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GPS AIDED IMU FOR UNMANNED AIR VEHICLES [1]

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Abstract: This paper presents the development of a strapdown navigation system to determine the pose (position and attitude) of unmanned air vehicles, using GPS, accelerometers, magnetometers and rate gyros triads. The current work resorts to complementary filtering to implement a navigation system developed on Earth coordinates and Euler angles. Special features include bias estimation and removal in inertial sensors. An attitude aiding device, referred to as Magneto-Pendular Sensor, is introduced and detailed. Multirate filter synthesis is outlined, and a time-varying attitude transformation is briefly discussed. The navigation system performance is evaluated in simulation using a typical model-scale helicopter maneuver.

Keywords: Navigation Systems, Complementary Filters, Time-varying Systems, Strapdown Systems

1. INTRODUCTION

Recent Unmanned Air Vehicles (UAVs) exhibit a high degree of operational reliability in the presence of dynamic, uncertain environments and challenging scenarios. Among the many UAV configurations available today, helicopters are one of the most maneuverable and versatile platforms. They can takeoff and land without a runway and can hover in place. These capabilities have brought about the use of unmanned helicopters as highly maneuverable sensing platforms, allowing for the access to remote and confined locations without placing human lives at risk.

One of the major stumbling blocks that have prevented UAVs from successfully executing their missions is the unavailability of low power, inexpensive and reliable onboard navigation systems capable of integrating the information from sophisticated sensor suites.

This paper presents the development of a GPS aided Inertial Measurement Unit (IMU) using complementary filters that explicitly tackle the problem of merging information provided by the vehicle sensor suite over distinct, yet complementary frequency regions. In this design the IMU provides the platform's pose (position and attitude). Fusion of these data with the linear position available from a low cost GPS receiver produces high rate, accurate position and attitude estimates that will be central to stabilize the platform and support the implementation of reliable trajectory tracking control strategies (Cunha *et al.*, 2003).

Due to physical constraints, the helicopter onboard sensors provide measurements at different

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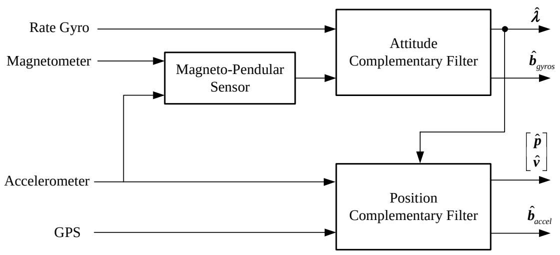

Fig. 1. Complementary Filter Block Diagram

sampling rates. A synthesis methodology to explicitly address the system's multirate nature is used, endowing this setup with properties associated to the single rate complementary filters (Pascoal *et al.*, 2000). The resulting navigation system is depicted in Fig. 1. The system produces estimates of the vehicle pose and removes the sensor bias that is a common source of drift error in navigation systems. The real time implementation in a DSP of the resulting filter structure is straightforward. Nowadays several navigation systems resort to Extended Kalman Filtering (EKF) to integrate the position measurements available from a GPS receiver with the position and attitude estimates given by the Inertial Navigation System (INS). While a unified error equations analysis for INS has been carried out in the literature (Britting, 1971), proposed navigation systems still differ due to the several filtering architectures that may be used. Recent work can be found in (Sukkarieh, 2000), (Gaylor, 2003), and (Whitmore *et al.*, 1997). Sukkarieh in (Sukkarieh, 2000) develops a general purpose, high integrity navigation system for land vehicles, that uses an EKF to integrate the information provided by a GPS and an INS. In this work the filter uses a simple vehicle dynamic model and a multi-path/signal blockage GPS error analysis is presented. Gaylor(Gaylor, 2003) also addresses GPS error analysis for spaceship docking and extends the sensor error correction by introducing sensor bias estimation in the EKF. Both rely on the assumption that an INS algorithm has been previously developed. Whitmore (Whitmore *et al.*, 1997) present an INS algorithm based on exact attitude update using quaternion dynamics.

The complementary filter structure, shown in Fig. 1, consists of an attitude filter and a position filter, both featuring bias removal. The attitude filter entries are the rate gyro readings and an attitude-aiding sensor based on magnetic-field and gravitational readings, thus referred as Magneto-Pendular Sensor (MPS). The position filter is based on the work presented in (Pascoal *et al.*, 2000), resorting to accelerometers readings and to GPS. Since gravity must be removed from accelerometers' specific force, the position filter uses the attitude estimates provided by the attitude filter.

This paper is organized as follows. Section 2 briefly introduces the complementary filters, provides the attitude complementary filter equations and develops the MPS attitude determination method. Section 3 addresses implementation issues, presents the position filter equations and the multirate filter that combines the low-rate GPS readings with the high-rate information provided by the remaining sensors. Section 4 presents and discusses the overall navigation system simulation results. Concluding remarks and future work are pointed out in section 5.

2. COMPLEMENTARY FILTERING. ATTITUDE DETERMINATION

The complementary filter theory is deeply rooted in the work of Wiener (Wiener, 1949). An unknown signal can be estimated using corrupted measurements from one or more sensors whose information naturally stands in distinct and complementary frequency bands (Brown, 1972), (Brown and Hwang, 1997). The minimum mean-square estimation (MMSE criteria) error was first solved by Wiener (Wiener, 1949), assuming that the unknown signal had noise-like characteristics, which usually does not fit the signal description. Complementary filtering explores redundancy to smooth measurement errors without prior knowledge about the signal.

Interestingly enough, the complementary filter does not apply any distortion to the signal (*distortionless* filtering), and can successfully reject the noise components. This approach proves to be a very efficient time-invariant solution, whose parameters are defined according to sensor's complementary bandwidths descriptions. In the linear time-invariant case the transfer function analysis can be done using conventional Bode plots.

The loss of optimality in complementary filters due to ignoring noise stochastic description is very slight and can even be beneficial for special cases where it is better to consider irregular measures that occur out of the expected variance, as convincingly argued in (Brown, 1972). To meet additional constrains, the filter order can be increased, as seen in (Pascoal *et al.*, 2000) where increasing the filter order enabled new degrees of freedom in the transfer function and enhanced the high frequency noise rejection without distorting the signal.

2.1 Attitude Filter

Euler angles are chosen as the state space representation for the rigid body attitude filter, due to its simplicity. A nonlinear transformation must be

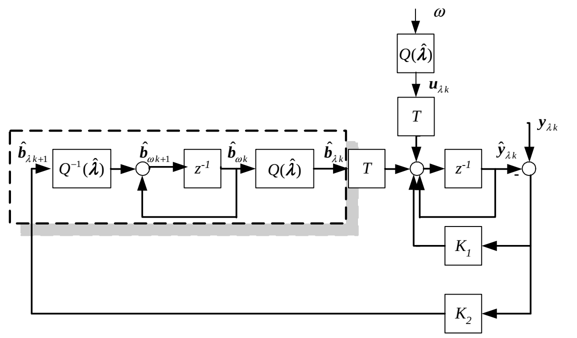

Fig. 2. Attitude Complementary Filter

applied to the rate gyros measurements to obtain Euler angles derivatives. In the case where bias terms are present in the sensor readings, time-invariance is lost and a new methodology is proposed to correctly estimate and compensate these sensor errors.

The complementary filter structure proposed requires also an attitude reading that is not directly available. To tackle this problem, a *Magneto-Pendular Sensor* (MPS) concept is also proposed, which assumes the role of an attitude aiding sensor.

Let $\boldsymbol{\lambda} = (\psi, \theta, \phi)'$ and $\mathbf{b}_\lambda = (b_\psi, b_\theta, b_\phi)'$ be the yaw, pitch and roll Euler angles, and the bias in each of the three channels of the angular rate sensors, respectively. Assuming the step invariant discrete time version of the attitude kinematics as the underlying model for the filter design, the attitude and biases estimates, $\hat{\boldsymbol{\lambda}}$ and $\hat{\mathbf{b}}_\lambda$ respectively, can be written as

$$\begin{cases} \begin{bmatrix} \hat{\boldsymbol{\lambda}}_{k+1} \\ \hat{\mathbf{b}}_{\lambda\, k+1} \end{bmatrix} = \begin{bmatrix} I_{3\times3} & I_{3\times3}T \\ 0 & I_{3\times3} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{\lambda}}_k \\ \hat{\mathbf{b}}_{\lambda\, k} \end{bmatrix} + \\ + \begin{bmatrix} I_{3\times3}T \\ 0 \end{bmatrix} \mathbf{u}_{\lambda\, k} + \begin{bmatrix} K_{1\lambda} \\ K_{2\lambda} \end{bmatrix} (\mathbf{y}_{\lambda\, k} - \hat{\mathbf{y}}_{\lambda\, k}) \\ \hat{\mathbf{y}}_{\lambda\, k} = \begin{bmatrix} I_{3\times3} & 0 \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{\lambda}}_k \\ \hat{\mathbf{b}}_k \end{bmatrix} \end{cases}, \quad (1)$$

where T represents the sampling interval, the index k abbreviates the time instant $t = kT$, $\mathbf{u}_\lambda$ is the vector of angular rate measurements, $\mathbf{y}_\lambda$ is the vector of angular measurements and $K_{1\lambda} = I_{3x3}k_{1\lambda}$, $K_{2\lambda} = I_{3x3}k_{2\lambda}$ are the Euler angles and biases estimation filter gain matrices, respectively. The block diagram of the proposed structure is shown in Fig. 2. Since rate gyros directly measure angular rates of body frame with respect to inertial frame expressed in the body coordinate system, denoted as ω, their readings must be converted to Euler angle rates using the relation $\mathbf{u}_\lambda = Q(\boldsymbol{\lambda})\,\omega$ where

$$Q(\boldsymbol{\lambda}) = \begin{bmatrix} 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \\ 0 & \cos\phi & -\sin\phi \\ 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \end{bmatrix}.$$

The same transformation $Q(\boldsymbol{\lambda})$ is implicitly applied to the rate gyro bias terms $\mathbf{b}_\lambda(t) = Q(\boldsymbol{\lambda})\mathbf{b}_\omega$, where $\mathbf{b}_\omega$ is the bias term in angular rate space, assumed as time-invariant.

The transformation $Q(\boldsymbol{\lambda})$ applied on rate gyro biases $\mathbf{b}_\omega$ issues a time-varying bias terms $\mathbf{b}_\lambda(t)$ in Euler angles, whose integration can not be directly computed from the feedback term $\hat{\mathbf{b}}_{\lambda\, k+1} = \hat{\mathbf{b}}_{\lambda\, k} + K_{2\lambda}\,(\mathbf{y}_{\lambda\, k} - \hat{\mathbf{y}}_{\lambda\, k})$.

The problem of properly integrating biases is solved resorting to a nonlinear space transformation, as depicted in the dotted box in Fig. 2. Stability and performance issues must be carefully examined in the near future to ensure reliability for operational applications. Nevertheless, exhaustive filter tests show no problems in the regular operational conditions, where the yaw angle is unconstrained and roll and pitch angles are not close to the non-singular configurations.

2.2 Magneto-Pendular Sensor

An indirect attitude measurement can be obtained given two vector readings in both body and inertial frames. This problem was first introduced by Wahba (Wahba, 1965) and several solutions have been proposed along time-spread articles. In the current work, the proposed solution may be considered as a TRIAD-like approach, see (Caruso, 1998). Earth's gravitational field available from the onboard accelerometer triad measurements is used to determine pitch and roll angles. The yaw angle is computed from the Earth's magnetic field measurements provided by a magnetometer triad.

If the gravity vector coordinates are known on both body and Earth frames, it is possible to determine pitch and roll angles resorting to the rotation between both frames, as expressed by the following relation

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \mathcal{R}^{-1}\,\mathbf{g} = \begin{bmatrix} -g\sin\theta \\ g\cos\theta\sin\phi \\ g\cos\theta\cos\phi \end{bmatrix}, \quad (2)$$

where $\mathbf{g} = (0, 0, g)'$ is the gravity vector in Earth frame coordinates, $\mathbf{a} = (a_x, a_y, a_z)'$ is the gravity vector expressed in body frame, g is the local gravitational acceleration and $\mathcal{R}$ is the shorthand for the rotation matrix between body and Earth frames, ${}^E_B\mathcal{R}(\boldsymbol{\lambda})$. The pitch and roll angles can be computed as

$$\begin{aligned} &\phi = \arctan\,(a_y, a_z) \\ &\begin{cases} \theta = \arctan 2(-a_x, \dfrac{a_y}{\sin\phi}), & \sin\phi \neq 0 \\ \theta = \arctan 2(-a_x, \dfrac{a_z}{\cos\phi}), & \cos\phi \neq 0 \end{cases}, \end{aligned} \quad (3)$$

where the four quadrant $\arctan 2$ was used. The yaw angle is determined by projecting the magnetic field reading on the x-y plane of the Earth

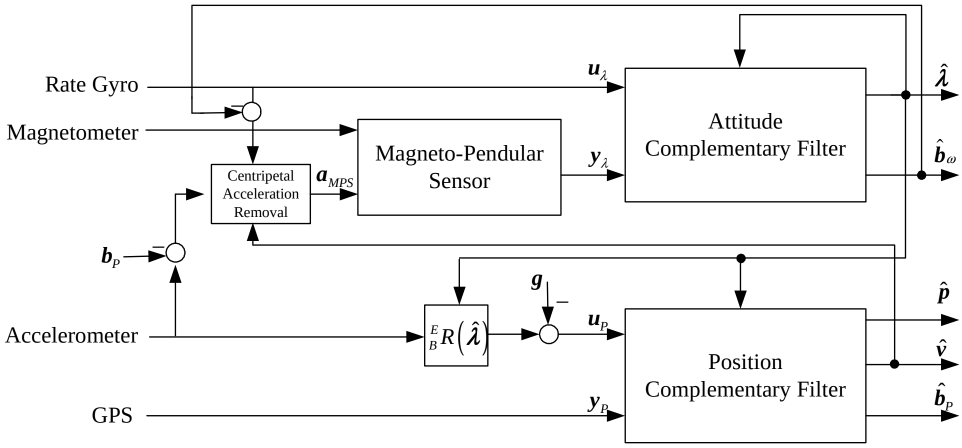

Fig. 3. Navigation System Architecture

frame according to ${}^{E_a}\mathbf{m} = \mathcal{R}_Y(\theta)\mathcal{R}_X(\phi)\mathbf{m}$, where $\mathbf{m}$ represents the Earth's magnetic field in body frame, ${}^{E_a}\mathbf{m}$ is the projection of the Earth's magnetic field, and $\mathcal{R}_Y(\theta)$, $\mathcal{R}_X(\phi)$ are pitch and roll elementary rotation matrices, respectively. The yaw angle is computed using $\psi = \arctan 2\left(-{}^{E_a}m_y, {}^{E_a}m_x\right) - \alpha_{dec}$ where α_{dec} is Earth's magnetic declination angle. This equation relies solely on the Earth's magnetic field projection ${}^{E_a}m$ and does not need any additional information on the magnetic field vertical component.

The computation of pitch and roll angles can become erroneous if external accelerations are present in the accelerometer readings, corrupting gravity vector estimation. Normal operating conditions seldom involve long time linear accelerations, so these disturbances only occur at high frequency, which is effectively removed due to the complementary filter lowpass characteristics. Regarding the centripetal accelerations, which can occur even in an uniform motion, they must be compensated according to the relation $\mathbf{a}_{MPS} = \mathbf{a} - \hat{\omega} \times^B \mathbf{v}$, where $\mathbf{a}_{MPS}$ is the actual gravity reading fed to (3), $\hat{\omega}$ is the angular rate drawn from the unbiased rate gyro measurement and ${}^B\mathbf{v}$ is the vehicle's velocity described in body frame, which can be provided by a velocity sensor or by the complementary position filter's velocity estimate.

3. IMPLEMENTATION

This section presents the overall navigation system architecture as depicted in Fig. 3 and points out some implementation issues.

The attitude complementary filter and MPS blocks have been described in the previous Section. The problem of estimating the velocity and position of an autonomous vehicle in the continuous time setup was dealt with in (Pascoal *et al.*, 2000) by resorting to time-varying filters.

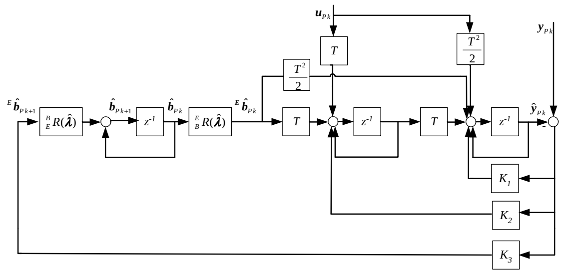

Fig. 4. Position Complementary Filter

Using linear differential inclusions, the stability of the resulting filters as well as their "frequency-like" performance can be assessed using efficient numerical analysis tools that borrow from convex optimization techniques. The proposed position complementary filter solution corresponds to the multirate version of the aforementioned filter, leading to the state model given by

$$\begin{cases} \begin{bmatrix} \hat{\mathbf{p}}_{k+1} \\ \hat{\mathbf{v}}_{k+1} \\ \hat{\mathbf{b}}_{P\,k+1} \end{bmatrix} = \begin{bmatrix} I_{3\times3} & I_{3\times3}T & I_{3\times3}\frac{T^2}{2} \\ 0 & I_{3\times3} & I_{3\times3}T \\ 0 & 0 & I_{3\times3} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{p}}_k \\ \hat{\mathbf{v}}_k \\ \hat{\mathbf{b}}_{P\,k} \end{bmatrix} + \\ + \begin{bmatrix} I_{3\times3}\frac{T^2}{2} \\ I_{3\times3}T \\ 0 \end{bmatrix} \mathbf{u}_{P\,k} + \begin{bmatrix} K_{1P} \\ K_{2P} \\ K_{3P} \end{bmatrix} (\mathbf{y}_{P\,k} - \hat{\mathbf{y}}_{P\,k}) \\ \hat{\mathbf{y}}_{P\,k} = \begin{bmatrix} I_{3\times3} & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{p}}_k \\ \hat{\mathbf{v}}_k \\ \hat{\mathbf{b}}_{P\,k} \end{bmatrix} \end{cases}, \quad (4)$$

where $\hat{\mathbf{p}} = (p_x, p_y, p_z)'$ and $\hat{\mathbf{v}} = (v_x, v_y, v_z)'$ are the position and velocity estimates in Earth frame, $\hat{\mathbf{b}}_P = (b_x, b_y, b_z)'$ is the bias estimate in each of the three acceleration measurements axis, $\mathbf{u}_P$ and $\mathbf{y}_P$ are the acceleration and position measurements, respectively. $K_{1P} = I_{3x3}k_{1P}$, $K_{2P} = I_{3x3}k_{2P}$, and $K_{3P} = I_{3x3}k_{3P}$ are the gain matrices related to $\hat{\mathbf{p}}$, $\hat{\mathbf{v}}$ and $\hat{\mathbf{b}}_P$, respectively, resulting in the block diagram shown in Fig. 4.

Position readings are available from a GPS receiver and acceleration is provided by an accelerometer triad after gravitational term compensation, as depicted in Fig. 3. Furthermore, an attitude estimate, provided by the attitude filter, is required to determine the position in Earth frame, since the strapdown mechanization provides sensor readings only in body frame.

The proposed position filter exhibits efficient bias estimation and compensation, displaying bounded position errors even in the presence of attitude estimation errors. As displayed in Fig. 3, attitude bias estimate is used to directly provide unbiased angular rate from rate gyros to the centripetal acceleration removal block. A similar accelerometer bias estimation feedback is unsuited since it leads to multiple equilibrium points for the underlying difference equations, see (Stovall, 1997) for details.

Complementary filter gains were obtained resorting to Kalman filter theory. For a discrete time stationary Kalman filter, the optimal gains are computed according to $K = P\,C_d^T(C_d\,P\,C_d^T + R)^{-1}$, where P is the error covariance matrix, determined by the solution of the algebraic Riccati equation $P = A_d\,P\,A_d^T - A_d\,P\,C_d^T\,(C_d\,P\,C_d^T + R)^{-1}C_d\,P\,A_d^T + Q$, where the matrices A_d and C_d are taken from the discrete state space kinematics model and Q and R are weight matrices that act as "tuning knobs" for gain determination.

Since the GPS sampling rate is much lower than the navigation system rate, the position filter is a multirate or periodic estimator. The filter gains can be obtained using the methodology presented in (Bittanti *et al.*, 1990), resulting in an optimal solution for the periodic estimator. For the current position filter, calculations show that optimal periodic position gains are only non-zero when a new GPS reading is available and, therefore, the filter will work open loop between GPS readings. Moreover, position filter transfer function analysis is not straightforward due to the periodic feature. See (Oliveira, 2002) for a multirate filter channel to channel frequency analysis methodology.

4. RESULTS

The navigation system was tested using a typical helicopter maneuver that is depicted in Fig. 5. The trajectory starts with a straight path, followed by an ascending helix. Inertial sensors and GPS update rates were set to 50 Hz and 1 Hz, respectively. The noise and bias characteristics of the sensors are presented in Table 1.

Sensor	Bias	Noise Variance (σ^2)
Rate Gyro	0.05 °/s	$(0.02\ °/\mathrm{s})^2$
Accelerometer	10 mg	$(0.6\ \mathrm{mg})^2$
Magnetometer	-	$(1\ \mu\mathrm{G})^2$
GPS	-	$10\ \mathrm{m}^2$

Table 1. Sensor Errors

The navigation system position results are shown in Fig. 6 and Mean Absolute Error values are presented in Table 2. Clearly, the position filter performance improves the simulated GPS accuracy and limits the IMU integration errors.

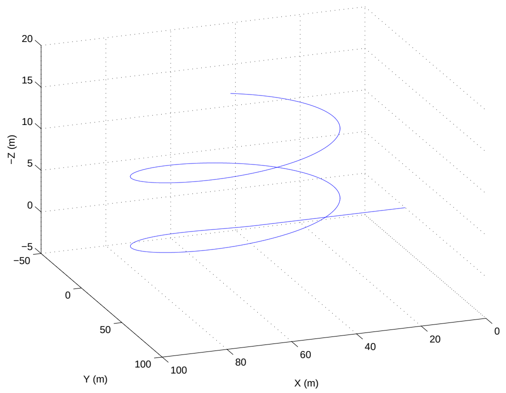

Fig. 5. Vehicle trajectory

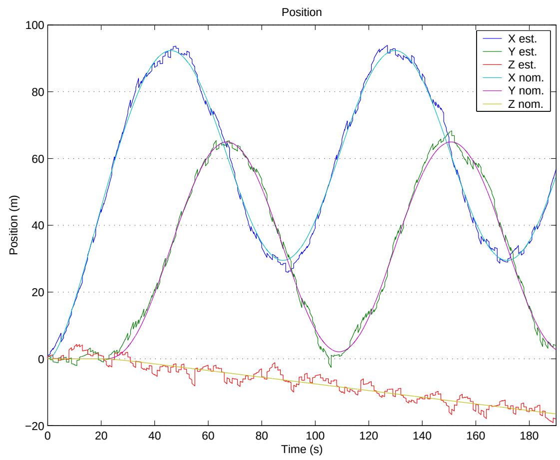

Fig. 6. Real and estimated position

	Coordinate	Mean Abs. Error $E(\lvert x - \hat{x}\rvert)$
Position Filter	X, Y, Z	1.39 m
Attitude Filter	Yaw	1.16×10^{-2} °
	Pitch	1.68×10^{-1} °
	Roll	1.01×10^{-1} °

Table 2. Filter Results

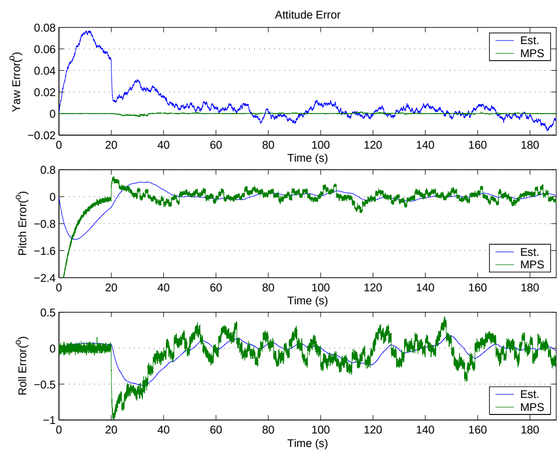

Fig. 7. Attitude Error

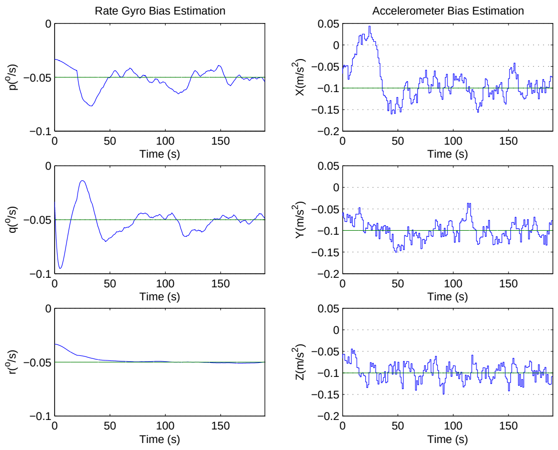

Fig. 8. Biases Estimation

The attitude filter errors, plotted in Fig. 7, are also bounded. The lowpass characteristics of the attitude complementary filter can be observed as it smooths out the high-frequency acceleration components during the linear path and the start of the loop (at 20 s). Due to these disturbances, yaw errors are smaller than pitch and roll errors since these are affected by the vehicle acceleration.

The sensor biases are correctly estimated through the high-pass filter rejection of low-frequency components in sensor measurements, as depicted in Fig. 8. In real applications, it is advisable to perform a careful calibration leading to good initial biases estimates. Since real biases have only small variations due to sensor's temperature changes and aging, small bias estimation gains can be used. Simulating such case yields about 1.2 m and $(8.3 \times 10^{-3}, 9.8 \times 10^{-2}, 3.4 \times 10^{-2})\,^\circ$ of mean absolute error in position and in Yaw, Pitch and Roll angles, respectively.

5. CONCLUSION

This paper presented a navigation system for an unmanned model-scale helicopter, based on complementary filtering. The navigation system featured distortionless filtering and good noise rejection. Bias compensation is also performed, further extending the typical IMU operation time. The whole navigation system structure proved to be simple and intuitive through its stationary setting, without resorting to more complex solution such as INS/EKF systems, providing good results both in attitude and position for typical vehicle maneuvers.

Future work involves the observability and stability issues related to the several feedback loops introduced in the proposed architecture. Additionally, filter performance can be enhanced by introducing new aiding sensors, such as a velocity sensor or a Laser Range Finder, and sensor errors should be extended beyond white noise model for a more realistic approach. Due to its low-cost, low-power and high-performance characteristics the proposed navigation system will be implemented in a DSP and tested in a real vehicle in the near future.

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