Fault Detection and Isolation for Dynamical Systems

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Abstract

Safety and reliability of a dynamical system is a concern that have always pursued designers in both academia and industry. The relevance of monitoring the health status of a system is even more relevant for safety critical applications, such as chemical and nuclear plants, medicine, transportation, and security systems. The occurrence of abnormal events on these processes may lead to malfunctions and disasters in ultimate fault conditions, as witnessed in the past. The paramount importance of the topic and the increasing interest in multiple-model approaches under the scope of on-line fault detection and isolation motivates this thesis. Initially, focus is given to classical multiple-model adaptive estimation (MMAE) in which an in-depth study is undertaken for the design of a scheme capable of determining the working regime of a system. This is done by identifying the region where the fault parameters lie under the associated uncertainty domain. The design procedure is built on a performance-based strategy, which ensures a well-defined level of state estimation performance despite the fault location. Due to the high computational complexity of the classical MMAE approach, in what follows we propose a novel bank design based on the combination of Kalman and robust $\mathcal{H}_2$ filters. This strategy leads to a substantial reduction on the number of estimators in the bank, while preserving the desired state estimation performance. In both approaches a prominent study on convergence properties is performed, so that robustness of the methods is guaranteed. Computational simulations based on a generic helicopter model are also executed to prove the potential of the strategies developed and provide a verification basis for the theoretical results achieved.
Keywords: Multiple-model adaptive estimation; model-based fault diagnosis; robust $\mathcal{H}_2$ filters; state estimation in uncertain systems;

1. Motivation for Fault Diagnosis

Safety has always been a critical factor in any technical application or process. Nowadays, more than ever before, human beings rely on control systems in their every-day life, either by stepping into an airplane or high-speed train, or in any other trivial actions such as baking a cake in a modern oven. Basically, automated systems are everywhere meaning that their reliability, safety, and efficiency play an important role for both the designer and end-user. This interest has brought about a considerable attention from the industry and academic research for the topic of on-line supervision and fault diagnosis.

The relevance of monitoring the health status of a system is even more relevant for safety critical applications, such as chemical and nuclear plants, medicine, transportation, and security systems. The occurrence of abnormal events on these processes may lead to malfunctions and disasters in ultimate fault conditions, as witnessed in the past. Several accidents in our history, specially during the 20^{th} century, due to the technological revolution, were caused by unexpected failures in control systems. Many claim that if proper diagnosis with an early fault detection have been undertaken several of this events could have been avoided by a simple advisory warning or at an advanced level a controller reconfiguration. Both from an economic perspective and even more importantly to avert the loss of lives, the topic of fault diagnosis has become a research priority across many fields of study. To strengthen the enunciated relevance of the subject, two examples of passed incidents in the interest field of this thesis are now provided:

- **X-15 Flight 3-65**: On November 15, 1967, X-15-3 was destroyed in flight due to a structural load exceedance precipitated by a loss of control. The causes of the accidents were attributed to an electrical anomaly associated to a test motor which resulted in instrumentation failures. The excessive demand for the pilot's awareness to troubleshoot the obvious malfunction and the extreme conditions of a ballistic flight regime culminated in a hypersonic spin and dive into the ground. It was also reported that the inability of the control system to deal with such failure prevented the pilot to manually recover the aircraft. The research pilot, USAF Major Michael J. Adams, did not survive the event. [1]

- **Copterline S-76 Flight 103**: On August 10, 2005, a helicopter Sikorsky S-76 crashed into the water of Tallin Bay, Estonia. The investigation commission declared that the accident occurred due to an uncommanded runaway of the main rotor actuator. As a consequence, the helicopter operated by Copterline entered in a an uncontrolled regime of pitch and roll manoeuvres. The 12 passengers and 2 pilot on board did not survive. [2]

In both described cases, system faults are identified as the primal cause of the accidents. In the early days, classical approaches based on hardware redundancy were the main tool to avoid catastrophes. This means that every mechanism, such as sensors or actuators, were double or tripled and subsequent voting schemes were applied to track the existence of faults. This strategy presents several limitations, namely the increase in system complexity, physical space and maintenance costs. The identified issues motivated the search for a novel strategy, which was firstly introduced in the 1970s by Beard [3], that suggested the replacement of hardware redundancy by analytical redundancy. The latter concept presupposes the use of the available signals, controller inputs and sensor outputs, in combination with a physical model of the system that enables to assess the health status of the system components. More than answering to the clear drawbacks of hardware redundancy, it also enabled the identification of more types of failures and malfunctions in dynamic processes.

2. Model-Based Fault Diagnosis Techniques

Analytical redundancy was introduced as an alternative for consistency checking of the system variables to achieve a fault diagnosis scheme. This type of analysis assumes the availability of some kind of mathematical relationships between those variables. In other words, we may refer to those relationships as a mathematical model which reflects the theoretically expected system behaviour under the physical laws applied. Therefore, analytical redundancy is also commonly referred as a model-based approach to fault diagnosis.

The idea behind the availability of a mathematical model is that one may compare the measured variables, with the aid of sensors, with the information provided by the model. If the mathematical relationships truly reflect the system behaviour, then a comparison can be achieved by the generation of a residual $r(t)$ in time which provides nothing else than a difference between the measured variables and the model variables.

In order to be applicable for fault detection, the residual is expected to satisfy the following properties:

1. Zero mean valued under no fault condition, i.e. $\mathbf{E}\{r(t)\} = 0$
2. Deviate from zero when a fault has occurred, i.e. $\mathbf{E}\{r(t)\} \neq 0$

Obviously, these properties are ideal and the assumption that a completely accurate system model is available is also unrealistic in practice. Models are always subject to uncertainties, and systems are affected by unpredictable noise and disturbances with unknown or partially unknown properties. This reasoning claims for robust fault diagnostic systems, which should be ideally insensitive to uncertainties, noise, and disturbances. Frank [4] states that "other than with modelling for the purpose of control, such discrepancies cause fundamental methodical difficulties in FDI applications. They constitute a source of false alarms which can corrupt the performance of the FDI system to such an extent that it may even become totally useless. The effect of modelling uncertainties is therefore the most crucial point in the observer-based FDI concept, and the solution of this problem is the key to its practical applicability."

The focus will be given now to observer-based methods which constitute part of the baseline of scientific research on this topic. Note that this thesis fall mainly on this type of approach. Other two relevant methods are parity relations, which we refer the reader to [5–7] for further details, and parameter estimation that is intensively studied in [8, 9]. Afterwards, a brief discussion on decision making tools for fault detection is undertaken.

2.1. Observer-based methods

Observer-based design constitute the development basis in FD research. The main idea behind this approach is to apply an observer, based on an available model of the system. The residual is then obtained by computing the difference between the observer outputs and measured signals. Several authors explore this method in a deterministic setting, through the so-called Luenberger Observer, as described in [3, 10] or in a stochastic fashion with the application of the Kalman filter [11–13]. It is straightforward to understand that if only one observer is put in practice, fault detection can be achieved but the isolation part becomes hard to solve. One possible alternative mentioned in the reviewed literature for sensor faults is the dedicated observer scheme, which suggests the development of a set of observers each of which driven by a specific measured output. In this way, if some sensor is faulty, the correspondent observer will have its residual deviated from the nominal behaviour. Usually, this causes the observer to be highly affected by model uncertainties and disturbances, being susceptible to false alarms. A second alternative is the generalized observer design, which also defines a set of observers but all driven by every output available except one. In this methodology the reasoning is opposed to the former scheme, i.e. all residuals except one are affected by a single fault. Although this method has its advantages in terms of robustness, it finds some drawbacks if one intends to detect multiple and independent faults. Moreover, the design of such structured residuals for actuator fault diagnosis is more challenging. For this case, alternatives like unknown input observers [14–16] and eigenstructure assignment [17, 18] are applicable. However, it is not always possible due to do so due to the observability properties of the system [19]. The basic idea behind the referred strategies is that by adapting the driven residual vector structure and the observer gain, it is possible to design an insensitive residual to some specific actuator fault. Other strategies that try to achieve fault isolation with only one observer were also a focus of study, namely the fault detection filter firstly introduced by Beard [3]. Such a filter is built with a gain design strategy which allows for the residual to react differently on the presence of distinct faults. Therefore, the residual properties along the time provide the isolation basis of the method. Multiple-model strategies may be interpreted as an extension of the observer-based methods, in the sense that all the information about the system process and admissible fault characteristics is used to build a set of filters, each of which designed for a particular fault scenario. A simple to describe this methodology is a system, that besides its nominal operation, can also operate at two other working points by the incidence of two distinct faults. This means that the uncertainty of this model, caused by the considered admissible faults, is defined by a discrete combination of three operating conditions. With a multiple-model approach, the designer uses three independent filters each tuned for one of the operating conditions. As a consequence, by the analysis of the residual sequences, fault detection and fault isolation may be achieved. Usually, the uncertainty caused by possible faults define an infinite set of operating points, rendering this method more challenging but still very useful. In fact, the multiple-model framework with application to fault diagnosis is going to be the focus on this thesis, thus extensively explored in the following chapters. A considerable research has been performed throughout the years upon this method mainly due to its flexible structure that allows intuitive modelling of faults [20] and higher support in modern computers for larger processing requirements. Note that one of the main drawbacks of this approach is its computational complexity, which increases in-line with the number of filters included in the bank. Some examples of successful applications may be found in [20–24]. A more recent variation of this method is the interacting multiple-model (IMM) design that considers an inter-dependent processing between the filters, what Ru and Li [23] suggest to lead to an enhanced performance in terms of detection time and proper identification. IMM-based fault diagnosis has attracted the interest of researchers in the last decades [25–27].

2.2. Residual Evaluation: Decision Making

Having discussed residual generation, the following step in the fault diagnosis process is devoted to residual evaluation which will enable to assess a fault occurrence. The most straightforward strategy is to define fixed residual thresholds which when crossed indicate a fault presence. Still, due to the inevitable system model uncertainties, disturbances and noise, the generated residuals will never be strictly null in a fault-free scenario. Similarly, with a fault occurrence it is probable that the expected characteristics of the residual signal are not met. As a consequence, the definition of thresholds is a challenging task while playing an important role on decision making [20]. Note that if small thresholds are assigned, false alarms are likely to occur, whereas large thresholds values may lead to missed detections, both of which deteriorate the fault diagnosis scheme. To overcome this limitation, one widely documented strategy is to use adaptive thresholds [28, 29]. Adaptive threshold techniques provide a methodology to compute threshold values in real-time based on the control activity, noise, and characteristics of the residual signal. This method enhances the decision making performance by decreasing the ratio of false alarms and missed detections.

It is worthwhile to mention that fault diagnosis schemes with multiple-model approaches are based on hypothesis testing. Each model considered in the bank of observers is assumed a hypothetical real model. This topic is going to be further explored in Section 5.

3. Research Proposal

In this thesis we focus on multiple-model estimation techniques with application to residual-based fault detection and isolation. More precisely, we intend to determine the working regime of the monitored plant under the uncertainty imposed by an unknown fault occurrence. Initially, the problem will be tackled through a classical approach using Kalman filters. In what follows, the study of robust H_2 filters designed for well-defined uncertainty domains is undertaken. In both approaches, we will discuss and explore the estimation stability properties and a provide a verification of the developed theory through several computational simulations, using a generic Helicopter dynamical model. We also intend to consider a mostly generic fault model, in opposition to what is found in great part of the dedicated literature where only specific faults are contemplated.

The decision of developing a research on multiple-model strategies lies mostly on the identified current trend studies on fault diagnosis. Additionally, and despite being focused on other applications, the work developed by other students and researchers at the Institute for System and Robotics on multiple-model adaptive estimation and control techniques [30–37] motivate us for the proposed approach.

4. Fault Model

Consider an LTI system of the form

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) \tag{1a}$$
$$\mathbf{y}(t) = C\mathbf{x}(t) + D\mathbf{u}(t) \tag{1b}$$

where $\mathbf{x}$ is the state, $\mathbf{u}$ the input, and $\mathbf{y}$ the output vectors of the system. Matrices A, B, C, and D correspond, respectively, to the state matrix, input matrix, output matrix and feedthrough matrix. Usually faults are modelled with a variation of the system parameters which directly affect the system matrices. Still, this multiplicative modelling fashion is more suitable for component faults, becoming restrictive if one intends to consider sensor or actuator faults. That is the case for an offset fault that imposes a bias in the state dynamics.

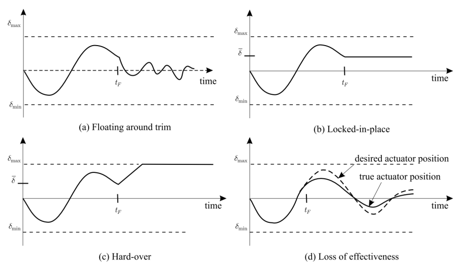


Figure 1: Types of actuator faults occurring after t_F. *Source: [24, pg. 6]*

In this thesis, the focus will be the study on actuator faults, which will require us to find an appropriate additive fault model. Despite this particularization, it is aimed that the developed research can also be applied to other types of faults by considering a dedicated fault model. In what follows, let us first reason that an actuator fault can be seen as a modification of the system input vector $\mathbf{u}$. Through the reviewed literature, but mainly based on [24], four major types of actuator faults can be identified. All of which are illustrated in Fig. 1. Except for fault type (a), the other three types of fault may be modelled by the combination of two scalar fault parameters: (i) an effectiveness parameter $\lambda \in [0, 1]$ and (ii) an offset parameter $u_0 \in [u_{\min}, u_{\max}]$ such that the system input may be given by

$$\mathbf{u}(t) = \Lambda\mathbf{u_c}(t) + \mathbf{u_0} \tag{2}$$

with

$$\Lambda = \operatorname{diag}\left([\lambda_1, \lambda_2, \ldots, \lambda_m]\right); \quad \mathbf{u_0} = [u_{01}, u_{02}, \ldots, u_{0m}]^T \tag{3}$$

where m is the number of system actuators and $\mathbf{u_c}$ the control input vector. The global actuator fault system model is then given by

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\left(\Lambda\mathbf{u_c}(t) + \mathbf{u_0}\right) \tag{4a}$$
$$\mathbf{y}(t) = C\mathbf{x}(t) + D\mathbf{u}(t) \tag{4b}$$

A fault domain representation is shown in Fig. 2 where each fault type region is indicated. Note that in nominal/fault-free condition the following fault parameters matrices hold

$$\Lambda = \operatorname{diag}\left(\mathbf{1_m}^T\right); \quad \mathbf{u_0} = \mathbf{0_m} \tag{5}$$

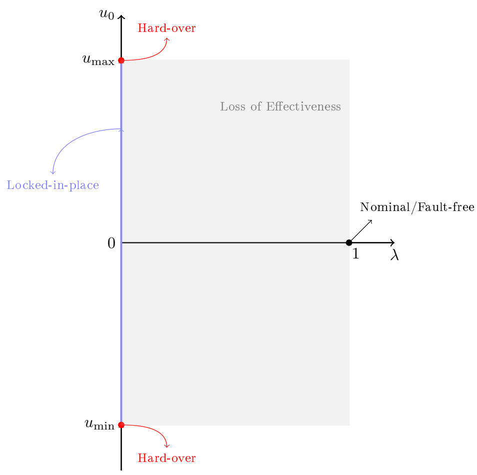


Figure 2: Types of actuator faults illustrated in fault parameters domain.

5. Multiple-Model Adaptive Estimation (MMAE)

The Multiple-Model Adaptive Estimation (MMAE) technique is a model-based estimation approach specially suitable for systems subject to parameter uncertainty. If some information is known about the uncertain parameter, such as its domain of uncertainty, then a multiple-estimator bank structure, composed by Kalman filters (KFs), may be designed covering an adequate range of possible models. A specific probability analysis tool may then be applied to analyse the local state-estimation and associated innovations, for a stochastic setting, generated by each estimator in order to obtain the optimal combined estimation. The architecture of the described technique is shown in Fig. 3.

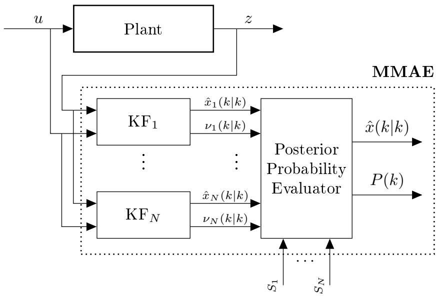


Figure 3: Multiple-model Adpative Estimation (MMAE) Architecture.

Note that we may interpret system faults as uncertainties in our model description, thus the MMAE turns out to be an interesting tool under the scope of our study in fault detection and isolation. Accordingly, consider an LTI MIMO system subject to actuator uncertainties in agreement to the actuator fault model described in Section 4

$$\mathbf{x}(\boldsymbol{k}+\mathbf{1}) = A\mathbf{x}(\boldsymbol{k}) + B\left(\Lambda^{\kappa}\mathbf{u}(\boldsymbol{k}) + \mathbf{u}_{\mathbf{0}}^{\boldsymbol{\kappa}}\right) + G\mathbf{w}(\boldsymbol{k}) \tag{6a}$$

$$\mathbf{z}(\boldsymbol{k}) = C\mathbf{x}(\boldsymbol{k}) + \mathbf{v}(\boldsymbol{k}) \tag{6b}$$

where $\mathbf{x}(\boldsymbol{k}) \in \mathbb{R}^n$ denotes the system state, $\mathbf{u}(\boldsymbol{k}) \in \mathbb{R}^m$ its control input, $\mathbf{z}(\boldsymbol{k}) \in \mathbb{R}^q$ the measured output, $\mathbf{w}(\boldsymbol{k}) \in \mathbb{R}^n$ the process noise input, and $\mathbf{v}(\boldsymbol{k}) \in \mathbb{R}^q$ the measurement noise. The noise vectors, which are white noise Gaussian sequences, obey the following relations

$$\begin{array}{ll} \mathbf{E}\left\{\mathbf{w}(\boldsymbol{k})\right\} = 0 & \mathbf{E}\left\{\mathbf{w}(\boldsymbol{k})\mathbf{w}(\boldsymbol{t})^T\right\} = Q\delta_{kt} \\ \mathbf{E}\left\{\mathbf{v}(\boldsymbol{k})\right\} = 0 & \mathbf{E}\left\{\mathbf{v}(\boldsymbol{k})\mathbf{v}(\boldsymbol{t})^T\right\} = R\delta_{kt} \end{array} \tag{7}$$

A, B, and C are the state, input and output matrix of appropriate dimensions, respectively. Matrix Λ^{κ} and vector $\mathbf{u_0}(\boldsymbol{k})$ are unknown and determine the uncertain parameters of system 6 that belong or are "close" to a finite discrete parameter set, $\kappa := \{\kappa_1, \kappa_2, \ldots, \kappa_{\mathrm{n}}\}$ indexed by $i \in \{1, 2, \ldots, N\}$. The MMAE approach suggests that the global estimate is given by

$$\hat{\mathbf{x}}(\boldsymbol{k}|\boldsymbol{k}) = \sum_{i=1}^{N} P_i(k)\hat{\mathbf{x}}_{\boldsymbol{i}}(\boldsymbol{k}|\boldsymbol{k}) \tag{8}$$

where $P_i(k)$ stands for the conditional posterior probability of $\kappa = \kappa_i$, i.e. that estimator i model matches the real system.

5.1. Posterior Probability Evaluator (PPE)

The central element of the MMAE is the previously referred Posterior Probability Evaluator (PPE) which is responsible for computing the posterior conditional probability of each model, at every instant, to match the real one. The recursive relation to compute those probabilities is given by

$$P_i(k+1) = \left(\frac{\zeta_i(k+1)e^{-\frac{1}{2}\omega_i(k+1)}}{\sum_{j=1}^{N}\zeta_j(k+1)e^{-\frac{1}{2}\omega_j(k+1)}P_j(k)}\right)\cdot P_i(k) \tag{9}$$

$$\text{with} \quad \zeta_i(k+1) \equiv \frac{1}{(2\pi)^{\frac{m}{2}}\sqrt{\det S_i(k+1)}}$$

$$\text{and} \quad \omega_i(k+1) \equiv \boldsymbol{\nu}_{\mathbf{i}}(\boldsymbol{k}+\mathbf{1})^T S_i(k+1)^{-1}\boldsymbol{\nu}_{\mathbf{i}}(\boldsymbol{k}+\mathbf{1})$$

for a given initial *prior* $P_i(0)$. A closer look at Eq. (9) reveals that, from an implementation point of view, one may not allow that any model κ_i has its probability down to 0 as it will cause the *posterior* to never recover, even if κ_i matches the real model. For further details on the deduction of result (9) consult the thesis document.

5.2. Convergence Properties

In the scope of the present thesis, it is specially relevant to explore the convergence result for the case that none of the models included in the bank of filters matches the real parameters. According to a theorem in [38, p. 274], the Baram Proximity Measure (BPM) given by

$$\beta_i^j = \ln\left(\det S_i\right) + \mathrm{Tr}\left(S_i^{-1}\Gamma_i^j\right) \tag{10}$$

holds the information concerning which model is converged under a certain fault scenario. In Eq. (10) Γ_i^j stands for the steady-state innovation mean-square matrix resultant from the estimation provided by filer i when the real model j holds, whereas S_i refers to the steady-state covariance matrix when $j = i$. In practice the model holding the lowest BPM will see its posterior probability tend do 1.

5.3. Bank of Kalman Filters Design Strategy

The actuator fault model which we consider provide us with an infinite uncertain parameters set. From this set one shall pick N admissible values which will be the tuning parameters of our N KFs. In what follows, two main questions arise in this design process

1. What should be the size of the representative parameter set given by N which define the number of KFs in the bank?

2. How can one establish the representative parameter set $\kappa := \{\kappa_1, \kappa_2, \ldots, \kappa_N\}$?

The design process considered has in its basis four relevant premises:

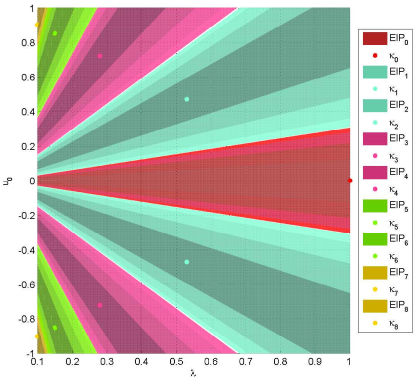
1. (Equivalent KF dynamics) It can be shown that the optimal estimation performance for $N \to \infty$ is invariant of the fault point considered.

2. (Independent Bank Design) The reason for this strategy lies in the convenience of performing the bank design in a $\mathbb{R}^2$ domain, rather than a larger dimension domain. Therefore, each actuator is considered separately.

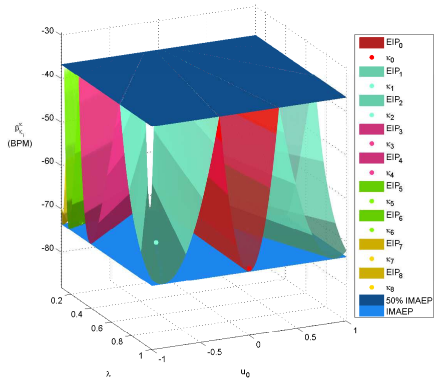
3. (Concept of EIP) Equivalently Identified Plants (EIP) define the regions in the uncertain parameter domain that are characterized by the model to which they will converge given all the admissible real parameter and the representative set κ. Those regions can be defined with the help of BPM.

4. (Concept of IMAEP) Infinite Model Adaptive Estimation Performance (IMAEP) is an index that provides the best performance in terms of Baram Proximity Measure considering an ideal bank design with $N \to \infty$.

Having introduced the previous premises, the design procedure can now be focused. It mostly inspired in [32], where the author suggests a performance-based model set design strategy for the MMAE. The performance criterion is defined as a percentage of the IMAEP corresponding to the worst admissible performance. The same approach was embraced but with several adaptations due to the bi-dimensional uncertainty domain considered. The outcome of the design was bank with 15 filters (80% IMAEP) and a second one with 9 filters (50%IMAEP). The latter bank EIP map representation is illustrated in Fig. 4.



(a) 2D view



(b) 3D view

Figure 4: Bank of Kalman Filters design for a 50% IMAEP minimum performance criterion.

5.4. Experiments on Simulation Environment

Several simulation were performed in order to assess: (i) the identifiability of the models by verifying the probability signals convergence in different scenarios; (ii) compare with the theoretical results found during the bank design; (iii) verify that the performance criterion was met. The most prominent conclusions of those experiments are now highlighted. Firstly, the MMAE approach allows to clearly identify different models under distinct fault occurrences. Nevertheless, some faults may not be detected if their fault parameters fall in the nominal EIP region. As a consequence, it must be assumed that the designed architecture is not ideal for fault detection and isolation, as it is susceptible to false alarms or missed detections. On the other hand, it is a powerful system for state estimation under parameter uncertainty. Finally, the results provided an indicative validation for the performance-based design of the estimators' bank. This is, indeed, a convincing argument for the application of the MMAE method along with the developed design strategy, since we are able ensure a well-defined a performance criterion for the state estimation.

5.5. Improving results: second filtering stage

Due to the oscillatory behaviour observed of the probability signals, a second filtering stage was developed based on an original algorithm that only sets a certain model probability to 1 if it meets a defined criterion. The same logic is applied to change the probability of some model back to 0. The outcome of the presented strategy, which was tested along the previous battery of simulations, is a well-defined identification of the models and smoother probability transitions.

6. Multiple-Model Adaptive Estimation (MMAE) with $\mathcal{H}_2$ Robust Filters

In the last chapter we focused our study on fault diagnosis in a multiple-model based approach which considered a bank of Kalman Filters, each specifically tuned for a fixed combination of actuator fault effectiveness and offset parameters. The developed strategy, which was built upon a well defined performance criterion, resulted in large banks of estimators capable of detecting and isolating faults effectively. To be more precise, the MMAE posterior probability evaluator could clearly indicate the real fault parameters region when under a fault occurrence or in a fault-free scenario.

One of the drawbacks identified of the accomplished designed was the requirement for a large number of Kalman Filters to achieve the performance criterion defined. Recall that under the most strict performance defined - 80% IMAEP - 15 filters needed to be included, whereas for the 50% IMAEP case 9 filters were required just for a single actuator monitoring. The use of a large number of estimators asks for substantial processing means which are not always available and may well be limited in real applications. This concern and the interesting studies about optimal linear filtering under parameter uncertainty reviewed on the literature ([39]) motivated the application of $\mathcal{H}_2$ robust filters under the scope of actuator fault diagnosis.

The goal is set to reduce the number of filters required, while meeting a certain worst-case performance. Note that the Kalman Filters designed in the previous chapter can be interpreted as $\mathcal{H}_2$ filters in the sense that they also minimize the 2-norm of the estimation error output, or in other words the steady-state estimation error covariance. The main difference between the two approaches is that with the $\mathcal{H}_2$ synthesis the dynamical model of the system does not have to be precisely known, allowing to cope with parameter uncertainties. Consequently, assuming that the estimation error depends on the unknown parameters, the performance index to be optimized is the upper bound of the mean-square estimation error, being valid for all admissible models [40].

6.1. $\mathcal{H}_2$ Robust Filter Design with LMI Convex Programming

The $\mathcal{H}_2$ robust filter was deduced, mainly based upon [39, 40]. The deduction consisted essentially of two main steps: (i) determination of the optimization problem that minimizes the upper bound of the mean-square estimation error, being valid for all admissible models and (ii) convert the previous problem to a convex LMI semi-definite programming, so that it can easily be solved with aid of available computational tools.
Integrating the described design procedure with actuator fault model was a challenging task mainly due to the inclusion of the offset type of faults, which can not be modelled by a simple modification of the system matrices. A solution to this problem was found by accounting the offset parameter fault as a white signal perturbation, obtaining a virtual augmented plant as depicted in Fig. 5.

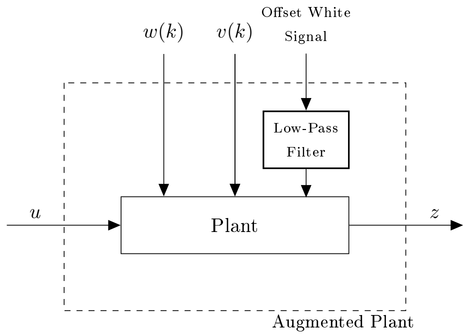


Figure 5: Augmented plant block diagram defining the offset as white perturbation.

6.2. Novel MMAE Bank Design

This section is focused on the MMAE filters' bank design with the inclusion of the $\mathcal{H}_2$ filter. However, before proceeding we would like to provide some remarks concerning the PPE formulation when including an $\mathcal{H}_2$ filter in the MMAE bank. Some discussion may arise in how $S_{i(k+1)}$, which relates to the recursive law (9), for the conditional posterior probability evaluation. Note that having a certain model κ_i matching the real plant loses significance for the $\mathcal{H}_2$ filter when optimized for a certain region. Still, it was discussed earlier that for any admissible model the optimal state estimation for each always yields the Kalman filter and associated steady-state residual covariance, given by $\tilde{S} \equiv C\tilde{\Sigma}C^T + R$, which is constant over the whole uncertainty domain. As a consequence, that value should also be applied for any $\mathcal{H}_2$ filter in bank independently of the optimization range. The supporting rationale for this choice is that the recursive function implemented in the PPE shall have a common optimal estimation reference for all filters in the bank, so that a fair comparison between residuals is attained. Furthermore, since the innovation ν has no meaning in the $\mathcal{H}_2$ description developed, we suggest the use of the residual $\mathbf{r}_{(k)} = \mathbf{z}_{(k)} - C\hat{\mathbf{x}}_{(k|k)}$ for any Kalman filter present in the bank and $\mathbf{r}_{(k)} = \mathbf{z}_{(k)} - C\hat{\mathbf{x}}_{(k)}$ for the $\mathcal{H}_2$ filter. Based on the developed results in Section 5.1, similarly it can be shown that with the use of $\mathbf{r}_{(k)}$ the PPE recursive law becomes

$$P_{i(k+1)} = \left(\frac{\beta_{i(k+1)} e^{-\frac{1}{2}\omega_{i(k+1)}}}{\sum_{j=1}^{N} \beta_{j(k+1)} e^{-\frac{1}{2}\omega_{j(k+1)}} P_{j(k)}} \right) \cdot P_{i(k)} \tag{11}$$

with $\beta_{i(k+1)} \equiv \dfrac{1}{(2\pi)^{\frac{m}{2}}\sqrt{\det \tilde{S}_i}}$

and $\omega_{i(k+1)} \equiv \mathbf{r_i}_{(k+1)}{}^T \tilde{S}_i^{-1} \mathbf{r_i}_{(k+1)}$

Rule (11) allow us to design our bank freely, which may only include Kalman filters, $\mathcal{H}_2$ filters or a combination of both. Following the same reasoning, the BPM may also be redefined by

$$\beta_j^i = \ln(\det \tilde{S}_i) + \mathrm{Tr}\left(\tilde{S}_i^{-1}\tilde{\Gamma}_j^i\right) \quad (12)$$

with $\tilde{\Gamma}_j^i \equiv \mathbf{E}\left\{\mathbf{r}(k)\mathbf{r}(k)^T\right\}$ as $k \to \infty$

In order to achieve a final bank design, let us first assess the RMS of the estimation error performance of the 50% IMAEP design and compare it to the proposed $\mathcal{H}_2$ filter as seen in Fig. 7. By analyzing the referred graph an observation of paramount importance must be emphasized, which is that a enhanced worst-case performance is achieved with a single H_2 filter when compared to the 9 KFs counterpart. Still, we highlight that this result is attained at the cost of lower performance in the KFs tuning points and their neighbourhoods. As we are dealing with faults, assumed not to be likely to occur in regular system operation, it becomes relevant to have an optimal state estimation performance at the nominal condition. Therefore, in this thesis, it is suggested the application of a combined filter structure for the MMAE bank with a Kalman filter tuned for the nominal parameters and an $\mathcal{H}_2$ filter optimized in the range $\lambda \in [0.1, 1]$, as illustrated in Fig. 6.

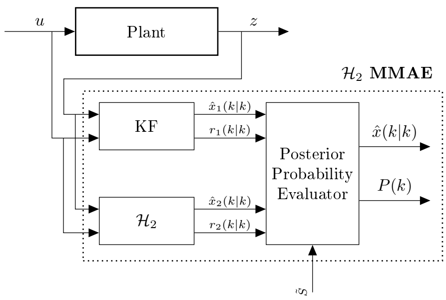


Figure 6: Novel MMAE block diagram.

6.3. Experiments on Simulation Environment

The same goals defined in Section 5.4, were settled here for the evaluation of the novel MMAE bank performance. From the obtained result, the most prominent conclusions were: (i) The novel MMAE bank, built upon the application of a nominal Kalman filter in combination with a $\mathcal{H}_2$ filter optimized in the effectiveness parameter uncertainty domain, showed an equivalent performance in terms of model identification considering the EIP regions obtained; (ii) The simulation results showed that

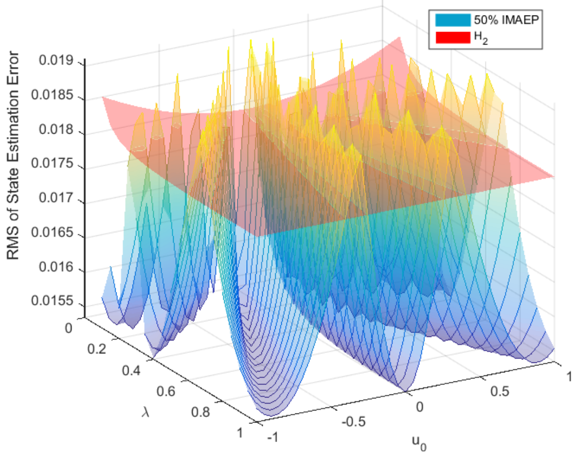


Figure 7: Performance comparison between $\mathcal{H}_2$ Filter and MMAE 50% IMAEP-based design.

the same level of worst-case state estimation performance is achievable with just two filters comparing to the 9 required in Section 5.4.

7. Conclusions

This thesis comprehended the study of multiple-model estimation methods applied to fault detection and isolation of linear dynamical systems. The paramount importance of safety and reliability of controlled systems, namely critical systems, and the accumulated experience at the Institute for System and Robotics (ISR) by other researchers on this methodology motivated this thesis and specific approach. By developing a MMAE architecture, the goal was set to identify the working regime of the plant leading to the detection of faults and identification of the operating region, which is determined by where the fault parameters lie in a known uncertainty domain. By developing the research based on MMAE an inherent focus of study was the accomplishment of a high performance state estimation, despite the uncertainty regime that stemmed from the faults occurrence.

The problem at hand was divided in two stages. Initially, a classic MMAE methodology based on Kalman filters was developed using a performance-based design for the bank. This permitted an intuitive determination of the filters' tuning point and, thus, the size of the bank. Considering the developed general actuator fault model, the design process was held in a bi-dimensional uncertainty domain, which accounted for an effectiveness and offset fault parameters. The computational simulations performed revealed that the developed system could effectively track the change of the working regime by convergence to a different filter, depending on the localization of the fault. However, if the fault was located in the nominal EIP region, a probability transition was no longer observed, as expected.

The performance-based design of the prequel strategy required the use of 9 KFs just for the monitoring of a single actuator, becoming computationally complex for real applications. This fact motivated the second addressed technique, which included a novel MMAE bank design in a combination of Kalman and robust $\mathcal{H}_2$ filters. At this point the goal was set to the reduction of filters in the bank, while preserving the state estimation performance previously attained. The study of the latter filters was challenging due to the interest in coping with a state estimation optimization in a polyhedral bounded domain and, simultaneously, account for the bias on the dynamics resultant from the offset fault parameter. With an equivalent performance, the outcome of this research was a bank size reduction to just 2 filters including a nominal Kalman filter and a robust $\mathcal{H}_2$ filter, optimized over the whole uncertainty domain defined by the faults' model.

Due to the high oscillatory behaviour of the conditional probability signals in both approaches, an independent filtering stage was also developed. The rationale behind the algorithm created was to attribute cumulative scores to the models depending on their probability at every instant. This way, the probability 1 was directly given to the first ranked filter. The result was a well-defined identification of the models with smoother probability sequences, turning the FDI scheme specially suitable for reconfiguration methods alike MMAC.

To conclude, it should be highlighted that despite the effective convergence to distinct models in both approaches developed, the multiple-model strategies alone may fall behind in what could be expected from a FDI scheme in terms of detection performance. Namely when the faults are located inside the nominal EIP region. A key for this drawback could be to add several filters in the neighbourhoods of the nominal model, but that would result in a very large bank that possibly could not meet the intended estimation performance criterion. Still, the attained results in terms of techniques for multiple-mode banks performance-based design can not be disregarded. Particularly, the state estimation performance with the proposed MMAE bank design, based on $\mathcal{H}_2$ filters, is indicative of the potentialities of this method to any application involving plant uncertainty constraints.

8. Future Work

After the last six months of research which resulted in this thesis, several topics in the scope of the study undertaken were left for future developments. Some of those are now highlighted.

Fault parameters identification: In the previous section, the drawbacks on fault detection performance from the MMAE system were emphasized. However, the possibility of guaranteeing a high level of estimation performance under a significant plant uncertainty, and at a minimized computational cost, motivates the search for a fault parameter identification scheme that could work in parallel or integrated in the developed MMAE architecture. This could be the key for an optimal strategy combining both detection and state estimation. In fact, preliminary studies on this topic were performed during the thesis but with low theoretical support. Therefore, it is now addressed for future development.

Real scenario experiments: After having completed a rather consolidated verification of the methods developed in a simulated environment, the natural step afterwards would be to execute trials on a real scenario. That would allow the validation of the method and strengthen the potentialities of the strategies designed.

Extension to sensor and component faults The thesis was focused on actuator fault detection and isolation. Nevertheless, the architectures designed are widely general and, thus, can easily be extended to other types of faults, namely sensor and component faults. Note that one of the most claimed advantages of the MMAE is the ease of inclusion and modelling of both additive and multiplicative faults.

$\mathcal{H}_\infty$ and $\mathcal{H}_2/\mathcal{H}_\infty$ synthesis: With the research undertaken it was proven that other types of filters, besides the classical ones, may be integrated in a MMAE scheme. Promising results are found in reviewed literature comprising the application of $\mathcal{H}_\infty$ estimators or in a combined synthesis $\mathcal{H}_2/\mathcal{H}_\infty$. Therefore, it would be deeply interesting to assess the applicability of these filters under the scope of robust fault diagnosis.

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