

# Fault Detection and Isolation for Inertial Measurement Units

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**Abstract**—This paper addresses the problem of Fault Detection and Isolation (FDI) for navigation systems equipped with sensors providing inertial measurements and vector observations. Two strategies are proposed. The first one takes advantage of existing hardware redundancy, providing sufficient conditions for the isolation of faults. The second approach exploits the analytical redundancy between the angular velocity measurements and the vector observations, by resorting to Set-Valued Observers (SVOs). The behavior of both strategies are illustrated in simulation.

## I. INTRODUCTION

The navigation system is a critical component in any aircraft or spacecraft. It provides key information – the attitude and position of the system – and, in case of failure, there is serious risk of damage or even human losses. In high reliability systems it is not only necessary to detect faults, but also to isolate the defective sensor. This has motivated a considerable amount of research to devise Fault Detection and Isolation (FDI) schemes for navigation systems – see for instance [19], [28] and references therein.

The field of FDI has been studied since the early 70's [29], and several techniques have, since then, been applied to different systems. For a survey of FDI methods in the literature, see, for instance, [13]. An FDI system must be able to bear with different types of faults in sensors and/or actuators, which can occur abruptly or slowly in time. Moreover, model uncertainty (such as unmodeled dynamics) and disturbances must never be interpreted as faults.

An active deterministic model-based Fault Detection (FD) system (see [9] for a description of the typical FD classes available in the literature) is usually composed of two parts: a filter that generates residuals that should be *large* under faulty environments; and a decision threshold, which is used to decide whether a fault is present or not – see [1], [2], [8], [9], [11], [18], [29] and references therein. The isolation of the fault can, in some cases, be done by using a similar approach, i.e., by designing filters for families of faults, and identifying the most likely fault as the one associated to the filter with the smallest residuals.

The FDI schemes for navigation systems available in the literature exploit two types of redundancy, namely, the hardware redundancy and the analytical or dynamic redundancy. The former takes advantage of the existing redundant

measurements to detect incoherences among them. In [12], an FDI solution is proposed that is based on algebraic invariants. Parity-based methods are proposed in [15], [27]. The work in [25] proposes a geometric method based on the singular value decomposition of the measurement matrix. A comparison between several FDI techniques using hardware redundancy is presented in [28]. The analytical redundancy emerges from the dynamic relationship between the sensor data. In [5], distributed Kalman filters are used. A solution based on parameter estimation, where the residuals are generated by least-squares estimation techniques, is presented in [14]. The work in [6] proposes two statistical schemes based on nonlinear autoregressive moving average. In [7], a left eigenvector assignment approach is developed to an aircraft accelerometer FD filter. A survey on FDI methods exploiting analytical redundancy can be found in [19].

The main contribution of the work presented in this paper is the development of FDI schemes for Inertial Measurement Units (IMUs) and vector observations, where the sensor measurements are assumed to be corrupted by bounded noise. Such bounds are suitable, for instance, in robust control designs, where worst-case guarantees are provided regarding the performance of the closed-loop system. We propose two schemes to exploit the different types of redundancy:

- hardware redundancy – resorting to intersection of sets;
- analytical redundancy – using SVOs to model the dynamic relation between the sensor measurements.

For further details on SVOs and SVO-based FDI, the interested reader is referred to [17], [21], [22], [24] and references therein. The proposed solutions guarantee that there will be no false alarms. Moreover, the computation of a decision rule, based on a threshold to be tuned, used to declare whether or not a fault has occurred in residual-based FDI approaches, is not needed.

The remainder of this article is organized as follows. In Section II, the problem of FDI in IMU measurements and vector observations is introduced. In Section III, a method for FDI that exploits sensor redundancy is proposed. The derivation of an FDI filter that takes advantage of the dynamic relation between the sensor measurements using the SVOs is presented in Section IV. In Section V, simulation results illustrating the performance of the proposed strategies are presented. Finally, some concluding remarks are discussed in Section VI.

## NOMENCLATURE

To enhance the readability of this paper, we introduce the following notation. The skew-symmetric operator in  $\mathbb{R}^3$  is denoted by  $[\cdot]_{\times}$  and satisfies  $[\mathbf{v}]_{\times} \mathbf{w} = \mathbf{v} \times \mathbf{w}$ ,  $\mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ . The real exponential function and exponential map  $a$  of matrix is denoted by  $\exp(\cdot)$ . The Kronecker product of matrices is denoted by  $\mathbf{A} \otimes \mathbf{B}$  (for further details see [31, p. 25]).

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This work was supported in part by Fundação para a Ciência e a Tecnologia (FCT) projects [PEst-OE/EEI/LA0009/2011], PTDC/EEA-CRO/102857/2008 SCARVE and PTDC/MAR/64546/2006 OBSERFLY, and by the EU project TRIDENT of the EC-FP7. The work of S. Brás was supported by the PhD grant SFRH/BD/47456/2008 from FCT.

The  $3 \times 3$  matrix whose elements are zeros except the element  $ij$  is denoted by  $\mathbf{E}_{i,j}$ . The maximum vector and matrix norms is denoted by  $\|\cdot\|_{\max}$  and is defined as the maximum of the absolute value of all vector and matrix elements, respectively, i.e.,  $\|\mathbf{x}\|_{\max} := \max\{|x_1|, \dots, |x_N|\}$  and  $\|\mathbf{A}\|_{\max} := \max\{|[\mathbf{A}]_{ij}|\}$ , where  $[\mathbf{A}]_{ij}$  denotes the element of column  $j$  and row  $i$  of matrix  $\mathbf{A}$ . Let a polytope be defined as  $\text{Set}(\mathbf{A}, \mathbf{b}) = \{\mathbf{x} \in \mathbb{R}^{n_x} : \mathbf{A}\mathbf{x} \leq \mathbf{b}\}$ .

## II. PROBLEM FORMULATION

In this paper, we assume that a craft is equipped with a strapdown navigation system comprising an IMU fixed in the body reference frame  $\{B\}$ . Vector observations that are constant in the inertial reference frame  $\{I\}$ , are also available to mitigate the errors associated with dead reckoning. Without loss of generality, throughout the remainder of this paper, assume that  $\{I\}$  shares the origin with  $\{B\}$ . We denote the angular velocity of  $\{B\}$  with respect to  $\{I\}$  and expressed in  $\{B\}$  as  $\boldsymbol{\omega} \in \mathbb{R}^3$ , and the specific force, which is the time-rate-of-change of the velocity of  $\{B\}$ , with respect to  $\{I\}$ , relative to a local gravitational space and expressed in  $\{B\}$ , as  $\mathbf{a}_{\text{SF}} \in \mathbb{R}^3$ . It is given by

$$\mathbf{a}_{\text{SF}} = {}^B\mathbf{a} - {}^B\mathbf{g}, \quad (1)$$

where  ${}^B\mathbf{a} \in \mathbb{R}^3$  corresponds to the linear acceleration term and  ${}^B\mathbf{g}$  corresponds to the gravity, both expressed in the body-fixed coordinates  $\{B\}$ . We will see later that, if the linear acceleration is negligible, the specific force can be regarded as a vector observation.

The IMU is composed of a set of rate gyros and a set of accelerometers. The ideal  $i$ -th rate gyro measures the projection of  $\boldsymbol{\omega}$  into its measurement axis,  $\mathbf{h}_{\Omega}^{(i)} \in \mathbb{R}^3$ , which is constant in  $\{B\}$ ,  $i = 1, \dots, N_{\Omega}$ ,

$$\Omega_r^{(i)} = \mathbf{h}_{\Omega}^{(i)T} \boldsymbol{\omega}. \quad (2)$$

However, the actual rate gyro measurements,  $\Omega_r^{(i)}$ , are corrupted by bias and noise, which are assumed to be bounded, i.e.,

$$\Omega_r^{(i)} = \Omega^{(i)} + b^{(i)} + \Delta_b^{(i)} + n_{\omega}^{(i)}, \quad (3)$$

where  $b^{(i)} \in \mathbb{R}$  and  $n_{\omega}^{(i)} \in \mathbb{R}$  denote the measurement bias and noise, respectively, and  $|\Delta_b^{(i)}| \leq \bar{\Delta}_b^{(i)}$ . The measurement noise is assumed to be bounded by a positive constant,  $|n_{\omega}^{(i)}| \leq \bar{n}_{\omega}^{(i)}$ . The bound  $\bar{\Delta}_b^{(i)}$  can be seen as a bias tolerance, which reflects the confidence that one has on  $b^{(i)}$  remaining constant. Let the measurement axis of the  $i$ -th accelerometer be given by  $\mathbf{h}_{\alpha}^{(i)} \in \mathbb{R}^3$ ,  $i = 1, \dots, N_{\alpha}$ , which is constant when expressed in  $\{B\}$ . This sensor ideally measures the specific force on its measurement axis  $\alpha^{(i)} = \mathbf{h}_{\alpha}^{(i)T} \mathbf{a}_{\text{SF}}$ . However, the sensed data are corrupted by bounded sensor noise  $\alpha_r^{(i)} = \alpha^{(i)} + n_{\alpha}^{(i)}$ , where  $|n_{\alpha}^{(i)}| \leq \bar{n}_{\alpha}^{(i)}$  denotes the measurement noise.

The following assumption guarantees that one can recover  $\boldsymbol{\omega}$  and  $\mathbf{a}_{\text{SF}}$  from  $\boldsymbol{\Omega} = [\Omega^{(1)} \dots \Omega^{(N_{\Omega})}]^T$  and  $\boldsymbol{\alpha} = [\alpha^{(1)} \dots \alpha^{(N_{\alpha})}]^T$ , respectively.

*Assumption 1:* Assume that the measurement axis of the rate gyros and accelerometers form a basis for  $\mathbb{R}^3$ , i.e.,

$$\text{span}\{h_{\omega}^{(1)}, \dots, h_{\omega}^{(N_{\Omega})}\} = \text{span}\{h_{\alpha}^{(1)}, \dots, h_{\alpha}^{(N_{\alpha})}\} = \mathbb{R}^3.$$

Note that, under this constraint, the following expression can be used to compute the angular velocity  $\boldsymbol{\omega} =$

$(\mathbf{H}_{\Omega}^T \mathbf{H}_{\Omega})^{-1} \mathbf{H}_{\Omega}^T \boldsymbol{\Omega}$ , where  $\mathbf{H}_{\Omega} = [h_{\Omega}^{(1)} \dots h_{\Omega}^{(N_{\Omega})}]^T$ , and  $\mathbf{a}_{\text{SF}} = (\mathbf{H}_{\alpha}^T \mathbf{H}_{\alpha})^{-1} \mathbf{H}_{\alpha}^T \boldsymbol{\alpha}$ , where  $\mathbf{H}_{\alpha} = [h_{\alpha}^{(1)} \dots h_{\alpha}^{(N_{\alpha})}]^T$ , can be used to compute the specific force. The matrices  $\mathbf{H}_{\Omega}$  and  $\mathbf{H}_{\alpha}$  are called measurement matrices of the angular velocity and angular acceleration, respectively.

To obtain estimates of position and attitude from the inertial data, it is necessary to integrate the measurements. This process introduces cumulative errors in the estimates. To correct them, it is typical to add sensors such as magnetometers, star trackers, and Sun sensors [10]. These sensors measure a vector expressed in  $\{B\}$ , which, for most practical purposes, can be considered constant in the inertial coordinates,  $\{I\}$ . These vectors satisfy the kinematic equation

$$\dot{\mathbf{v}} = -[\boldsymbol{\omega}]_{\times} \mathbf{v}, \quad (4)$$

where  $\mathbf{v} \in \mathbb{R}^3$  denotes a generic vector observation expressed in  $\{B\}$ . We assume that sensors provide uncertain data of  $N_{\mathbf{v}}$  vector observations in the form

$$\nu_{ri} = \mathbf{H}_{\nu i} \mathbf{v}_i + \mathbf{n}_{\nu i}, \quad i = 1, \dots, N_{\mathbf{v}}, \quad (5)$$

where  $\mathbf{H}_{\nu i}$  is the measurement matrix for the vector  $\mathbf{v}_i$ , and  $\mathbf{n}_{\nu i}$  is the measurement noise vector. Each component of this vector, denoted by  $\mathbf{n}_{\nu i}^{(j)}$ , satisfies

$$|\mathbf{n}_{\nu i}^{(j)}| \leq \bar{n}_{\nu i}^{(j)}, \quad (6)$$

where  $\bar{n}_{\nu i}^{(j)} \in \mathbb{R}^+$ ,  $i = 1, \dots, N_{\mathbf{v}}$  identifies the different vector observations and  $j \in \mathbb{N}$  identifies the  $j$ -th component.

In many practical applications, the external accelerations can be neglected when compared with the gravity. Under this assumption, the dynamics of (1) can be rewritten as

$$\dot{\mathbf{a}}_{\text{SF}} = [\boldsymbol{\omega}]_{\times} {}^B\mathbf{g}.$$

Apart from the sign, the specific force has a similar behaviour to a vector observation as described in (4), and, hence, the faults in the accelerometers can be treated as the faults in vector observations.

A comprehensive study on the faults affecting mechanical rate gyros is present in [30]. In this work, we follow the characterization of faults described in [16], separating them into *hard* and *soft* faults. The hard faults include step-type failures, such as zero output and *stuck at* faults. Changes in noise level and bias variation are typical examples of soft faults.

In this paper, we propose a novel technique based on Set-Valued Observers (SVOs) to detect and isolate faults in sensors of navigation systems, namely, rate gyros, accelerometers, and sensors providing vector observations, such as magnetometers, Sun sensors, and star trackers. As described in the sequel, this is done by building upon recent results that extend the applicability of SVOs for FDI [3], [22].

## III. FDI USING HARDWARE REDUNDANCY

In this section, we describe a technique to detect faults on sensor measurements using hardware redundancy. We illustrate the method using rate gyros measurements, although it is equally fitted to exploit redundancy in other sensors, such as accelerometers and magnetometers.

The optimal sensor configuration depends on how many sensors are available. In [26], the optimal configuration using different number of redundant sensors is studied assuming

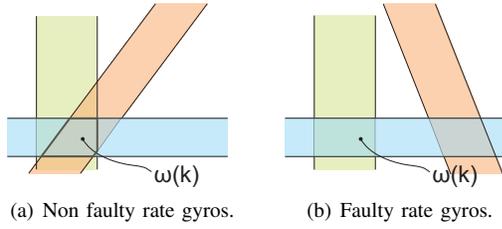


Fig. 1. Illustration of the proposed FD method to exploit hardware redundancy.

equal probabilistic properties for the noise of each sensor. It is shown that the optimal configuration is obtained when the measurement matrix satisfies  $\mathbf{H}^T \mathbf{H} = \frac{N}{3} \mathbf{I}_3$ , where  $N$  is the number of available sensor measurements.

From the model of the rate gyros measurements in (3) and the boundedness of the measurement noise, we have that  $\Omega$  satisfies the following inequality

$$\begin{bmatrix} \mathbf{I}_{N_\Omega} \\ -\mathbf{I}_{N_\Omega} \end{bmatrix} \Omega \leq \begin{bmatrix} \Omega_r - \mathbf{b} + \delta_\Omega \\ -\Omega_r + \mathbf{b} + \delta_\Omega \end{bmatrix},$$

where  $\mathbf{I}_{N_\Omega}$  is the  $N_\Omega \times N_\Omega$  identity matrix,  $\Omega_r = [\Omega_r^{(1)} \dots \Omega_r^{(N_\Omega)}]^T \in \mathbb{R}^{N_\Omega}$ ,  $\delta_\Omega = [\delta_\Omega^{(1)} \dots \delta_\Omega^{(N_\Omega)}]^T \in \mathbb{R}^{N_\Omega}$ ,  $\mathbf{b} = [b^{(1)} \dots b^{(N_\Omega)}]^T \in \mathbb{R}^{N_\Omega}$  and

$$\delta_\Omega^{(i)} = \bar{\Delta}_b^{(i)} + \bar{n}_\omega^{(i)}. \quad (7)$$

Therefore,

$$\Omega \in \text{Set}(\mathbf{M}_\Omega, \mathbf{m}_\Omega), \quad (8)$$

where  $\mathbf{M}_\Omega = [\mathbf{I}_{N_\Omega} \quad -\mathbf{I}_{N_\Omega}]^T$ ,  $\mathbf{m}_\Omega = \begin{bmatrix} \Omega_r - \mathbf{b} + \delta_\Omega \\ -\Omega_r + \mathbf{b} + \delta_\Omega \end{bmatrix}$ . The matrix form of (2) is given by

$$\Omega = \mathbf{H}_\Omega \omega. \quad (9)$$

From (8) and (9), we conclude that  $\omega$  must satisfy

$$\omega \in \text{Set}(\mathbf{M}_\Omega \mathbf{H}_\Omega, \mathbf{m}_\Omega). \quad (10)$$

*Definition 1:* A rate gyro is faulty if its measurements does not satisfy the relations (2)-(3).

The following proposition characterizes the proposed FD method, illustrated in Fig. 1(b), which exploits the existence of redundant sensor measurements.

*Proposition 1:* Consider the rate gyros model (3) and the linear transformation between the ideal sensor measurements  $\Omega$  and the angular velocity  $\omega$  given in (9). Then, if  $\text{Set}(\mathbf{M}_\Omega \mathbf{H}_\Omega, \mathbf{m}_\Omega) = \emptyset$ , there exists at least one faulty rate gyro.

*Proof:* Assume that all rate gyros are healthy and that  $\text{Set}(\mathbf{M}_\Omega \mathbf{H}_\Omega, \mathbf{m}_\Omega) = \emptyset$ . Since all rate gyros are healthy, the rate gyros model (3) holds and  $\Omega \in \text{Set}(\mathbf{M}_\Omega, \mathbf{m}_\Omega)$ . Then, from the linear transformation (9), we have that  $\omega \in \text{Set}(\mathbf{M}_\Omega \mathbf{H}_\Omega, \mathbf{m}_\Omega)$ . But this contradicts the initial assumption that  $\text{Set}(\mathbf{M}_\Omega \mathbf{H}_\Omega, \mathbf{m}_\Omega) = \emptyset$ . Thus, we conclude that there must be at least one faulty sensor. ■

This strategy has the advantages of not requiring the tuning of a limit threshold and that it is guaranteed that no false alarms are issued.

The proposed scheme for fault isolation consists in evaluating the emptiness of  $S_i = \text{Set}(\mathbf{M}_\Omega \mathbf{H}_{\Omega \setminus \{i\}}, \mathbf{m}_\Omega)$ , where

$$\mathbf{H}_{\Omega \setminus \{i\}} = [\mathbf{h}_\Omega^{(1)} \dots \mathbf{h}_\Omega^{(i-1)} \quad \mathbf{h}_\Omega^{(i+1)} \dots \mathbf{h}_\Omega^{(N_\Omega)}]^T.$$

If only for one  $i$ ,  $S_i$  is non-empty, the faulty measurement is  $\Omega_r^{(i)}$ . If more than one  $S_i$  is non-empty, it is not possible

to isolate the fault. The following proposition provides sufficient conditions on the magnitude of a detected fault that ensure that isolation is feasible.

*Proposition 2:* Let the model of the faulty rate gyro  $i$  be given by

$$\Omega_r^{(i)} = \Omega^{(i)} + b^{(i)} + \Delta_b^{(i)} + n_\omega^{(i)} + \varepsilon,$$

where  $b^{(i)} \in \mathbb{R}$ ,  $|n_\omega^{(i)}| \leq \bar{n}_\omega^{(i)}$ ,  $|\Delta_b^{(i)}| \leq \bar{\Delta}_b^{(i)}$ , and  $\varepsilon \in \mathbb{R}$  encodes the measurement error resulting from a detected fault. If

$$|\varepsilon| > 4\sigma_{\max}(\mathbf{H}_\Omega) \|\delta_\Omega\|,$$

then the proposed FDI scheme is able to isolate non-simultaneous faults.

*Proof:* Start by noting that, for any  $\mathbf{x}_1, \mathbf{x}_2 \in \text{Set}(\mathbf{M}_\Omega, \mathbf{m}_\Omega)$ ,  $\|\mathbf{x}_1 - \mathbf{x}_2\| \leq \rho$ , where  $\rho = \|2\delta_\Omega\|$ . Moreover,  $\|\mathbf{H}_\Omega \mathbf{x}_1 - \mathbf{H}_\Omega \mathbf{x}_2\| \leq \sigma_{\max}(\mathbf{H}_\Omega) \rho$ , where  $\sigma_{\max}(\mathbf{H}_\Omega)$  denotes the maximum singular value of  $\mathbf{H}_\Omega$ , i.e., the maximum distance between two points belonging to the polytope defined by

$$P_H = \text{Set}(\mathbf{M}_\Omega \mathbf{H}_\Omega, \mathbf{m}_\Omega)$$

is given by  $\sigma_{\max}(\mathbf{H}_\Omega) \rho$ . On the other hand, any polytope  $P$  that can be enclosed in a ball of radius  $\sigma_{\max}(\mathbf{H}_\Omega) \rho$  and that contains the fixed point  $\omega$ , is also enclosed in a ball of radius  $2\sigma_{\max}(\mathbf{H}_\Omega) \rho$  centered at  $\omega$ . Consider such a polytope,  $P$  and note that if a hyperplane intersecting to the boundary of  $P$  is displaced by  $2\sigma_{\max}(\mathbf{H}_\Omega) \rho$  from its initial position, it will no longer intersect  $P$ .

Since that  $P_H$  satisfies the constraints assumed for  $P$ , and the hyperplanes that define the polytope  $P_H$  are given by

$$\{\mathbf{x} \in \mathbb{R}^3 : \mathbf{h}_\Omega^{(i)T} \mathbf{x} = \mathbf{m}_\Omega^{(i)}\}, \quad i = 1, \dots, N_\Omega,$$

where, for all  $i$ ,  $\|\mathbf{h}_\Omega^{(i)}\| = 1$ , we conclude that any hyperplane

$$\{\mathbf{x} \in \mathbb{R}^3 : \mathbf{h}_\Omega^{(i)T} \mathbf{x} = \mathbf{m}_\Omega^{(i)} + \varepsilon\}, \quad i = 1, \dots, N_\Omega,$$

with  $\varepsilon = 2\sigma_{\max}(\mathbf{H}_\Omega) \rho$ , does not intersect  $P_H$ . ■

*Remark 1:* To isolate non-simultaneous faults it is required, at least, five sensors. With more sensors and the appropriate modifications, the proposed method is also suitable to isolate simultaneous faults of two or more sensors.

#### IV. FDI USING ANALYTICAL REDUNDANCY AND SVOs

In this section, we present an FDI filter for rate gyros measurements and vector observations. We propose a technique based on SVOs [23] that takes advantage of the analytical redundancy between the sensed data to detect and isolate sensor faults.

##### A. Fault Detection

As most physical phenomena, the kinematic model described in (4) is in continuous time and, hence, not in the desired discrete-time framework of the SVOs. In the following, we devise a discrete-time approximation of the model based on the knowledge of upper bounds on the magnitude of the angular acceleration.

The solution of the differential equation (4) is given by

$$\mathbf{v}(t) = \exp\left(-\int_{t_0}^t [\omega(\tau)]_\times d\tau\right) \mathbf{v}(t_0), \quad (11)$$

where  $t_0 < t$ . Using the Mean Value Theorem and (11), we can rewrite (11) at the discrete-time instants as

$$\mathbf{v}((k+1)T) = \exp\left(-\frac{T^2}{2}[\dot{\boldsymbol{\omega}}(\xi)]_{\times} - T[\boldsymbol{\omega}(kT)]_{\times}\right) \mathbf{v}(kT), \quad (12)$$

for some  $\xi \in [kT, (k+1)T]$ .

In (12), the angular velocity and the angular acceleration are not fully known and hence are not suitable to be used in the SVO. On the other hand, from (10) the angular velocity satisfies

$$\boldsymbol{\omega}(kT) \in \text{Set}(\mathbf{M}_{\boldsymbol{\omega}}(kT), \mathbf{m}_{\boldsymbol{\omega}}(kT)), \quad (13)$$

where  $\mathbf{M}_{\boldsymbol{\omega}}(kT) = \mathbf{M}_{\Omega}(kT)\mathbf{H}_{\Omega}$  and  $\mathbf{m}_{\boldsymbol{\omega}}(kT) = \mathbf{m}_{\Omega}(kT)$ . Since (13) defines a convex polytope, the center of the polytope, denoted by  $\boldsymbol{\omega}_r(kT)$ , can be computed by resorting to a linear optimization problem. Define  $\bar{\omega}_r = \|\boldsymbol{\omega}_r(kT)\|_{\max}$  as the maximum distance between the center and the border of the polytope, and define the uncertainty in the angular velocity as  $\boldsymbol{\delta}_{\boldsymbol{\omega}}(kT)$ , such that

$$\boldsymbol{\delta}_{\boldsymbol{\omega}}(kT) = \boldsymbol{\omega}(kT) - \boldsymbol{\omega}_r(kT). \quad (14)$$

The uncertainty  $\boldsymbol{\delta}_{\boldsymbol{\omega}}(kT)$  satisfies  $\boldsymbol{\delta}_{\boldsymbol{\omega}}(kT) \in \text{Set}(\mathbf{M}_{\boldsymbol{\omega}}(kT), \mathbf{m}_{\boldsymbol{\omega}}(kT) - \mathbf{M}_{\boldsymbol{\omega}}(kT)\boldsymbol{\omega}_r(kT))$ , which denotes a polytope centred at the origin, and let the maximum distance along any major axis to the boundary of this polytope be given by

$$\bar{\delta}_{\boldsymbol{\omega}} = \|\boldsymbol{\delta}_{\boldsymbol{\omega}}(kT)\|_{\max}. \quad (15)$$

The angular acceleration is inherently bounded due to the limitations on the energy that can be provided to any physical system. Moreover, in many applications, either due to constraints on the thrusters, or due to the action of friction, it is in fact possible to derive an upper bound on the magnitude of the angular acceleration. Hence, we pose the following assumption.

*Assumption 2:* Assume that the magnitude of the angular acceleration is bounded by a known (but possibly conservative) positive scalar  $\bar{\alpha}$ , i.e.,

$$\|\dot{\boldsymbol{\omega}}\|_{\max} \leq \bar{\alpha}, \quad \bar{\alpha} \in \mathbb{R}^+. \quad (16)$$

For simplicity of notation, in the remainder of this paper the time dependence of the variables will be simply denoted by  $k$ ,  $k \in \mathbb{N}$ .

Using the result from [4, Appendix A] and the magnitude bounds on the uncertainty of the angular velocity measurements (16) and on the angular acceleration (14), we devise the following relation for each element of the dynamic matrix

$$\begin{aligned} & [\exp(-T[\boldsymbol{\omega}_r(k)]_{\times} + \frac{T}{2}\dot{\boldsymbol{\omega}}(\xi) + \boldsymbol{\delta}_{\boldsymbol{\omega}}(k)]_{\times})]_{ij} \\ &= [\exp(-T[\boldsymbol{\omega}_r(k)]_{\times})]_{ij} + \epsilon \Delta_{\epsilon}(i, j), \end{aligned}$$

for some  $|\Delta_{\epsilon}(i, j)| \leq 1$ , where

$$\epsilon = \frac{1}{2} (\exp(2T(\bar{\omega}_r + \bar{\delta}_{\boldsymbol{\omega}} + \frac{T}{2}\bar{\alpha})) - \exp(2T\bar{\omega}_r)). \quad (17)$$

With this construction, we have obtained a discrete-time approximate system that depends solely on sensor data in the framework of the SVOs [4], [21], [22], [24]. The upper bound on the error of the approximation,  $\epsilon$ , can be handled by the same framework. Hence, an SVO can be designed to the system

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}_0(k)\mathbf{x}(k) + \mathbf{A}_{\Delta}(k)\mathbf{x}(k) \\ \mathbf{y}(k) = \mathbf{C}(k)\mathbf{x}(k) + \mathbf{n}(k) \end{cases}, \quad (18)$$

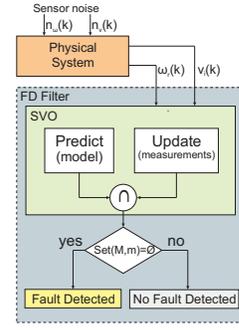


Fig. 2. Proposed fault detection (FD) filter for navigation systems.

where  $\mathbf{x}(k+1) = [\mathbf{v}_1^T \dots \mathbf{v}_{N_v}^T]^T$ ,  $\mathbf{A}_0(k) = \mathbf{I}_{N_v} \otimes \exp(-T[\boldsymbol{\omega}_r(k)]_{\times})$ ,  $\mathbf{A}_i(k) = \epsilon \mathbf{E}_{m,n}$ ,  $m = 1, \dots, 3$ ,  $n = 1, \dots, 3$ ,  $i = m + 3(n-1)$ ,  $\mathbf{C}(k) = \text{blkdiag}(\mathbf{H}_{v_1}, \dots, \mathbf{H}_{v_{N_v}})$ , and  $\mathbf{n} = [\mathbf{n}_{v_1}^T \dots \mathbf{n}_{v_{N_v}}^T]^T$ .

If, at some point, the set containing the state,  $\text{Set}(\mathbf{M}(k), \mathbf{m}(k))$ , degenerates into the empty set, we conclude that the model no longer describes the system and sensor data, and hence a fault has occurred. The proposed FD architecture is illustrated in Fig. 2 and its main property is formally stated in the following proposition

*Proposition 3:* Consider the model of the rate gyros (3) and the model of the vector observations (5), which are dynamically related by the model (4), and the corresponding SVO described in (18). Then, if  $\text{Set}(\mathbf{M}(k), \mathbf{m}(k)) = \emptyset$ , for some  $k \geq 0$ , a fault has occurred at some time instant  $k_f \leq k$ .

*Remark 2:* The proposed FD filter guarantees that there will be no false alarms. However, it may not be able to detect some sensor faults. This may be due to severe sensor noise or to the conservatism added to the model in (18). This problem is related with the concept of indistinguishability. The interested reader is referred to [20].

*Remark 3:* This method may lead to some implementation issues, since it might not be suitable for systems with very low computational power. However, nowadays, many aircrafts and spacecraft are equipped with powerful state-of-the-art computers. In addition, the proposed solution is highly parallelizable and thus can take advantage of the recent multi-core and multi-processor systems.

## B. Fault Isolation

To design fault-tolerant systems, it is required not only to detect that a fault has happened, but also to determine its exact location. Therefore, if redundant sensors exist, the system should be reconfigured, so that the normal operation can be resumed using the remaining healthy sensors.

In this work, we adopt the strategy proposed in [22] and illustrated in Fig. 3. This strategy relies on the concept of model invalidation. A bank of SVOs is designed modeling each different fault, and another one modeling the nominal (non-faulty) system. Since only one model is consistent with the sensor data, all the others SVOs will be invalidated, i.e., their estimated sets containing the state will degenerate into the empty set. The remaining SVO, if any, identifies the fault.

To spare unnecessary computational burden, and since the faults can occur at any time, we use the following scheme.

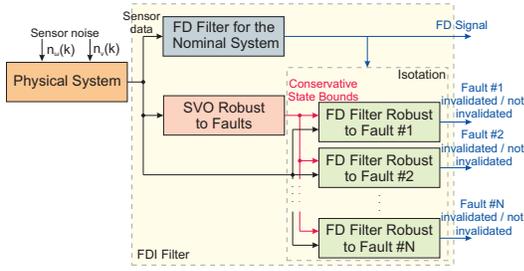


Fig. 3. Proposed fault detection and isolation (FDI) filter for navigation systems.

Firstly, only the nominal FD filter and one SVO robust to all faults are active, i.e., the set estimated by the robust SVO will always include the true state of the system, even if a fault in the sensors has occurred. If, at some point, the FD filter for the nominal system is invalidated, a fault has occurred. Hence, the bank of FD filters modeling the faults is initialized with the set estimated by the robust SVO. Once all the filters describing faults that did not occur have been invalidated, we have isolated the fault.

After this general description of the proposed FDI filter, we are now in condition of stating how this can be applied to inertial measurements and vector observations.

1) *Faults in the Vector Observations*: The faults in the vector observations can be modeled directly in the SVO. The hard faults considered are the zero output and the *stuck at* types of faults. The zero output fault is modeled by zeroing the row in the measurement matrix corresponding to the faulty sensor, whereas, a *stuck at* type of fault is modeled assuming a constant value in the sensor measurements. Thus, the SVO for this fault, performs the intersection of the set obtained from the measurements that contains  $\mathbf{v}^{(i)}$  at successive time instants, neglecting the dynamics of the system (for this sensor). The soft faults are modeled as an unexpected increase in the magnitude of the sensor noise, i.e., a greater value  $\bar{n}_{\mathbf{v}_i}^{(j)}$  in (6).

2) *Faults in the Rate Gyros*: The kinematics of the rigid body attitude depends nonlinearly on the angular velocity. For that reason, this method is not suitable to isolate faults that affect only one rate gyro. On the other hand, it can isolate faults affecting all the rate gyros. A higher noise magnitude in the rate gyros bias is modeled using an SVO with greater  $\bar{\delta}_\omega$  in (15). A bias variation greater than what was anticipated can be modeled by a greater  $\Delta_b^{(i)}$  in (7) and, consequently, a greater  $\bar{\delta}_\omega$  in (17). Since these two sources of uncertainty influence the dynamics in a similar way, they are indistinguishable (see [20]). As a consequence, we can only design an SVO that is tolerant to both faults. The hard faults – zeroing the measurements and the *stuck at* type of faults – invalidate any information regarding the model. Hence, to isolate a hard fault in the rate gyros it is necessary to design a different SVO for each faulty rate gyro assuming constant measurements as model.

## V. SIMULATIONS RESULTS

In this section, we present some simulation results illustrating the performance of the two proposed FDI schemes. We consider two different system configurations: i) five rate

TABLE I  
NUMBER OF ITERATIONS NECESSARY TO DETECT AND ISOLATE THE FAULTS WITH FIVE GYROS AND FIVE SENSORS PER VECTOR

Fault #	$k_d$ HW	$k_i$ HW	$k_d$ An.	$k_i$ An.
1	0	0	1	7
2	0	0	1	5
3	3	10	3	-
4	54	99	55	-
5	37	51	11	22
6	10	19	2	12
7	11	17	10	19

gyros, and two vector observations, each of which with five sensors, with installation matrices described by

$$\mathbf{H}_\Omega = \mathbf{H}_{\mathbf{v}_1} = \mathbf{H}_{\mathbf{v}_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.47 & 0.47 & 0.75 \\ -0.64 & 0.17 & 0.75 \\ 0.17 & -0.64 & 0.75 \end{bmatrix},$$

and satisfying  $\mathbf{H}^T \mathbf{H} = \frac{N}{3} \mathbf{I}$ ; ii) three rate gyros, and two vector observations, each of which with three sensors, with installation matrices given by  $\mathbf{H}_\Omega = \mathbf{H}_{\mathbf{v}_1} = \mathbf{H}_{\mathbf{v}_2} = \mathbf{I}$ . In both cases, we assume that the sensors are installed onboard a vehicle describing oscillatory angular movements characterized by the following angular velocity vector

$$\boldsymbol{\omega}(t) = \begin{bmatrix} 40 \sin(2\pi 0.05t) \\ -29 \sin(2\pi 0.04t) \\ 34 \sin(2\pi 0.02t) \end{bmatrix} \text{ deg s}^{-1}.$$

It is assumed that, under normal operation, the rate gyros measurements are corrupted by uniform noise with amplitude of  $0.2 \text{ deg s}^{-1}$  and for the bias calibrated at the beginning of the mission, we assume a tolerance of  $|\Delta_b^{(i)}| = 0.06 \text{ deg s}^{-1}$ ,  $i = 1, \dots, 5$ . Each vector has unit norm, and each sensor measurement is corrupted by uniform noise with maximum amplitude of  $0.1$ . The sampling period of all sensors is set to  $T = 0.1 \text{ s}$ .

We assume that one of the following seven faults can occur:

- 1) a *stuck at* type of fault has occurred in rate gyro one;
- 2) rate gyro #2 has been badly damaged, producing a measurement of zero;
- 3) the maximum amplitude of the noise in the rate gyro #3 increases ten times;
- 4) the bias in rate gyro #3 changes more than ten times the specified tolerance;
- 5) a *stuck at* type of fault has occurred in first sensor of vector #1;
- 6) the second component of vector #1 starts proving only the output zero;
- 7) the maximum amplitude of the noise in the third sensor of vector #2 increases three times.

Table I provides the number of iterations, i.e., the number of sampling periods, required to detect and isolate each fault using the methods for hardware redundancy (HW) and analytical redundancy (An.) for the case where there are five rate gyros and five sensors for each vector. In this table,  $k_d$  and  $k_i$  stand for the number of iterations for detection and isolation, respectively. The number of iterations required to detect and isolate each fault for the case where there are three rate gyros and five sensors for each vector is presented in Table II. Note that, since there are no redundant sensors

TABLE II

NUMBER OF ITERATIONS NECESSARY TO DETECT AND ISOLATE THE FAULTS WITH THREE GYROS AND THREE SENSORS PER VECTOR

Fault #	$T_d$ An.	$T_i$ An.
1	8	16
2	7	17
3	76	-
4	42	-
5	4	34
6	3	17
7	0	9

is not possible to detect or isolate faults using the method that exploits the hardware redundancy.

The results presented in Table I show that both methods are able to detect the considered faults. The method based on SVOs is not able to isolate the faults 3 and 4 since, as discussed in Section IV-B.2, they impact the model in a similar way and hence are indistinguishable. It is apparent from the results that no method is superior in detecting all faults. The method based on hardware redundancy has the advantage of requiring less computational power than the method based on SVOs, while the latter, by exploiting the dynamic relation between sensor measurements, has the advantage of not requiring redundant sensors. Table II shows that, as expected, with less sensors the detection and isolation of faults requires more iterations. It should be also noted that, even with a sampling time set to  $T = 0.1$  s, the two proposed methods are able to detect and isolate all the distinguishable faults in less than 10 s.

## VI. CONCLUSIONS

In this paper, we have proposed two novel FDI methodologies for IMUs and vector observations. The first scheme takes advantage of hardware redundancy in the sensor measurements to detect incoherences between them. Sufficient conditions have been provided that depend on the measurement matrix and on the measurements uncertainty that guarantees isolation of non-simultaneous faults. To exploit the dynamic relation between the angular velocity and the vector measurements, a second methodology was proposed based on set-valued state estimates provided by SVOs, which can be used to validate or falsify different models of faults. Neither solution generates false detections, as long as the non-faulty model of the system remains valid. Simulation results show that the detection and isolation of the faults take, in general, only a few iterations.

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