

Decentralized Range-based Linear Motion Estimation in Acyclic Vehicle Formations with Fixed Topologies

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Abstract—The problem of decentralized state estimation in formations of autonomous vehicles is addressed in this paper. A reduced number of agents in the formation have access to absolute position measurements, while the rest must rely on range measurements to neighboring agents and local sensing and communication capabilities to estimate their own position and velocity. A method for designing local state estimators for each of the agents is presented such that the resulting decentralized state estimator features globally exponentially stable error dynamics when the graph associated with the formation is acyclic. Realistic simulation results are presented and discussed to assess the performance of the proposed solution under the influence of measurement noise.

I. INTRODUCTION

There are many applications where the use of formations composed by multiple autonomous vehicles working cooperatively is advantageous or, in some cases, crucial. To list only a few examples, formations of Autonomous Underwater Vehicles (AUVs) can be used for applications such as minesweeping and oceanographic sampling over large areas [1][2], and close formation flight of Unmanned Aerial Vehicles (UAVs) allows for more efficient fuel usage [3][4]. As a result of that, the field of control and estimation in formations of autonomous agents has been the subject of extensive research in the last decade, see e.g. [5], [6], [7], [8], [9], and [10].

Conceptually, the easiest way to tackle control and estimation problems in formations of vehicles is to design a centralized solution, in which a central processing node receives all relevant information from the agents, performs all the computations, and spreads the results through the formation by communication. However, the implementation of a centralized solution poses problems in practical settings, as both the heavy computational load in the central processing computer and the extensive communication that are required may cause unacceptable delays and high communication loads in the formation. The alternative is to design a decentralized or distributed solution, in which each agent in the formation only concerns itself with a fraction of the computations, and relies only on locally available data (provided by a combination of sensor measurements and/or communication with other agents in the formation). Although a decentralized solution will fall behind a centralized one

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in terms of performance, the much lower computational and communication load makes it very attractive for practical applications, where communication, computational, and energy constraints are commonplace.

The problem addressed in this paper is the design of a decentralized state observer to estimate linear motion quantities in a formation of vehicles with fixed topology moving in a fluid with constant drift velocity. In order to achieve a decentralized architecture, each agent in the formation must rely only on locally available measurements and limited communication with other agents in the vicinity to estimate its inertial position and velocity. In the scenario envisioned in this paper, some agents in the formation have access to measurements of their absolute position, while the rest rely on measurements of their distance, or range, to other agents in the vicinity, as well as the position estimates of those agents, received through communication. Additionally, to provide awareness of its movement, it is assumed that each agent has access to a measurement of its velocity relative to the fluid, provided e.g. by a Doppler Velocity Log (DVL), measurements of its attitude, through an Attitude and Heading Reference System (AHRS), and a sensor which provides accurate estimates of its depth or altitude. Our approach to this problem can be divided into two main parts:

- 1) The design of a local state observer for each agent, which features Globally Exponentially Stable (GES) error dynamics when the position estimates received from neighboring agents are exact. This problem is tackled by obtaining, through state and output augmentation, a Linear Time-Varying (LTV) system which mimics exactly the nonlinear dynamics of a given agent in the formation. Two cases are considered, depending on the number of range measurements available to the agent, and sufficient conditions for observability of the LTV system are derived for each case. This reduces the problem of designing the local state observers to a classical linear state observer design problem.
- 2) The stability analysis of the decentralized state estimator composed by the local state observers of all the agents in the formation. It is shown that, when the directed graph associated with the formation is acyclic, and under mild assumptions, the estimation error of the decentralized state estimator converges globally exponentially to zero.

The rest of the paper is organized in the following way: Section II poses the dynamics of the agents in the formation as nonlinear systems, while Section III details their observability analysis. In Section IV, the stability of the resulting decentralized estimator is analyzed. Section V presents sim-

ulation results and, finally, Section VI summarizes the main conclusions of the paper.

A. Notation

Throughout the paper the symbol $\mathbf{0}$ denotes a matrix (or vector) of zeros and \mathbf{I} an identity matrix, both of appropriate dimensions. A block diagonal matrix is represented as $\text{diag}(\mathbf{A}_1, \dots, \mathbf{A}_n)$. The superscript x^i is used to identify the i -th component of a vector \mathbf{x} .

II. PROBLEM STATEMENT

Consider a formation composed by N agents, evolving in 3-D and under the influence of an unknown constant current. To discriminate between different agents, a distinct integer label between 1 and N is assigned to each one. It is assumed that each agent has access to either:

- An absolute position measurement, provided e.g. by a Long Baseline (LBL) positioning system [11] or a GPS; or
- Range measurements to one or more neighboring agents, as well as position estimates communicated by those agents.

Additionally, it is assumed that the sensor suite mounted onboard each agent has a DVL, which provides the velocity of the agent relative to the fluid, a sensor which provides accurate measurements of its depth/altitude (which is readily available in most situations: depth cells for underwater vehicles, pressure sensors for aerial vehicles), and an AHRS to measure its attitude.

The first of two problems considered here is the design of local state estimators for each agent to accurately estimate its position and velocity, relying only on locally available measurements and limited communication with neighboring agents. During this design process, it will be assumed that the position estimates received through communication are correct. This process will result in a decentralized state estimator, in which each agent only concerns itself with a fraction of the total information available in the formation to estimate its state. The second problem to address is that of establishing conditions that guarantee globally exponential stability of the decentralized estimator, as the position estimates each agent receives through communication are in fact not exact, but provided by each local estimator.

A. Motion Kinematics and Measurements

Let $\{I\}$ denote an inertial reference coordinate frame and $\{B_i\}$ a coordinate frame attached to agent i , which is usually denominated as the body-fixed coordinated frame. The linear motion of agent i follows $\dot{\mathbf{p}}_i(t) = \mathbf{R}_i(t)\mathbf{v}_i(t)$, where $\mathbf{p}_i(t) \in \mathbb{R}^3$ is its inertial position, $\mathbf{v}_i(t) \in \mathbb{R}^3$ its velocity relative to $\{I\}$, expressed in body-fixed coordinates, and $\mathbf{R}_i(t) \in SO(3)$ is the rotation matrix from $\{B_i\}$ to $\{I\}$. For the agents that have access to absolute position measurements, the design of a navigation system is straightforward and it is not elaborated here (for further details, see [11][12][13]).

On the other hand, for the agents that only have access to range measurements, the conditions for observability, as well as the design of stable state observers, are considerably more complex, and are henceforth the main focus of this section and the next. The DVL provides a measurement of

the velocity relative to the fluid in body-fixed coordinates, so it makes sense to divide $\mathbf{v}_i(t)$ into $\mathbf{v}_{ri}(t) \in \mathbb{R}^3$ (velocity of the agent relative to the fluid, in body-fixed coordinates) and $\mathbf{v}_f(t) \in \mathbb{R}^3$ (velocity of the fluid relative to $\{I\}$, in inertial coordinates), yielding

$$\dot{\mathbf{p}}_i(t) = \mathbf{R}_i(t)\mathbf{v}_{ri}(t) + \mathbf{v}_f(t).$$

A range measurement made by agent i to agent j will be denoted as $r_{ij}(t) = \|\mathbf{p}_j(t) - \mathbf{p}_i(t)\| \in \mathbb{R}$, and the depth measurement of agent i is $z_i(t) = p_i^3(t) \in \mathbb{R}$.

B. Agent Dynamics

To proceed from the "point of view" of some agent i , define the following nonlinear system by joining the linear motion kinematics and the available measurements:

$$\begin{cases} \dot{\mathbf{p}}_i(t) = \mathbf{R}_i(t)\mathbf{v}_{ri}(t) + \mathbf{v}_f(t) \\ \dot{\mathbf{v}}_f(t) = \mathbf{0} \\ z_i(t) = p_i^3(t) \\ r_{ij}(t) = \|\mathbf{p}_j(t) - \mathbf{p}_i(t)\|, \quad j \in D_i \end{cases},$$

where D_i is the set of other agents to whom agent i measures its range. Define

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \end{bmatrix} = \begin{bmatrix} \mathbf{p}_i(t) \\ \mathbf{v}_f(t) \end{bmatrix} \in \mathbb{R}^6,$$

and let $\mathbf{u}(t) = \mathbf{R}_i(t)\mathbf{v}_{ri}(t) \in \mathbb{R}^3$. Suppose that agent i has access to measurements of its range to M other agents in its vicinity and, for the sake of simplicity, take indexes from 1 to M to identify each of those agents. Additionally, suppose that they transmit both $\mathbf{u}_j(t)$ and an estimate of $\mathbf{p}_j(t)$ to agent i . Then, the system takes the form

$$\begin{cases} \dot{\mathbf{x}}_1(t) = \mathbf{x}_2(t) + \mathbf{u}(t) \\ \dot{\mathbf{x}}_2(t) = \mathbf{0} \\ z(t) = x_1^3(t) \\ r_j(t) = \|\mathbf{p}_j(t) - \mathbf{x}_1(t)\|, \quad j = 1, 2, \dots, M \end{cases}. \quad (1)$$

The first problem addressed in the paper is the analysis of the observability of the nonlinear system (1). As the observability analysis is constructive, the estimator design follows naturally. Two cases are considered: in the first, the agent has access to only one range measurement. In the other, it has access to $M \geq 3$ range measurements. For the second case, the motion of the corresponding agents verifies the following assumption:

Assumption 1: The projections on the "horizontal" plane (any plane orthogonal to the depth/altitude axis) of the position of at least 3 of those agents are non-collinear for all $t \in [t_0, t_f]$, $t_0 < t_f$.

Remark 1: In (1), $\mathbf{x}(t)$ is used to denote the state of agent i instead of $\mathbf{x}_i(t)$ or some other form of indexation, as it simplifies the notation in the rest of the paper considerably.

III. OBSERVABILITY ANALYSIS

This section details the observability analysis of the nonlinear system (1). The method applied here consists in obtaining, through state augmentation, a LTV system that mimics exactly the dynamics of (1), allowing to carry out the observability analysis in a linear system framework. Throughout this section, the estimates of $\mathbf{p}_j(t)$, $j = 1, 2, \dots, M$ received through communication, which are used in building the dynamics of the problem, are assumed to be exact.

A. State augmentation

Define an additional state variable, $\mathbf{x}_3 \in \mathbb{R}^M$, with each component equal to one of the range measurements, and let $\Delta \mathbf{u}_j(t) := \mathbf{u}_j(t) - \mathbf{u}(t) \in \mathbb{R}^3$. Then, it is straightforward to show that the dynamics of the augmented system can be described by the following LTV system:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t) \begin{bmatrix} \mathbf{u}(t) \\ \mathbf{p}_1(t) \\ \vdots \\ \mathbf{p}_M(t) \end{bmatrix}, \\ \mathbf{y}(t) = \mathbf{C}_A \mathbf{x}(t) \end{cases}, \quad (2)$$

where

$$\mathbf{A}(t) = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ -\frac{\Delta \mathbf{u}_1^T(t)}{r_1(t)} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{\Delta \mathbf{u}_M^T(t)}{r_M(t)} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \end{bmatrix} \in \mathbb{R}^{(6+M) \times (6+M)},$$

$$\mathbf{B}(t) = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \frac{\Delta \mathbf{u}_1^T(t)}{r_1(t)} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \frac{\Delta \mathbf{u}_M^T(t)}{r_M(t)} \end{bmatrix} \in \mathbb{R}^{(6+M) \times 3(1+M)},$$

and $\mathbf{C}_A = \begin{bmatrix} [0 \ 0 \ 1] & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \in \mathbb{R}^{(1+M) \times (6+M)}$.

Regarding specifically the second case, note that

$$r_j(t) - r_k(t) + \frac{2[\mathbf{p}_j(t) - \mathbf{p}_k(t)] \cdot \mathbf{x}_1(t)}{r_j(t) + r_k(t)} = \frac{\|\mathbf{p}_j(t)\|^2 - \|\mathbf{p}_k(t)\|^2}{r_j(t) + r_k(t)},$$

for all $j, k = 1, 2, \dots, M$, $j \neq k$. The right side is assumed to be known, so the left side can be included as an additional output. Doing all combinations provides $(M-1)M/2$ new outputs to the system, leading to the following LTV system:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t) \begin{bmatrix} \mathbf{u}(t) \\ \mathbf{p}_1(t) \\ \vdots \\ \mathbf{p}_M(t) \end{bmatrix}, \\ \mathbf{y}(t) = \mathbf{C}_B(t)\mathbf{x}(t) \end{cases}, \quad (3)$$

where

$$\mathbf{C}_B(t) = \begin{bmatrix} [0 \ 0 \ 1] & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{C}_1(t) & \mathbf{0} & \mathbf{C}_2 \end{bmatrix} \in \mathbb{R}^{[1+(M+1)/2] \times (6+M)},$$

in which

$$\mathbf{C}_1(t) = \begin{bmatrix} \frac{2(\mathbf{p}_1(t) - \mathbf{p}_2(t))^T}{r_1(t) + r_2(t)} \\ \frac{2(\mathbf{p}_1(t) - \mathbf{p}_3(t))^T}{r_1(t) + r_3(t)} \\ \vdots \\ \frac{2(\mathbf{p}_{M-1}(t) - \mathbf{p}_M(t))^T}{y_{M-1}(t) + y_M(t)} \end{bmatrix} \in \mathbb{R}^{[(M-1)M/2] \times 3}$$

and

$$\mathbf{C}_2 = \begin{bmatrix} 1 & -1 & 0 & 0 & \dots & \dots & \dots & 0 \\ 1 & 0 & -1 & 0 & \dots & \dots & \dots & 0 \\ \vdots & & & & & & & \\ 0 & \dots & \dots & 0 & 0 & 1 & -1 \end{bmatrix} \in \mathbb{R}^{[(M-1)M/2] \times M}.$$

Remark 2: Note that, even though $\mathbf{A}(t)$, $\mathbf{B}(t)$, and $\mathbf{C}_B(t)$ depend on the system's input and output, the systems (2) and (3) can still be regarded as LTV systems for analysis, as all involved quantities are known, see [14, Lemma 1].

B. Observability of the LTV system

The following result provides a sufficient condition for the observability of the LTV system (2), when there is only one range measurement:

Theorem 1: Suppose that the functions in the set

$$\mathcal{F} = \{\Delta u_1^1(t), \Delta u_1^2(t), (t - t_0)\Delta u_1^1(t), (t - t_0)\Delta u_1^2(t)\} \quad (4)$$

are linearly independent in $[t_0, t_f]$. Then, the LTV system with one range measurement (2) is observable on $[t_0, t_f]$.

The following result provides a sufficient condition for the observability of the LTV system (3) when there are $M \geq 3$ range measurements.

Theorem 2: If Assumption 1 holds, then the LTV system (3) is observable on $[t_0, t_f]$, $t_0 < t_f$.

C. Observability of the nonlinear system

The following result presents a sufficient condition for the observability of the nonlinear system (1) when there is only one range measurement available, as well as a method for designing state observers for that system with globally asymptotically stable (GAS) error dynamics.

Theorem 3: If the set of functions (4) is linearly independent, then the nonlinear system (1) with one range measurement is observable on $[t_0, t_f]$, $t_0 < t_f$, in the sense that, given the outputs $z(t)$ and $r_1(t)$ and the input $\mathbf{u}(t)$, the initial state $\mathbf{x}(t_0)$ is uniquely defined. Moreover, a state observer for the LTV system (2) with GAS error dynamics is also a state observer for the nonlinear system (1), with GAS error dynamics.

The following result presents a sufficient condition for the observability of the nonlinear system (1) when there are $M \geq 3$ range measurements, as well as a method for designing state observers for that system with globally asymptotically stable (GAS) error dynamics.

Theorem 4: If Assumption 1 holds, the nonlinear system (1) with $M \geq 3$ range measurements is observable on $[t_0, t_f]$, $t_0 < t_f$, in the sense that, given the outputs $z(t)$ and $r_j(t)$, $j = 1, 2, \dots, M$, and the input $\mathbf{u}(t)$, the initial state $\mathbf{x}(t_0)$ is uniquely defined. Moreover, a state observer for the LTV system (3) with GAS error dynamics is also a state observer for the nonlinear system (1), with GAS error dynamics.

Remark 3: For the case where the agent has access to two range measurements, if the sufficient condition for observability of Theorems 1 and 3 holds for at least one of the two agents corresponding to the range measurements, then the nonlinear system (1) with two range measurements

is observable. Future work will focus on deriving a less restrictive sufficient condition for observability for that case.

Remark 4: The proofs of the results presented in this section had to be left out due to space constraints. For reference, Theorems 1 and 3 can be proved following a method similar to the one used in [14], while Theorems 2 and 4 can be proved applying the same reasoning that was employed in proving the results in [15].

IV. DECENTRALIZED STATE ESTIMATION

The previous section established two distinct sufficient conditions for the observability of the nonlinear system (1) depending on the number of range measurements available to the agent, assuming that the position estimates received through communication are exact. Following this, it is straightforward to design local state observers for each agent that will feature GES error dynamics when the received state estimates are exact [16]. If the agent has only one range measurement, its local state observer will be designed based on the LTV system (2), and its dynamics follow

$$\dot{\hat{x}}(t) = \mathbf{A}(t)\hat{x}(t) + \mathbf{B}(t) \begin{bmatrix} \mathbf{u}(t) \\ \mathbf{p}_1(t) \end{bmatrix} + \mathbf{K}(t)[\mathbf{y}(t) - \mathbf{C}_A\hat{x}(t)], \quad (5)$$

where $\hat{x}(t)$ is the state estimate, and $\mathbf{K}(t)$ is a time-varying matrix of output injection gains. On the other hand, if the agent has range measurements to $M \geq 3$ other agents, its local state observer will be based on (3), and its dynamics are described by

$$\dot{\hat{x}}(t) = \mathbf{A}(t)\hat{x}(t) + \mathbf{B}(t) \begin{bmatrix} \mathbf{u}(t) \\ \mathbf{p}_1(t) \\ \vdots \\ \mathbf{p}_M(t) \end{bmatrix} + \mathbf{K}(t)[\mathbf{y}(t) - \mathbf{C}_B(t)\hat{x}(t)]. \quad (6)$$

However, when taking the formation as a whole, the resulting decentralized estimator is not guaranteed to be stable, as the state estimates the agents receive through communication are not exact, since they are generated by the state observers of other agents. This section presents a method to design such a decentralized estimator that guarantees global exponential stability for the overall decentralized estimator error dynamics.

Before presenting the main result of this section, it is convenient to introduce some concepts on graph theory [17][18], as formations of agents can be handily described by a directed graph. A directed graph, or digraph, $\mathcal{G} := (\mathcal{V}, \mathcal{E})$ is composed by a set \mathcal{V} of vertices and a set of directed edges \mathcal{E} , which are represented by ordered pairs of vertices. Such an edge can be expressed as $e = (a, b)$, meaning that edge e is incident on vertices a and b , directed towards b . A directed cycle in \mathcal{G} is a sequence $(v_0, e_1, v_1, e_2, v_2, \dots, e_n, v_0)$ of distinct vertices (with the exception of the first and the last) and edges of \mathcal{G} such that $e_i = (v_{i-1}, v_i)$. A directed graph is called acyclic if it contains no directed cycles. If a directed graph \mathcal{G} is acyclic, it can be represented graphically by a tiered drawing such as the one depicted in Fig. 1, that is, the drawing is divided in K hierarchical tiers, in which tier 0 is composed of the vertices with no edges directed towards them while, for a vertex in tier $k > 0$, all directed paths ending in that vertex start in a node of a lower tier.

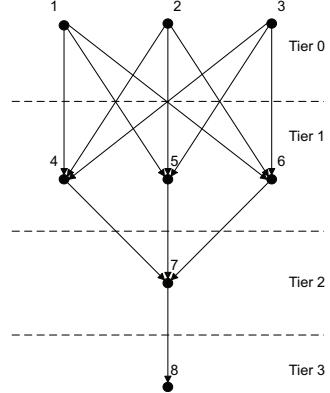


Fig. 1. Example of an acyclic directed graph divided in tiers

Now, consider the agent formation described in Section II. This kind of formation can be associated with a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where each vertex represents a distinct agent, and an edge (a, b) means that agent b measures its range to agent a . The nodes with no edges directed towards them refer to agents with access to measurements of their own absolute position.

In order to derive the main result of this section, the following assumption is required:

Assumption 2: The quantities $\mathbf{x}_1(t)$, $\mathbf{x}_2(t)$, and $\mathbf{u}(t)$ associated with each agent in the formation are bounded. Furthermore, the range measurements obey $r_i(t) > \epsilon$, $\forall t$, where ϵ is a positive scalar constant.

Note that, in practical terms, this assumption is very mild, as the state $\mathbf{x}(t)$ will be bounded by the specifications of the mission scenario, and $\mathbf{u}(t)$ is limited by the maximum velocity of the agents. Besides that, violation of the constraint on the ranges would translate into agents colliding with each other. Furthermore, the particular values of the bounds are not required for observer or filter design purposes.

The following result establishes a sufficient condition for global exponential stability of the estimation error for the distributed state estimator.

Theorem 5: Consider a formation composed by N agents, as described in Section II, whose linear motion verifies Assumption 2, and assume that:

- 1) The directed graph associated with the formation is acyclic;
- 2) The agents in tier 0 implement GES state observers to estimate their position and velocity from the absolute position measurements available to them [12], [13];
- 3) The agents in the other tiers implement state observers for the LTV system (2) or (3), depending on the number of range measurements available to them, that feature GES error dynamics when the position estimates received through communication are exact;
- 4) The sufficient condition for observability of the LTV system associated with each agent in the formation holds (Theorem 1 for the system (2) and Theorem 2 for the system (3));

Then, the estimation error of the decentralized state estimator composed by the local state observers of all the agents in the formation converges globally exponentially to zero.

Proof: Due to space constraints, it is only possible to present a sketch of the full proof here.

Since the directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ associated with the formation is acyclic, consider its drawing with K tiers, and notice that since the local state observer of an agent in a given tier only depends on estimates received from lower tiers, its properties can be studied disregarding the dynamics of agents in higher tiers as well as those of other agents in the same tier.

Consider an agent i in tier $k > 0$, and assume that the local observers of each agent in tiers 0 to $k - 1$ have GES error dynamics. Then, it is possible to show that the estimation error dynamics of agent i are input-to-state stable (ISS) with the error of the received state estimates as inputs, by application of Lemma 4.6 from [19]. Special care must be taken when agent i has access to $M \geq 3$ range measurements: as $\mathbf{C}_B(t)$ is built using received state estimates, the errors on those state estimates will have impact on the computation of $\mathbf{C}_B(t)$ and $\mathbf{K}(t)$ by the local state observer, which can be dealt with by applying Lemma 9.1 from [19]. Since it is assumed that the errors of the received state estimates converge exponentially fast to zero, it then follows that the estimation error dynamics of agent i are GES, see [20, Theorem 5].

So, as the dynamics of agents in the same tier have no mutual impact, this means that, since the local observers of each agent in the tiers 0 through $k - 1$ have GES error dynamics, the local observers of the agents in tier k will also feature GES error dynamics. Considering this, plus the fact that the local observers of the agents in tier 0 feature GES error dynamics, it follows by induction that all local state observers in the formation feature error dynamics that converge globally exponentially to the origin, that is, the decentralized estimator that results when considering all the local estimators as a whole has GES error dynamics. ■

Remark 5: This method can also be used to design a stable state observer when the formation graph \mathcal{G} is cyclic, by removing edges from the graph until it is no longer cyclic, while making sure to never remove the last edge directed towards a vertex. It seems naturally advantageous to remove as few edges as possible, therefore this procedure could be restated as that of finding the maximum acyclic subgraph of \mathcal{G} [21], with the added restriction that the last edge directed towards a vertex may not be removed. This straightforward approach is then applied to the local observers by disregarding the range measurements referring to edges which were removed during this process.

V. SIMULATION RESULTS

This section presents simulation results to assess the performance of the proposed decentralized estimation solution in the presence of measurement noise.

In the simulations detailed here, a formation composed by 8 agents is considered. The graph associated with this formation is the one depicted in Fig. 1. The agents are assumed to be evolving in a fluid with constant inertial velocity $\mathbf{v}_f = [0.4 \ -0.2 \ 0]^T$ (m/s), and their initial positions and the formation trajectory are depicted in Fig. 2. To guarantee observability, agent 8, which only has access to one range measurement, performs oscillations around

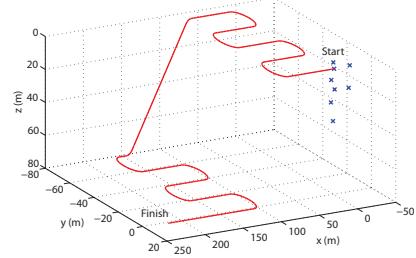


Fig. 2. Initial positions of the agents and trajectory followed by the formation

the nominal trajectory of Fig. 2. To simulate noise in the measurements, a zero-mean, uncorrelated and normally distributed perturbation was added to the range, depth, and velocity measurements, with standard deviation of 0.2 m for the range and depth measurements, and 0.01 m/s for the velocity measurements. For the absolute position measurements available to agents 1 through 3, as the noise in absolute positioning systems such as the GPS is usually area-dependent, they were corrupted by additive, zero-mean white Gaussian noise with standard deviation of 0.1 m , and some correlation between the measurements was added, resulting in the following covariance matrix:

$$\mathbf{R}_{abs} = 0.01 \times \begin{bmatrix} 1 & 0.1 & 0.1 \\ 0.1 & 1 & 0.1 \\ 0.1 & 0.1 & 1 \end{bmatrix} \otimes \mathbf{I}_3.$$

In addition to these perturbations, noise was also simulated in the attitude measurements required for computing the inputs of each system. To this effect, a zero-mean, uncorrelated white Gaussian perturbation was added to the roll, pitch, and yaw Euler angles used to parametrize attitude, with standard deviation of 0.03° for the roll and pitch, and 0.3° for the yaw.

Kalman filtering theory [22] was applied to design a local state estimator for each agent, and as such Theorem 5 guarantees GES error dynamics for the resulting decentralized state estimator. For agents 1 through 3, which have access to absolute position measurements, linear time-invariant Kalman filters were implemented, and tuned with covariance matrices $\mathbf{\Xi}_0$ and $\mathbf{\Theta}_0$ for the process and observation noise, respectively, defined as

$$\begin{cases} \mathbf{\Xi}_0 = 0.01 \times \text{diag}(1, 1, 1, 0.001, 0.001, 0.001) \\ \mathbf{\Theta}_0 = \mathbf{I} \end{cases} .$$

As agents 4 through 7 each have access to 3 range measurements, a Kalman filter was implemented for each one based on the LTV system (3), and tuned with the covariance matrices

$$\begin{cases} \mathbf{\Xi}_1 = 0.01 \times \text{diag}(1, 1, 1, 0.001, 0.001, 0.001, 1, 1, 1) \\ \mathbf{\Theta}_1 = \text{diag}(1, 1, 1, 1, 2, 2, 2) \end{cases} .$$

Finally, as agent 8 only has access to one range measurement, the Kalman filter was based on the LTV system (2), with associated covariance matrices

$$\begin{cases} \mathbf{\Xi}_2 = 0.01 \times \text{diag}(1, 1, 1, 0.001, 0.001, 0.001, 1) \\ \mathbf{\Theta}_2 = \text{diag}(1, 1, 1, 1) \end{cases} .$$

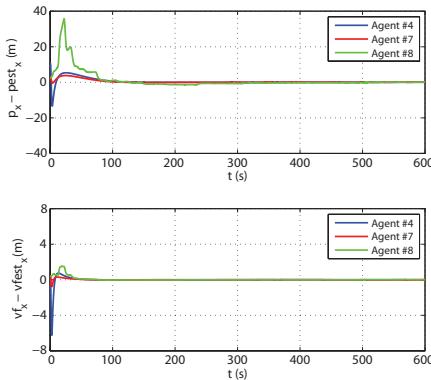


Fig. 3. Initial evolution of the estimation error of selected state variables of agents 4, 7, and 8

The initial values for all state estimates in each filter were set to zero.

TABLE I
STANDARD DEVIATION OF STEADY-STATE ESTIMATION ERROR

Variable	σ of agent 4	σ of agent 7	σ of agent 8
$p^x(t)$ [m]	6.01×10^{-3}	8.73×10^{-3}	5.92×10^{-2}
$p^y(t)$ [m]	1.19×10^{-2}	1.25×10^{-2}	4.81×10^{-2}
$p^z(t)$ [m]	5.78×10^{-3}	5.79×10^{-3}	5.78×10^{-3}
$v_f^x(t)$ [m/s]	1.35×10^{-4}	1.80×10^{-4}	2.94×10^{-4}
$v_f^y(t)$ [m/s]	1.76×10^{-4}	2.12×10^{-4}	3.07×10^{-4}
$v_f^z(t)$ [m/s]	1.62×10^{-4}	1.62×10^{-4}	1.61×10^{-4}
$x_3^1(t)$ [m]	5.20×10^{-3}	5.28×10^{-3}	6.07×10^{-2}
$x_3^2(t)$ [m]	5.03×10^{-3}	5.26×10^{-3}	—
$x_3^3(t)$ [m]	5.07×10^{-3}	5.27×10^{-3}	—

The results of the simulations are depicted in Fig. 3 and Table I. Figure 3 depicts the initial evolution of the first coordinate of the estimation error of $p(t)$ and $v_f(t)$ for agents 4, 7, and 8. As it can be seen, after the initial transients caused by mismatches in initial conditions, the estimation error variables converge to the vicinity of zero (they do not converge to zero only due to the presence of noise in the measurements). To complement the graphical data, Table I details the measured standard deviation of the steady-state estimation error variables at agents 4, 7, and 8, averaged over 500 different runs of the simulation. The data shows that even in the presence of measurement noise with realistic intensity, the estimation error remains confined to tight intervals around zero. A clear degradation in performance can be observed at agent 8 when compared to the others, which is to be expected given that agent 8 is not only at the “tail” of the formation, but also has access to only one range measurement.

VI. CONCLUSIONS

The problem of decentralized state estimation in formations of autonomous vehicles with fixed topologies was addressed in this paper. In the envisioned mission scenario, only a reduced number of agents in the formation have access to absolute position measurements, while the rest must rely on range measurements to neighboring agents and local sensing and communication capabilities to estimate

their own position and velocity. A method for designing local state estimators for each of the agents was presented such that the resulting decentralized state estimator features GES error dynamics when the graph associated with the formation is acyclic. Realistic simulation results were presented and discussed to assess the performance of the proposed solution under the influence of measurement noise.

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