\mathbf{y}(t)$ is the observation vector, $v(t)$ is a continuous-time noise process and $\mathbf{C}(t)$ is the matrix that obtains the observations from the states.

In the present problem, tracking a moving target involves the estimation of its position and velocity. Additionally, the estimation of its acceleration may be advantageous to reduce errors in the velocity estimate. Firstly, only the position and velocity of the target are estimated, this gives the problem state

$$\mathbf{x}(t)^{K-V} = \begin{bmatrix} \mathbf{p}_t(t)^T & \mathbf{v}_t(t)^T \end{bmatrix}^T \ . \tag{41}$$

Let

$$\dot{\mathbf{p}}_t(t) = \mathbf{v}_t(t) \tag{42}$$

and

$$\dot{\mathbf{v}}_t(t) = \mathbf{a}_t(t) \ . \tag{43}$$

Then,

$$\begin{aligned} \dot{\mathbf{x}}(t)^{K-V} &= \begin{bmatrix} \dot{\mathbf{p}}_t(t)^T & \dot{\mathbf{v}}_t(t)^T \end{bmatrix}^T \\ &= \begin{bmatrix} \mathbf{v}_t(t)^T & \mathbf{a}_t(t)^T \end{bmatrix}^T \ , \end{aligned} \tag{44}$$

and the matrix that satisfies (39) is

$$\mathbf{A}(t)^{K-V} = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{I}_3 \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix} \ . \tag{45}$$

Since, by definition of the problem the input in the dynamic equation of the target is unknown, it is assumed non-existent, $\mathbf{B}(t)\mathbf{u}(t)$ disappears and the dynamic equation is fully defined. The row of zeros in the $\mathbf{A}(t)$ matrix appears because $\mathbf{a}_t(t)$ is not a part of the states considered above and thus the dynamic equation evaluates $\dot{\mathbf{v}}_t(t)$ as zero which corresponds to the modeled assumption of an unvarying velocity. This solution will be referred to as the Velocity Estimating Kalman Filter (Kalman-V).

However, the unvarying velocity assumption may not be a good enough approximation to the measured reality. Thus, in order to remove this modeled constraint from the dynamic equation, the addition of the target acceleration in the problem states is considered, resulting in

$$\mathbf{x}(t)^{K-A} = \begin{bmatrix} \mathbf{p}_t(t)^T & \mathbf{v}_t(t)^T & \mathbf{a}_t(t)^T \end{bmatrix}^T \ . \tag{46}$$

Take once again (42) and (43), and now let

$$\begin{aligned} \dot{\mathbf{x}}(t)^{K-A} &= \begin{bmatrix} \dot{\mathbf{p}}_t(t)^T & \dot{\mathbf{v}}_t(t)^T & \dot{\mathbf{a}}_t(t)^T \end{bmatrix}^T \\ &= \begin{bmatrix} \mathbf{v}_t(t)^T & \mathbf{a}_t(t)^T & \dot{\mathbf{a}}_t(t)^T \end{bmatrix}^T \ , \end{aligned} \tag{47}$$

the matrix that satisfies (39) is now

$$\mathbf{A}(t)^{K-A} = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{I}_3 & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{I}_3 \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix} \ . \tag{48}$$

There is still a row of zeros in the $\mathbf{A}(t)$ matrix, now representing a lack of model for the variation of acceleration based on the estimated states. This solution evaluates $\dot{\mathbf{a}}_t(t)$ as zero which is somewhat analogous to modeling an unvarying acceleration and will be henceforth referred to as the Accelleration Estimating Kalman Filter (Kalman-A).

Finally, for the considered conditions the measurements available must be related to the problem states in order to complete the $\mathbf{C}_t(t)$ matrix of (40). However, prior to the investigation of these solutions, some alterations to the way the state-space system is treated will be considered.

Any linear discrete-time system may be represented as

$$\mathbf{x}_k = \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{G}_{k-1}\mathbf{u}_{k-1} + w_{k-1} \tag{49a}$$

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + v_k \tag{49b}$$

where, $\mathbf{F}_{k-1}$ is the state transition matrix for the previous state, $\mathbf{H}_k$ is the observation model matrix, $\mathbf{y}_k$ is the measurement vector, $\mathbf{u}_{k-1}$ is the input in the previous moment, $\mathbf{G}_{k-1}$ is the matrix that describes the effect of the previous moment inputs on the state, and w_{k-1} and v_k are AWGN processes with covariance matrices defined respectively as $\mathbf{Q}_k$ and $\mathbf{R}_k$. The same reasoning applies to the $\mathbf{G}_{k-1}$ matrix and $\mathbf{u}_{k-1}$ vector as for the continuous time $\mathbf{B}(t)$ and $\mathbf{u}(t)$ elements, respectively, and accordingly will be assumed zero and omitted.

In order to present the current system in discrete-time form, the $\mathbf{A}$ and $\mathbf{C}$ matrices of the continuous model must be discretized into the $\mathbf{F}_{k-1}$ and $\mathbf{H}_k$ matrices respectively. Resorting to the Step Invariant method, for the $\mathbf{A}$ matrix this is done through the matrix exponential, shown in (50), where Δt is the time difference between steps $k-1$ and k. Applying (50) to (45) and (48), results in (51) and (52), which are the discrete-time state transition matrices for the Kalman-V and Kalman-A approaches respectively. In the case of the $\mathbf{C}$ matrix, no discretization is required since the measurement model does not represent a dynamic relation but rather a relation between the state and the measurements, and thus is the same in discrete or continuous-time, simply changing denomination to $\mathbf{H}_k$.

$$\mathbf{F}_{k-1} = e^{\mathbf{A}\Delta t} \tag{50}$$

$$\mathbf{F}_{k-1}^{K-V} = \begin{bmatrix} \mathbf{I}_3 & \Delta t\mathbf{I}_3 \\ \mathbf{0}_{3\times3} & \mathbf{I}_3 \end{bmatrix} \tag{51}$$

$$\mathbf{F}_{k-1}^{K-A} = \begin{bmatrix} \mathbf{I}_3 & \Delta t\mathbf{I}_3 & \frac{1}{2}\Delta t^2\mathbf{I}_3 \\ \mathbf{0}_{3\times3} & \mathbf{I}_3 & \Delta t\mathbf{I}_3 \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{I}_3 \end{bmatrix} \tag{52}$$

Taking the discrete-time Kalman filter [3] equtions as derived and described in [4], the algorithm is the application of (53) to (57)

$$\hat{\mathbf{x}}_k^- = \mathbf{F}_{k-1}\hat{\mathbf{x}}_{k-1}^+ \ , \tag{53}$$

$$\mathbf{P}_k^- = \mathbf{F}_{k-1}\mathbf{P}_{k-1}^+\mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1} \ , \tag{54}$$

$$\mathbf{K}_k = \mathbf{P}_k^-\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_k^-\mathbf{H}_k^T + \mathbf{R}_k)^{-1} \ , \tag{55}$$

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k(\mathbf{y}_k - \mathbf{H}_k\hat{\mathbf{x}}_k^-) \ , \tag{56}$$

$$\mathbf{P}_k^+ = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_k^-(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)^T + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^T \ . \quad (57)$$

The only remaining step required is to define the measurement model matrix $\mathbf{H}_k$, the initial state estimate $\hat{\mathbf{x}}_0^+$, initial estimation-error covariance matrix $\mathbf{P}_0$, and the measurement noise covariance $\mathbf{R}$ and model noise covariance $\mathbf{Q}$ matrices. During the simulation phase, the initial estimates and noise matrices were given initial plausible values which were refined through repeated simulation runs.

As previously accomplished in the RLS estimator, a loosely coupled solution will be developed using the position results from the algebraic solution. Accordingly, the measurements for this approach will be the position results of the algebraic solution, $tilde\mathbf{p}_t(t)$. Defining the measurement

$$\mathbf{y}(t) = \mathbf{p}_{mt}(t) \quad (58)$$

with

$$\mathbf{p}_{mt}(t) = \mathbf{p}_t(t) + v, \quad (59)$$

then for the Kalman-V case results in the measurement model matrix

$$\mathbf{C}_{K-V} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{0}_{3\times3} \end{bmatrix} \quad (60)$$

or, for the Kalman-A case

$$\mathbf{C}_{K-A} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix}. \quad (61)$$

With the measurement model completely defined in continuous-time, in discrete-time the matrix changes denomination but maintains its contents. These discrete-time measurement models are then

$$\mathbf{H}_k^{K-V} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{0}_{3\times3} \end{bmatrix} \quad (62)$$

for the Kalman-V method, and for the Kalman-A

$$\mathbf{H}_\mathbf{k}^\mathbf{K-A} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix}. \quad (63)$$

The Kalman Filter algorithm has now all of its elements available to implement in simulation.

After the development of the loosely coupled solution, it is desirable to pursue a filter that would extract as much information from the measurements as possible while at the same time removing the need for external computations. For these reasons, a tightly coupled Kalman filter was designed in order to extract the state information directly from the sensor measurements.

Firstly, the system states must be chosen and the system model described. For the following designs, the model assumptions remain the same as those in the previous Kalman filters namely the creation of two distinct filters, the velocity estimating filter and the acceleration estimating filter. As a starting point, the states and state propagation are assumed the same as in the previous filters.

Following, the available sensor measurements will be expressed as functions of the state variables. The available measurements are the elements $\mathbf{d}_{p_t}$, $\mathbf{d}_{p_b}$, $\mathbf{d}_{p_{tb}}$, and $\|\mathbf{p}_b\|$ from Figure 2. Still from the figure, remembering that $\mathbf{d}_{p_t}$ is the direction cosine of the target position, $\mathbf{p}_t$, it can be expressed as

$$\mathbf{d}_{p_t} = \frac{\mathbf{p}_t}{\|\mathbf{p}_t\|} \ . \quad (64)$$

Analogously, for the transponder position, $\mathbf{p}_b$, and the target position in relation to the transponder, $\mathbf{p}_{tb}$, their direction cosines, $\mathbf{d}_{p_b}$ and $\mathbf{d}_{p_{tb}}$, can be expressed as

$$\mathbf{d}_{p_b} = \frac{\mathbf{p}_b}{\|\mathbf{p}_b\|} \quad (65)$$

and

$$\mathbf{d}_{p_{tb}} = \frac{\mathbf{p}_{tb}}{\|\mathbf{p}_{tb}\|} \ . \quad (66)$$

Finally, the direction cosine $\mathbf{d}_{p_{tb}}$ may be expressed as a function of of $\mathbf{p}_t$ and $\mathbf{p}_b$. Attending to figure 2, and since the $\mathbf{p}_t$, $\mathbf{p}_b$ and $\mathbf{p}_{tb}$ vectors form a closed path, $\mathbf{p}_{tb}$ can be expressed as the vector difference (67)

$$\mathbf{p}_{tb} = \mathbf{p}_t - \mathbf{p}_b \ . \quad (67)$$

Expressing the $\mathbf{p}_{tb}$ vector by the product between its length and direction cosine in (67) obtains (68)

$$\|\mathbf{p}_{tb}\|\mathbf{d}_{p_{tb}} = \mathbf{p}_t - \mathbf{p}_b \ . \quad (68)$$

Using (67) and (68), $\mathbf{d}_{p_{tb}}$can be given by

$$\mathbf{d}_{p_{tb}} = \frac{\mathbf{p}_t - \mathbf{p}_b}{\|\mathbf{p}_t - \mathbf{p}_b\|} \ , \quad (69)$$

With the four sensor measurements now expressed as function of the state, the measurement vector $\mathbf{y}_k$ is built from (64), (65) and (69) as

$$\begin{aligned} \mathbf{y}_k &= \begin{bmatrix} \mathbf{d}_{p_t} & \mathbf{d}_{p_b} & \mathbf{d}_{p_{tb}} & \|\mathbf{p}_b\| \end{bmatrix}^T \\ &= \begin{bmatrix} \frac{\mathbf{p}_t}{\|\mathbf{p}_t\|}^T & \frac{\mathbf{p}_b}{\|\mathbf{p}_b\|}^T & \frac{\mathbf{p}_t - \mathbf{p}_b}{\|\mathbf{p}_t - \mathbf{p}_b\|}^T & \|\mathbf{p}_b\| \end{bmatrix}^T \end{aligned} \quad (70)$$

This measurement vector is only related to the target position state with the necessity to have the calculation of the transponder position. In order to circumvent this, and to add a degree of precision to the estimates, the state shall be augmented with the transponder's position, velocity and acceleration.

Furthermore, the measurement vector is time-varying and not a linear combination of the states, thus the filter cannot be a linear Kalman filter. A simple way of overcoming this problem is to transform the filter into an extended Kalman filter. The principle of the EKF is to linearize the non-linear system around the *a posteriori* estimate at the previous time step before applying the state-update step and around the *a priori* estimate at the present time step before the measurement-update step of the regular Kalman filter. Here is presented the series of steps required to produce the filter.

Firstly, the system must be presented as the common system and measurement equations as follows. For the present case, the known inputs are considered non-existent as previously justified and shall be omitted. Taking (49a) and (49b), linearizing $\mathbf{x}_k$ around the a posteriori estimate $\mathbf{x}_{k-1} = \hat{\mathbf{x}}_{k-1}^+$ gives

$$\mathbf{x}_k = \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{L}_{k-1}w_{k-1} \ , \quad (71)$$

with

$$\mathbf{F}_{k-1} = \frac{\partial f_{k-1}}{\partial \mathbf{x}}\Big|_{\hat{\mathbf{x}}_{k-1}^{+}} \tag{72a}$$

$$\mathbf{L}_{k-1} = \frac{\partial f_{k-1}}{\partial \mathbf{w}}\Big|_{\hat{\mathbf{x}}_{k-1}^{+}} , \tag{72b}$$

and expanding $\mathbf{y}_k$ around the a priori estimate $\mathbf{x}_k = \hat{\mathbf{x}}_k^-$ gives

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{M}_k v_k \ , \tag{73}$$

with

$$\mathbf{H}_k = \frac{\partial h_k}{\partial \mathbf{x}}\Big|_{\hat{\mathbf{x}}_k^-} \tag{74a}$$

$$\mathbf{M}_k = \frac{\partial h_k}{\partial \mathbf{v}}\Big|_{\hat{\mathbf{x}}_k^-} . \tag{74b}$$

It is assumed that both noise processes are additive in their respective equations and thus the $\mathbf{L}_{k-1}$ and $\mathbf{M}_k$ matrices are appropriately sized identities. This means that the regular linear Kalman filter equations may be applied as they were presented. Since the state-space system is already linear, the $\mathbf{F}_{k-1}$ matrix remains as in (52) and the time update equations (53) and (54) may be applied. To obtain the time-variant $\mathbf{H}_k$ matrix from the non-linear measurement, the Jacobian

$$\mathbf{H}_k = \frac{d\mathbf{y}_k}{d\mathbf{x}} , \tag{75}$$

is applied to (70).

With this result, the measurement-update equations (55), (56) and (57) may be applied as previously described.

IV. SIMULATION RESULTS

A. Setup

In order to validate the developed solutions, numerical simulations were carried out covering a range of model behaviors assumed to be valid for the targets. In all situations, the measurements are assumed to be corrupted with zero-mean Additive White Gaussian Noise (AWGN) with standard deviations given in Table I.

Measurement	AWGN Standard deviation
α angle	0,5 [deg]
β angle	0,5 [deg]
$\|\mathbf{d}_{p_b}\|$	1 [m]
$\Delta\tau$	100 [μs]

TABLE I: Measurements standard deviation

For this purpose, a sinusoidal velocity situation, shown in Fig. 4 is considered. This situation is modeled for a mission time of 150 seconds with a time step of 1 second.

The transponder is fixed at $\mathbf{p}_b = \begin{bmatrix} 15 & 10 & 2 \end{bmatrix} m$ and the target starts at $\mathbf{p}_t = \begin{bmatrix} 30 & 0 & -1 \end{bmatrix} m$. The velocity starts as $\mathbf{v}_3 = \mathbf{v}_1$ and its v_{3y} component varies with $v_{3y} = \cos 2\pi f t \ ms^{-1}$ with a frequency $f = 0.015 \ Hz$. The speed of sound (v_s) in the medium is also assumed known and constant, $v_s = 1560 \ ms^{-1}$.

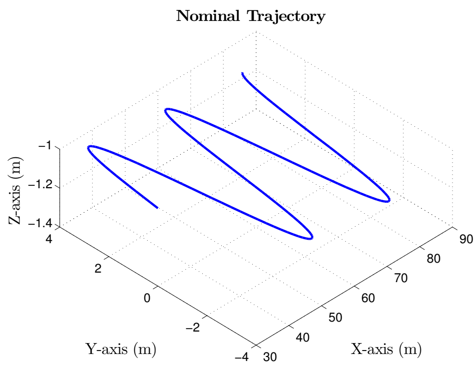

Fig. 4: Sinusoidal velocity trajectory

B. Results

In order to evaluate and compare the simulation results, an error was defined as the norm of the vector difference between the different position estinmates of the various solutions and the real position of the target.

Firstly, the algebraic solutions are compared in Fig. 5, then the results for the RLS estimators are shown in Fig. 6. Figure 7 presents the results of the linear Kalman filters, and Fig. 8 reveals the estimates of the EKF methods.

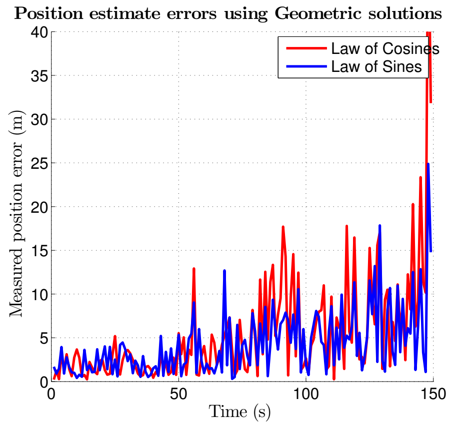

Fig. 5: Position estimate errors of the Geometric solutions for the sinusoidal velocity trajectory

Table II compares the RMS errors of the various proposed estimation methods.

Estimator	Simulation RMS error
Law of Sines	7.8186 (m)
Law of Cosines	13.7981 (m)
RLS-V	3.1984 (m)
RLS-A	3.3307 (m)
Kalman-V	3.2238 (m)
Kalman-A	3.9639 (m)
EKF-V	1.0006 (m)
EKF-A	1.2815 (m)

TABLE II: Estimate RMS error comparisons

Finally, in order to have a visual representation of the comparative levels of performance of the proposed methods,

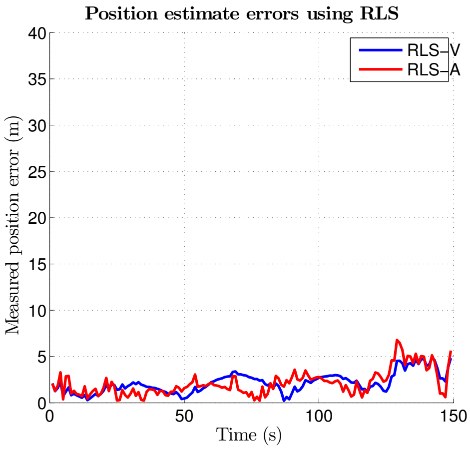

Fig. 6: Position estimate errors of the RLS estimators for the sinusoidal velocity trajectory

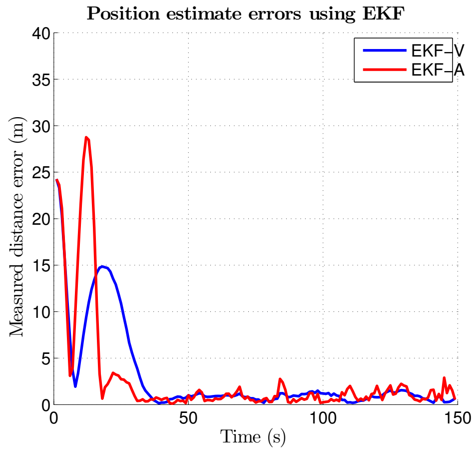

Fig. 8: Position estimate errors of the Extended Kalman filters for the sinusoidal velocity trajectory

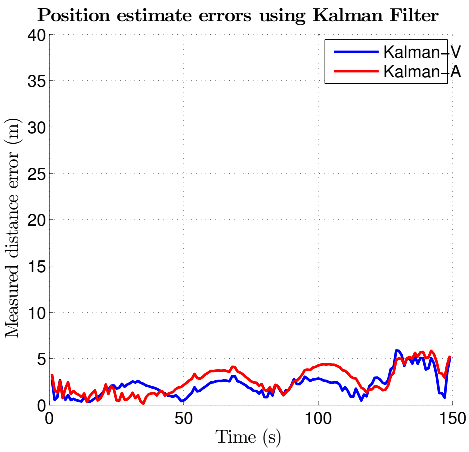

Fig. 7: Position estimate errors of the Kalman filters for the sinusoidal velocity trajectory

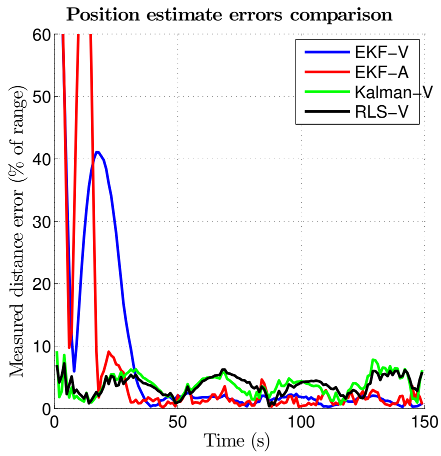

Fig. 9: Position estimate errors comparison for the sinusoidal velocity trajectory

Fig. 9 presents the target position errors of the four solutions with the lowest RMS errors as a percentage of the true range of the target.

V. CONCLUSIONS

This document proposes two novel methods of algebraically finding a moving target based on spacial information obtained from two USBL receiver arrays. A Least Squares estimation technique is derived and applied in a Recursive Least Squares algorithm, and Kalman filtering methods are implemented in a linear, lossely-coupled, Kalman filter and a non-linear, tightly-coupled, Extended Kalman Filter. All developed estimation schemes are evaluated and compared in numerical simulation.

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