

Torque Vectoring for a Formula Student Prototype

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Abstract. Torque Vectoring (TV) has the objective to substitute the need of a mechanical differential, while improving the handling and response of the wheeled vehicle. This work addresses the design of a torque vectoring system in an rear wheel driven formula student prototype. The proposed solution resorts to a PID controller for yaw rate tracking with an evenly distributed torque to each wheel. Also an LQR scheme is discussed, for tracking the yaw rate and the lateral velocity. To assess and design, first a 7 degree of freedom (DOF) non linear model is constructed, followed by a linear 2 DOF model, both validated with real data. The linear model, is used to design and simulate the proposed controllers. When the controller is within the desired parameters it is tested in the non linear model. Tests with the vehicle are performed to verify the contribution of the controller to the overall performance of the vehicle.

Keywords: Torque vectoring · Formula student · PID controller · LQR controller · Vehicle model

1 Introduction

Each year that goes by sees an increase in sales of personal use of electric vehicles (battery EVs, plug-in hybrids and regular hybrids), all of which rely on electric motors as a basis or an aid to propulsion. It also opens the opportunity for vehicle stability control systems directly at the motors, like Electronic Stability Program (ESP) and Anti-lock Braking System (ABS) which gives the vehicle better stability and maneuverability. This paper addresses another type of vehicle stability control called Torque Vectoring (TV). By controlling the amount of torque distributed to each driven wheel, the system has the potential to improve both the stability and response of a vehicle without compromising safety and drivability.

Developing a torque vectoring system can be tackled resorting to several different approaches. The most common [6, 8] use the yaw rate of the vehicle as the reference for the controller. More advanced solutions [7, 9, 11] use the sideslip

angle and/or a combination of yaw rate and sideslip angle. The choice on the strategy will depend on the available sensors.

A variety of control system design approach can be used. The most basic method is to distribute the left and right torque, proportional to the amount of steering input $\Delta T = f(\delta)$, where δ is the steering wheel angle. The proportional integral derivative (PID) controller is the classic control structure and the most commonly used in practical applications. It is a straightforward method to implement and tune [9, 12]. Sliding mode control is a non linear control design methodology used by several researchers to achieve the objectives of tracking the yaw rate and slip angle [9, 11]. Predictive control estimates the future states of the vehicle in order to find the best control input [3, 4]. Some authors also try to implement Fuzzy control to create a set of rules for the allocation of the torque [14].

2 Vehicle Model

Vehicle dynamics is the area devoted to the development of models that describe the behavior of a vehicle for any given set of inputs and disturbances. Modeling this type of system is a very complex and challenging task. A lot of different approaches and models can be needed. A complex multi-body system (with 20+ degrees of freedom), or a simple two degree of freedom model can be representative [5, p. 6].

The development of the torque vectoring controller will be tested in the FST06e. The FST06e is a prototype electric vehicle powered by two *Siemens* permanent magnetic synchronous motors each one with an RPM range of 0 to 8000, and producing a maximum torque of 107 Nm. With a planetary gear set fixed with a gear ratio of 4.1:1, amplify at the wheel with a total of 876 Nm.

2.1 Non Linear Model

The vehicle model used to study a torque vectoring control will typically have seven degrees of freedom. The lateral and longitudinal velocities of the vehicle (v_x and v_y respectively) and the yaw rate $\dot{\psi}$ constitute three degrees of freedom related to the vehicle body. The wheel velocities of the four wheels, the front left wheel (w_{fl}), front right wheel (w_{fr}), rear left wheel (w_{rl}), and rear right wheel (w_{rr}) constitute the other four degrees of freedom [10].

The full model consists in a horizontal model which describes the model position and orientation of the vehicle. A model of the steering kinematics, which describes the relation between the steering wheel and the actual steering of each wheel. A simplified tire model to calculate the forces acting on the wheels. A balance at the wheel between the applied torque and the tire longitudinal force. Lateral and longitudinal weight transfer, to model the loads at each wheel during cornering, braking and acceleration. Figure 1 shows the block diagram of the model, where some of the physical variables can be found in [2].

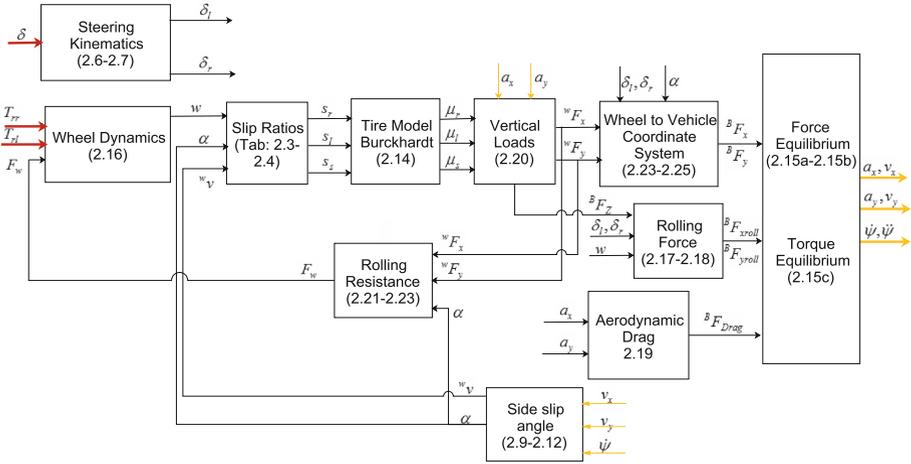


Fig. 1. Simulink Schematic of Non Linear Model

2.2 Linear Model

The linear model, is one of the most common models. The linear bicycle model is a simplification in which it is only being considered the lateral velocity v_y and yaw rate $\dot{\psi}$ on the model [5, 6, 8, 14]. The way the model is presented, the equations only generate lateral force by the steering input. The goal with the torque vectoring is to generate yaw moment based on controlling the torque (longitudinal force) at the driven wheels. For this it is necessary to introduce a new term M_z that represents the additional yaw moment. Table 1 summarizes the values from the FST06e need for the model.

$$\dot{x} = Ax + Bu_1 + Eu_2$$

$$\dot{x} = \begin{bmatrix} \dot{v}_y \\ \dot{\psi} \end{bmatrix}; \quad u_1 = M_z; \quad u_2 = \delta$$

$$A = \begin{bmatrix} -\frac{C_{y,f} + C_{y,r}}{mv_{x0}} & -\frac{l_f C_{y,f} + l_r C_{y,r}}{mv_{x0}} - v_{x0} \\ -\frac{l_f C_{y,f} + l_r C_{y,r}}{I_{zz} v_{x0}} & -\frac{C_f l_f^2 + C_r l_r^2}{I_{zz} v_{x0}} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \\ I_{zz} \end{bmatrix} \quad E = \begin{bmatrix} C_{y,f} \\ mv_{x0} \\ l_f C_{y,f} \\ I_{zz} \end{bmatrix} \quad (1)$$

2.3 Validation

The constant radius turn (skidpad) is defined by ISO 4138 [1]. According to the norm, the test should be performed with a minimum circle radius of 30 m. For the FST06e, the available test track is at the campus in front of the main build

Table 1. Linear model values

Term	Symbol	Value	Units	Term	Symbol	Value	Units
Yaw rate	$\dot{\psi}$	-	$[rads^{-1}]$	Rear wheelbase	l_r	0.717	$[m]$
Velocity	v_{x0}	[0,40]	$[ms^{-1}]$	Steering angle	δ	[-3.3,3.3]	$[rad]$
Rear Stiffness	$C_{y,r}$	21429	$[Nrad^{-1}]$	Yaw moment	M_z	-	$[Nm]$
Front Stiffness	$C_{y,f}$	15714	$[Nrad^{-1}]$	Lateral velocity	v_y	-	$[ms^{-1}]$
Inertia moment	I_{zz}	120	$[Kgm^2]$	Gear ratio	G_r	4.4	-
Mass	m	356	$[Kg]$	Half track	t_r	0.65	m
Front wheelbase	l_f	0.873	$[m]$	Wheel Radius	R_w	0.265	m

in the parking lot, with a maximum circle radius of 7.5 m. Another test track is at the International Kartodromo in Palmela, with a maximum radius of 15 m.

In the Formula Student competition, one of the disciplines is to perform a constant radius turn in a track with a radius of 8.75 m. To be more close to the competition conditions, the tests are modified and performed with a range of 5-9 m radius.

In total three data sets are available. “Test 1” and “Test 2” are both done at the campus. The radius of the circle is the same, but the driver varied the velocity. “Test 3” is a test with a bigger radius done in Palmela. Table 2 summarizes the tests.

Table 2. Test setups

	Test 1	Test 2	Test 3	Unit
Radius of circle	5.62	5.62	9.02	$[m]$
Global velocity	7	8.5	9.3	$[m/s]$
Steering angle	110	100	71	$[deg]$
Measured yaw rate	72.3	72	58.6	$[deg/s]$

During the tests some data are being monitored. The inputs are the global velocity (v_{CG}), the steering wheel angle (δ). The output values are the longitudinal, lateral acceleration (a_x, a_y) and yaw rate ($\dot{\psi}$). Although, many more data can be logged from the car, these are sufficient to conclude if the models are accurate enough [13, p. 345]. Figure 2 shows the comparison between the yaw rate from the vehicle and the yaw rate from both linear and non linear simulations. The simulated values are very close to the real data. The data presented illustrates that the vehicle made 4 turns. Two to the left and two to the right (\approx 5s each). Negative values of yaw rate represent the car cornering to the right (in a clockwise way) while positive values represents the car cornering left (counter clockwise).

3 Control System Design

Based on the availability, sampling time and quality of the sensors, the strategy for the calculation of the reference value is presented next, followed by the choice and tuning of the controllers.

3.1 Reference Value

Before the introduction of the control algorithm, it is necessary to define a reference signal. Various authors propose the calculation of the reference based on the velocity and steering angle of the car. It is assumed that the vehicle is in a steady state [10].

$$K_u = \frac{l_r m}{C_{y,f}(l_f + l_r)} - \frac{l_f m}{C_{y,r}(l_f + l_r)} \tag{2}$$

If the under-steer gradient, K_u is positive ($K_u > 0$): The car is said to have an under-steer behavior (under yaw rate).

If K_u is negative ($K_u < 0$): The car is said to have an over-steer behavior (over yaw rate). And if $K_u = 0$, it means that the car is neutral-steer (ideal yaw rate).

A neutral-steer vehicle has the smallest possible turning radius for a given velocity, which corresponds to optimal performance. Therefore, it would be assumed that a neutral-steered vehicle should be chosen as the reference. However, this approach would put the car on the verge of over-steer instability, so a slightly under-steered is taken as reference. This reference is much closer to neutral-steered than the actual car, so the controller can improve the yaw rate.

Given the velocity and steering angle of the car and with the known steer gradient and wheelbase it is possible to know the turning radius, combining with

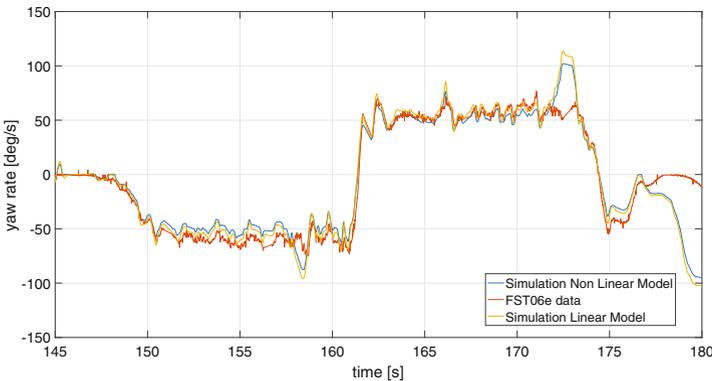


Fig. 2. Comparison between real data form the FST06e and both linear and non linear simulation during a skidpad to the left and right from test1

the under-steer gradient and the road radius, gives the desired yaw reference.

$$\dot{\psi}_{desired} = \frac{v_{CG}}{(l_r + l_f) + K_u v_{CG}^2} \delta \quad (3)$$

The desired yaw rate is a function of the velocity, steering and characteristic of the car. The under-steer gradient K_u can be tuned in for each driver preference. The smaller the under-steer gradient the bigger the difference between the desired and actual yaw rate, more will the car have neutral steer characteristics.

3.2 Maximum Yaw Value

With all the various possible implementations and control strategies, some limitations are valid for all torque vectoring controllers. These limitations are related to the physical properties of the vehicle like, the maximum yaw rate possible, the maximum tire adhesion, microprocessor computing time, etc. Depending on the entry speed of the car, it will be able or not to achieve the desired yaw. If entering in a corner too fast the road may be unable to provide the necessary tire forces, and the car just goes forward, thus under-steering. The solution is to bound the force according to the tire-road coefficient.

$$v_{CG} \dot{\psi} + a_x \beta + \frac{v_{CG} \dot{\beta}}{\sqrt{1 + \tan^2 \beta}} \leq \mu g \quad (4)$$

In Eq. (4), if considering that the car has a small heading angle, the equation can be further simplified and reduced to:

$$\dot{\psi}_{max} = \sigma \frac{\mu g}{v_{CG}} \quad (5)$$

where σ represents a tunability factor to take into account changes in the friction coefficient from different types of pavement. The yaw rate reference is used as long as it doesn't pass the maximum possible yaw rate.

$$X(m, n) = \begin{cases} \dot{\psi}_{des}, & |\dot{\psi}_{des}| \leq \dot{\psi}_{max} \\ \pm \dot{\psi}_{max}, & \text{otherwise} \end{cases} \quad (6)$$

3.3 Proposed Controller

The added momentum results from the difference between the left wheel torque T_{rl} and the right wheel torque T_{rr} . This difference multiplied by the half track of the car will be the additional yaw momentum M_z in the model. If the right wheel has more torque than the left wheel the car will have a positive yaw momentum, thus turning to the left, if the opposite happens the car will have a negative momentum and will turn right.

$$M_z = (T_{rr} - T_{rl}) t_f \quad (7)$$

Table 3. PI values for different velocities values

Vx (m/s)	P	I
7	296.29	12716.7
10	392.23	12492.5
13	421.72	12040
16	479.87	11536.07
19	396.22	11058.5
22	404.79	13650

But the torque at the wheel is not the same torque at the motor. Between the motor and the wheel there is a planetary gear set, this gear set multiplies by a gear ratio G_r the torque from the motor $T_{wheel} = Gr * T_{motor}$. Then the torque at the wheel has to be divided by the wheel radius R_w to obtain the torque at the ground. These values can be found in Table 3.

Putting together all this information and with Eq. 7 the yaw moment from the difference in torque is given by.

$$\Delta T = \frac{R_w}{2t_r G_r} M_z = \frac{0.265}{2 * 0.65 * 4.4} M_z = 0.05 M_z \tag{8}$$

Thus rewriting the state space equation from Eq. 1 in order to the delta torque instead of the yaw moment the new input matrix is:

$$B = \begin{bmatrix} 0 \\ 1 \\ \frac{1}{0.05 * I_{zz}} \end{bmatrix} \tag{9}$$

Taking into consideration that the linear model is dependent on the velocity, the design of one controller will not be sufficient to ensure the control for full range of operation. So a gain scheduled controller is implemented. Six different setting points are chosen. The main disadvantage of having a small number of points is that when the gains change the driver could sense an unexpected yaw rate change. If this occurs more setting points are necessary and thus the same procedure is refined or points are interpolated.

3.4 LQR

Linear quadratic regulator (LQR) is an optimal control strategy for linear systems. In the design of this type of control an optimal gain K is calculated based on the performance index J . The advantage of the PI controller is its simplicity

in implementation and understanding of what is happening in terms of allocated torque, but this simplicity also has its drawbacks. If the tire-road friction is wrong or the calculated reference yaw rate is excessive for the current state of the vehicle, the vehicle behavior may become unstable. To further improve the torque control, a LQR controller is presented. The controller will also use both models, both lateral velocity v_y and yaw rate $\dot{\psi}$ are considered state variables and ΔT the control input. The difference is that now it will also be monitored the lateral velocity, which will give a more robust control.

$$\Delta T = K_r \dot{\psi} + K_v v_y \quad (10)$$

The performance index may be written in the following way:

$$J(u) = \frac{1}{2} \int_{t_0}^{t_f} [(X_d - X)^T Q (X_d - X) + u^T R u] dt \quad (11)$$

where:

- u - control effort, $u = M_z$
- R - weight factor of the control effort
- Q - Penalization matrix for the states, lateral velocity v_y , yaw rate $\dot{\psi}$ and desired yaw rate $\dot{\psi}_{des}$

$$J = \int_{t_0}^{t_f} \left[\frac{1}{2} (\dot{\psi} - \dot{\psi}_{des})^2 + \frac{1}{2} w \Delta T^2 \right] dt \quad (12)$$

where $\dot{\psi}_{des}$ is the desired yaw rate of the vehicle. Minimizing this will lead to a vehicle with very close to neutral steer behaviour. Not forgetting that as discussed, the control effort ΔT must be constrained both due to the maximum torque possible and the tires limit.

The K gains are calculated by solving the Riccati equation, choosing appropriate values of Q and R . The controller is tuned by varying both values, First R and then Q .

$$A^T P + P A - P B R^{-1} B^T P + Q = 0 \quad (13)$$

Solving for our case will be:

$$\begin{aligned} 2a_{11}p_{11} + 2a_{21}p_{12} - R^{-1} \frac{p_{12}^2}{I_{zz}} &= Q_{11} \\ (a_{11} + a_{22})p_{12} + a_{21}k_{22} + a_{12}k_{11} - R^{-1} \frac{p_{12}p_{22}}{I_{zz}} &= Q_{12} \\ 2a_{12}p_{12} + 2a_{22}p_{22} - R^{-1} \frac{p_{22}^2}{I_{zz}} &= Q_{22} \end{aligned} \quad (14)$$

This system can be solved and the optimal feedback gain matrix K will be:

$$K = R^{-1}B^T P = R^{-1} \begin{bmatrix} 0 & \frac{1}{I_{zz}} \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} = R^{-1} \begin{bmatrix} p_{12} & p_{22} \\ I_{zz} & I_{zz} \end{bmatrix} \quad (15)$$

Equation 14 is solved for $Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10^6 \end{bmatrix}$ and a value of $R = 10^{-6}$.

4 Results

The results of the implemented controller are discussed. The car is tested in the same conditions as in Sect. 2.3. The driver performs a skidpad with a radius of 5 m trying to match the same conditions that are used to validate the model (same speed and steering angle). With the data acquired with the different controllers, a comparison is made, and the gain in vehicle performance evaluated.

4.1 Without Torque Vectoring Controller

Figure 3 shows the variables, desired yaw rate (blue color), current yaw rate (red color), and the speed of the vehicle (yellow color). The driver does 5 laps of approximately 5 s each (80s-110s) to the left, cornering in a counter-clockwise way, and from 110s-120s the driver exists and starts cornering in the opposite way in a clockwise way, from (120s-150s). The vehicle speed is also plotted in the graph to confirm if the driver is maintaining its speed, recalling Eq. 3 the reference is calculated based on the velocity and the steering angle, if both are constant then the desired yaw will also be constant.

Focusing now just on the current yaw rate value and desired yaw rate value it can be observed that the current value of the yaw rate is 70 deg/s, and the desired yaw rate is 81deg/s, which means that with torque vectoring there can be a possible gain of 11deg/s. Also, it is important to see that if we look at test1 from Table 4, the yaw rate is quite similar, which makes sense because the test track is the same.

4.2 With Torque Vectoring Controller

After the baseline test is performed, the test with the controllers are performed. Figure 4 shows the same variables as in Fig. 3, but this time the torque vectoring controller is acting on the vehicle. The data presented is from the car cornering to the right. As the driver is starting the corner, like in Fig. 3 at first (525s-535s) the driver is inconsistent but starts to become consistent the more time he is in the corner (535s-555s). That is when it can be seen that the controller is improving the yaw rate of the car.

Table 4 shows that the proposed controller contributes to an increase in lateral performance, the vehicle as more yaw rate, allowing to achieve a higher cornering speed, which translates in a reduction of 7.6% of lap time.

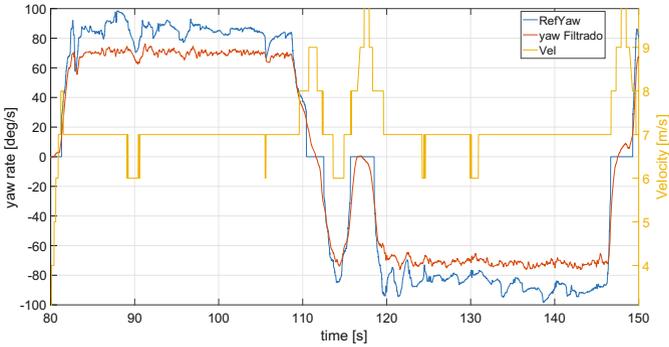


Fig. 3. Data logged from the vehicle during the test with no torque vectoring. The variables presented are: Yaw rate reference, yaw rate, and global velocity

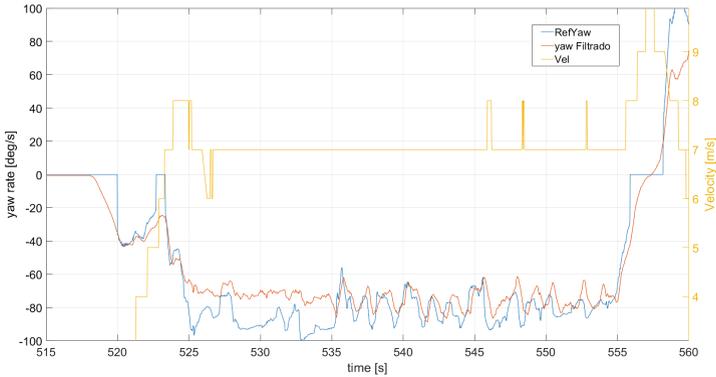


Fig. 4. Data logged from the vehicle during the test with the PI controller. The variables presented are: Yaw reference, yaw rate, and global velocity

Table 4. Comparison between torque vectoring and no torque vectoring for the PI controller

	Yaw rate [deg/s]	Velocity [m/s]	Time [s]
No TV	70	8.4	4.97
TV	74	8.8	4.59

4.3 LQR

Once the tests of the PI controller are done, the LQR controller is tested. Due to tire wear and electronic failure it was not possible to test the LQR controller in the car. Figure 5 compares the real data with the PI controller and the simulation of the LQR controller, it can be seen that the LQR controller response would be very similar to that of the PID controller.

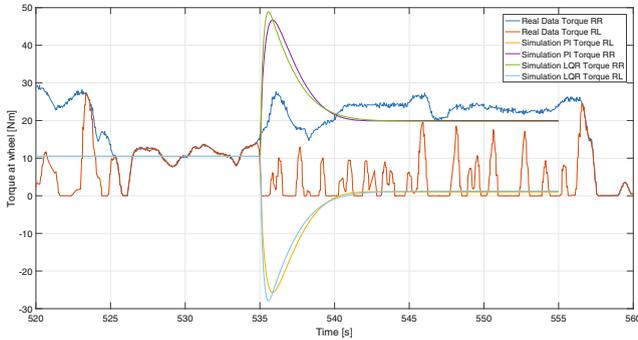


Fig. 5. Comparison of torques between simulation and real data from the car for the LQR controller and PI controller

4.4 Conclusion

A nonlinear model for simulation has been presented, as well as a linearized model for yaw rate control through the application of a differential torque. Both models are validated with logged data from the electric prototype.

Two control strategies are presented; (i) A gain scheduling PI controller for controlling the wheel torques based on the yaw rate, (ii) and an LQR approach for controlling the wheel torques based on the yaw rate and lateral velocity. It was concluded that for low velocities the models, are not much affected by the lateral velocity gain, and in terms of response it was quite similar to the PI controller. The main advantage of the LQR is its robustness and lack of a gain scheduling when compared to the base solution.

The microcontroller developed by the team proved sufficient for receiving and filtering the data, and also run the controller every 0.2s. The fail safe system was crucial to ensure that the tests with the controller could be executed in safe conditions.

Looking at all the different test procedures, it is clear that the implementation of the torque vectoring has an effect in the vehicle behaviour. When the controller is used a gain of 7.6% is achieved, which translates in a reduction of 0.38s in a skidpad when compared to the same situation but without the torque vectoring.

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