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Filter Design for Localization aided by Direction and Doppler Measurements

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***Abstract*— This paper presents a localization solution for the study of tagged marine animals (sources) based on direction and Doppler measurements. The source localization, in 3-D, is supported by a Portable Underwater Robotic Tool (PURT), comprising a Ultra-Short Baseline (USBL) aided INS navigation system, carried by a diver (agent). Furthermore, a Surface Robotic Tool (SRT) provides the agent with its absolute position expressed in inertial coordinates. The observability of the system is studied and realistic simulation results are presented, in the presence of measurement noise.**

I. INTRODUCTION

During the last decades, efforts have often been concentrated in large scale observation of marine species, as reported in [1] and [2]. In order to attain satisfiable localization results, whether absolute or relative, the species under study should spend most of their time at the surface, which is a non-controllable and quite unpredictable behavior of the animals. In addition, marine animal studies require an innate need for human approach, i.e. consolidated strategies that allow the biologist to work underwater and to be provided with knowledge of the surroundings. Notwithstanding, the absolute position of the agent could also be a leading requirement, where acoustic positioning methods based on time-of-flight measurements [3] or Long Baseline systems [4] are common solutions.

In another approach, the problem of source localization may be complemented with a navigation scenario including path following, where the follower is the agent and the path is outlined by a surface vehicle, the latter being the source. Assuming the agent is an Autonomous Underwater Vehicle (AUV), a solution was recently proposed in [5] for the navigation problem, whereas in [6] a new method for simultaneous source localization and navigation is presented. The previous solutions were shown to be globally asymptotically stable (GAS) under a persistent excitation condition. Interestingly, the same condition is also evidenced in the works of Antonelli *et. al.* [7], despite a trend towards the use of robotic vehicles presented therein.

Nevertheless, the key aspect in studies has been the identification of the position of the study targets, hereinafter designated as sources and, in spite of the everyday contributions for Navigation Systems based on Global Positioning System (GPS), the underwater environment remains incompatible with the application of such methods, namely RF communications. Instead, USBL acoustic positioning systems have become reliable tools, even if with lower quality performances, though with satisfiable accuracy results, which can be improved in the aftermath of the application of Kalman filters and INS tightly-coupled integration techniques [8].

Moreover, with proper processing of acoustic signals, one can extract more information besides the position, for instance an unambiguous tag identification or relative velocity. Recent developments in acoustic transmitters, mainly the de facto standard from VEMCO©, have even allowed to incorporate, through time delay codification coding, further characteristics of the environment like temperature and pressure, with respect to the location of the source. These transmitters, with long periods of autonomy, are an affordable and effective solution for underwater fields of research.

Recently, under the scope of the FCT funded MAST/AM project, an apparatus was designed and built that presents a convenient solution for maritime biologists (agents) to carry into the underwater environment. The full set of equipment includes a Surface Robotic Tool (SRT), presented in [9], and recent developments allowed to reach a final stage on the Portable Underwater Robotic Tool (PURT) construction. The latter is essentially an extension of the body of the diver, providing a high degree of maneuverability in order to track moving sources (commonly fishes) equipped with (commercially available) low power consumption acoustic signal transmitters.

The main contribution of this paper is to provide the scientific community with a new filter design for source localization problems based on direction and Doppler measurements. The observability of the system is studied and a time-varying Kalman filter with GAS error dynamics is proposed, without any system linearization. Proper simulations sustain the validity of the projected filter, thus by the end of the paper a novel source localization methodology is ready to attain the imposed requirements by the MAST/AM project.

This paper is organized as follows. Section II describes the framework of the problem and outlines the system dynamics. The filter design together with the proof of observability are presented in Section III, while Section IV includes simulation results along with discussions. Conclusions and future work are reported in Section V.

A. Notation

Throughout the paper, a bold symbol stands for a multi-dimensional variable, the symbol **0** denotes a matrix of zeros

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and $\mathbf{I}$ an identity matrix, both of appropriate dimensions. A block diagonal matrix is represented as diag($\mathbf{A}_1$,...,$\mathbf{A}_n$) and the set of unit vectors on $\mathbb{R}^3$ is denoted by $S(2)$. $\delta(t)$ represents the Dirac delta function. A positive-definite matrix $\mathbf{M}$ is identified as $\mathbf{M} \succ \mathbf{0}$.

II. PROBLEM STATEMENT

A. Motivation and Framework

The MAST/AM project, whose mission scenario is depicted in Fig. 1, consists of a SRT, commonly a Surface Buoy (SB) with USBL aided INS and GPS based navigation system, which may or not be stationary; an agent carrying a PURT also equipped with a USBL system; lastly, a source with an attached low power consumption acoustic signal transmitter. The SRT collaborates with the agent, therefore it is possible to assume that the agent knows its own current position $\mathbf{p}(t)$ expressed in inertial coordinates, which is transmitted by the SRT. The goal of the agent is to determine in the inertial framework the source position $\mathbf{s}(t)$ and the source velocity $\mathbf{v}_s(t)$. The Doppler measurements, v^d, are proportional to the relative velocity between the agent and the source, hence one needs to estimate the agent velocity $\mathbf{v}_p(t)$ to fully characterize the system. Overall, the relative position is, truly speaking, the main objective of the MAST/AM project.

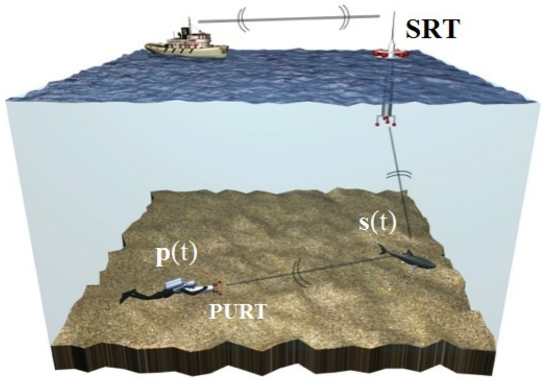


Fig. 1: MAST/AM mission scenario.

The Doppler velocity measurements are obtained through post processing of the incoming signals. With the knowledge of their natural frequency of transmission, which is an immutable specification, and by determining the Fast Fourier Transform (FFT), it is possible to calculate the frequency shift, which is directly proportional to the absolute relative velocity between the agent and the source.

B. System Dynamics

Consider a PURT carried by a diver (the full set may be interpreted as the agent) moving in an underwater scenario where there is also a moving source, which can be for example a fish under a study routine, equipped with a device that transmits, with a fixed ratio, a known acoustic signal, allowing the agent to determine the direction and the Doppler velocity according to its own body frame, $\mathcal{B}$, whose point of origin is placed in a suitable mechanical location, typically the centroid of the USBL array. Further, the agent interrogates the SRT, thereupon it obtains its position expressed in an inertial frame $\mathcal{I}$.

To conclude the definition of the framework, let $\mathbf{p}(t) \in \mathbb{R}^3$ be the agent position in inertial coordinates, while $\mathbf{s}(t) \in \mathbb{R}^3$ denotes the source position also in inertial coordinates. The velocity of the agent, which is unknown, is thus given by $\mathbf{v}_p(t) = \dot{\mathbf{p}}(t) \in \mathbb{R}^3$ and the source is moving with velocity $\mathbf{v}_s(t) = \dot{\mathbf{s}}(t) \in \mathbb{R}^3$; both velocities are expressed in inertial coordinates. The first mild assumption here is that both agent and source have constant velocity, therefore $\dot{\mathbf{v}}_s(t) = \mathbf{0}$ and $\dot{\mathbf{v}}_p(t) = \mathbf{0}$.

The agent measures the direction to the source expressed in $\mathcal{B}$, $\mathbf{d}^{\mathcal{B}}(t) \in \mathbb{R}^3$, however, after interrogating the SRT, the agent learns its body orientation according to $\mathcal{I}$, wherewith he may obtain the direction of the source expressed in $\mathcal{I}$. It results

$$\mathbf{d}(t) = {}^{\mathcal{I}}_{\mathcal{B}}\mathcal{R}(t)\mathbf{d}^{\mathcal{B}}(t) \in S(2), \tag{1}$$

where ${}^{\mathcal{I}}_{\mathcal{B}}\mathcal{R}(t) \in SO(3)$ is the rotation matrix used to perform the rotation in the Euclidean space from $\mathcal{B}$ to $\mathcal{I}$. Another way to denominate the direction is

$$\mathbf{d}(t) = \mathbf{r}(t)\|\mathbf{r}(t)\|^{-1}, \tag{2}$$

with $\mathbf{r}(t) := \mathbf{s}(t) - \mathbf{p}(t) \in \mathbb{R}^3$. The scalar Doppler velocity, $v^d \in \mathbb{R}$, is seen as the subtraction of the projected velocity vectors over the radial direction that unifies the agent and the source, i.e. the direction of the agent to the source. Thus, it is possible to write

$$v^d = (\mathbf{v}_s^T(t) - \mathbf{v}_p^T(t))\mathbf{d}(t).$$

The system dynamics can then be written as

$$\begin{cases} \dot{\mathbf{s}}^T(t) &= \mathbf{v}_s^T(t) \\ \dot{\mathbf{p}}^T(t) &= \mathbf{v}_p^T(t) \\ \dot{\mathbf{v}}_s^T(t) &= \mathbf{0} \\ \dot{\mathbf{v}}_p^T(t) &= \mathbf{0} \\ \mathbf{d}(t) &= (\mathbf{s}(t) - \mathbf{p}(t))\|\mathbf{s}(t) - \mathbf{p}(t)\|^{-1} \\ v^d &= (\mathbf{v}_s^T(t) - \mathbf{v}_p^T(t))\mathbf{d}(t) \end{cases} \tag{3}$$

and the problem is to estimate the position of the source $\mathbf{s}(t)$ and its velocity $\mathbf{v}_s(t)$, even if the latter is not a primary requirement for the project, and finally the velocity of the agent $\mathbf{v}_p(t)$. In other words, it is intended to design an estimator for the non-linear system (3).

III. LOCALIZATION FILTER DESIGN

This section presents a filter design methodology for the problem stated in Section II. First, a linear time-varying system is introduced in Section III-A. Afterwards, in Section III-B the observability of this system is analysed. The filter design is then discussed in Section III-C.

A. System States

As seen in (3), the agent velocity has to be estimated so that the source velocity can also be estimated. That means the agent position, although a known variable, will also be part of the states vector. Furthermore, being a transmitted information from the SRT into the PURT, the agent position is affected by sensor noise, thus becoming a measurement for all purposes.

The state vector is

$$\mathbf{x}(t) = [\mathbf{s}^T(t) \quad \mathbf{p}^T(t) \quad \mathbf{v}_s^T(t) \quad \mathbf{v}_p^T(t)]^T \quad \in \mathbb{R}^{12}.$$

However, this will not be enough to design an estimator for the nonlinear system dynamics, having considered the direction to the source as part of the measurements. To circumvent this impasse, add the distance between the agent and the source as one state, resulting in

$$\mathbf{x}(t) = [\mathbf{s}^T(t) \quad \mathbf{p}^T(t) \quad \mathbf{v}_s^T(t) \quad \mathbf{v}_p^T(t) \quad \|\mathbf{r}(t)\|]^T \quad \in \mathbb{R}^{13}.$$

Next, the derivative of the range is given by

$$\begin{aligned}\frac{d}{dt}\|\mathbf{r}(t)\| &= \frac{d}{dt}\sqrt{[\mathbf{s}^T(t)-\mathbf{p}^T(t)]\,[\mathbf{s}(t)-\mathbf{p}(t)]}\\ &= \left(\dot{\mathbf{r}}^T(t)\mathbf{r}(t)+\mathbf{r}^T(t)\dot{\mathbf{r}}(t)\right)(2\|\mathbf{r}(t)\|)^{-1}\\ &= \left(\mathbf{v}_s^T(t)-\mathbf{v}_p^T(t)\right)\mathbf{d}(t) = v^d,\end{aligned}$$

which comes as no surprise, since the distance between the source and the agent is expected to remain constant if $v^d = 0$. Still, this does not necessarily mean that both agent and source are at rest; one of them may be stopped while the other travels around the first in perfect circles, i.e. the internal product between the direction and the velocity of the moving target returns zero, such as they can also be both moving with the same speed and orientation. These situations must be avoided, leading to the requirement of some kind of persistent excitation condition.

The states vector derivative can be written as

$$\dot{\mathbf{x}}(t) = [\mathbf{v}_s^T(t) \quad \mathbf{v}_p^T(t) \quad \mathbf{0} \quad \mathbf{0} \quad v^d]^T \quad \in \mathbb{R}^{13}.$$

Since there exists no inputs in neither the agent nor the source, the system dynamics can be written as an LTV system expressed by

$$\begin{cases}\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t)\\ \mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t)\end{cases}, \tag{4}$$

where

$$\mathbf{A}(t) = \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & 0\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & 0\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0\\ \mathbf{0} & \mathbf{0} & \mathbf{d}^T(t) & -\mathbf{d}^T(t) & 0\end{bmatrix} \quad \in \mathbb{R}^{13\times 13}.$$

Exploiting the fact of adding the range as a state, a subtle trick can be made in order to avoid the nonlinearity caused by direction measurements.

From (2), it can be written

$$\mathbf{0} = \mathbf{s}(t) - \mathbf{p}(t) - \|\mathbf{s}(t)-\mathbf{p}(t)\|\mathbf{d}(t),$$

and the measurements vector, $\mathbf{y}(t) \in \mathbb{R}^7$, becomes

$$\mathbf{y}(t) = \begin{bmatrix}\mathbf{p}(t)\\ \mathbf{0}\\ v^d\end{bmatrix} = \begin{bmatrix}\mathbf{s}(t)-\mathbf{d}(t)\|\mathbf{s}(t)-\mathbf{p}(t)\|\\ \mathbf{s}(t)-\mathbf{p}(t)-\mathbf{d}(t)\|\mathbf{s}(t)-\mathbf{p}(t)\|\\ (\mathbf{v}_s(t)-\mathbf{v}_p(t))\cdot\mathbf{d}(t)\end{bmatrix}, \tag{5}$$

hence the system matrix $\mathbf{C}(t)$ follows as

$$\mathbf{C}(t) = \begin{bmatrix}\mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{I} & -\mathbf{I} & \mathbf{0} & \mathbf{0} & -\mathbf{d}(t)\\ \mathbf{0} & \mathbf{0} & \mathbf{d}^T(t) & -\mathbf{d}^T(t) & 0\end{bmatrix} \quad \in \mathbb{R}^{7\times 13}.$$

B. Observability Analysis

The observability of the problem of source localization with Doppler velocity readings and direction measurements is studied in this section. The following proposition [Proposition 4.2, [10]] is useful in the sequel.

Proposition: Let $\mathbf{f}(t) : [t_0, t_f] \subset \mathbb{R} \rightarrow \mathbb{R}^n$ be a continuous and i-times continuously differentiable function on $\mathcal{T} := [t_0, t_f], T := t_f - t_0 > 0$, and such that

$$\mathbf{f}(t_0) = \dot{\mathbf{f}}(t_0) = ... = \mathbf{f}^{i-1}(t_0) = \mathbf{0}.$$

Further assume that there exists a nonnegative constant C such that $\|\mathbf{f}^{(i+1)}(t)\| \leq C$ for all $t \in \mathcal{T}$. If there exist $\alpha > 0$ and $t_1 \in \mathcal{T}$ such that $\|\mathbf{f}^{(i)}(t_1)\| \geq \alpha$ then there exist $0 < \delta \leq T$ and $\beta > 0$ such that $\|\mathbf{f}(t_0+\delta\| \geq \beta$.

To prove the observability is to guarantee the outputs are enough to determine the correspondent current state. That said, a generic response $\mathbf{y}(t)$ must determine uniquely an initial state $\mathbf{x}(t_0)$.

From the Peano-Baker series it follows the transition matrix associated to $\mathbf{A}$, denoted by $\boldsymbol{\phi}(t,t_0) \in \mathbb{R}^{13\times 13}$, and given by

$$\boldsymbol{\phi}(t,t_0) = \begin{bmatrix}\mathbf{I} & \mathbf{0} & (t-t_0)\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I} & \mathbf{0} & (t-t_0)\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \int_{t_0}^{t}\mathbf{d}^T(\sigma)d\sigma & -\int_{t_0}^{t}\mathbf{d}^T(\sigma)d\sigma & 1\end{bmatrix}.$$

The condition to guarantee the observability of the LTV expressed in (4) is to ensure that the Observability Gramian is invertible, which is defined by

$$\boldsymbol{\mathcal{W}}(t_0,t_f) := \int_{t_0}^{t_f}\boldsymbol{\phi}^T(\tau,t_0)\mathbf{C}^T(\tau)\mathbf{C}(\tau)\boldsymbol{\phi}(\tau,t_0)d\tau.$$

Theorem: The LTV system (4) is observable on $\mathcal{T}$ if and only if the direction vector $\mathbf{d}(t)$ does not remain constant between two successive moments. Mathematically speaking,

$$\exists_{t_1\in\mathcal{T}} : \mathbf{d}^T(t_0)\mathbf{d}(t_1) < 1. \tag{6}$$

Proof: Obviously, if the direction remained constant the internal product between the two vectors would wind up to be the norm of $\mathbf{d}$ raised to second power, i.e. 1.

Let $\mathbf{c} = [\mathbf{c}_1^T \ \mathbf{c}_2^T \ \mathbf{c}_3^T \ \mathbf{c}_4^T \ c_5]^T \in \mathbb{R}^{13}$, with $\mathbf{c}_i \in \mathbb{R}^3$, for $i = 1,2,3,4$, and $c_5 \in \mathbb{R}$, be a unit vector. Since it must be $\|\mathbf{c}\| = 1$ for all $\mathbf{c}$,

$$\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0,t_f)\mathbf{c} \neq 0$$

in order to the LTV system be observable. Then,

$$\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0,t_f)\mathbf{c} = \int_{t_0}^{t_f} \|\mathbf{f}(\tau)\|^2 d\tau,$$

where

$$\mathbf{f}(\tau) = \begin{bmatrix}\mathbf{f}_1^T(\tau) & \mathbf{f}_2^T(\tau) & f_3(\tau)\end{bmatrix}^T \quad \in \mathbb{R}^7, \tau \in \mathcal{T},$$

with

$$\mathbf{f}_1(\tau) = \mathbf{c}_2 + (\tau - t_0)\mathbf{c}_4 \in \mathbb{R}^3,$$

$$\mathbf{f}_2(\tau) = \mathbf{c}_1 - \mathbf{c}_2 + \\ + \left[(\tau - t_0)\mathbf{I} - \mathbf{d}(\tau)\int_{t_0}^{\tau} \mathbf{d}^T(\sigma)d\sigma\right](\mathbf{c}_3 - \mathbf{c}_4) - \mathbf{d}(\tau)c_5 \in \mathbb{R}^3,$$

and

$$f_3(\tau) = \mathbf{d}^T(\tau)(\mathbf{c}_3 - \mathbf{c}_4) \in \mathbb{R}.$$

As for the first two derivatives of $\mathbf{f}(\tau)$ in order to τ, they are given by

$$\frac{d\mathbf{f}(\tau)}{d\tau} = \begin{bmatrix}\mathbf{c}_4 \\ \frac{d\mathbf{f}_2(\tau)}{d\tau} \\ \dot{\mathbf{d}}^T(\tau)(\mathbf{c}_3 - \mathbf{c}_4)\end{bmatrix} \quad \in \mathbb{R}^7,$$

with $d\mathbf{f}_2(\tau)/d\tau$ resulting in

$$\frac{d\mathbf{f}_2(\tau)}{d\tau} = \left[\mathbf{I} - \dot{\mathbf{d}}(\tau)\int_{t_0}^{\tau} \mathbf{d}^T(\sigma)d\sigma - \mathbf{d}(\tau)\mathbf{d}^T(\tau)\right](\mathbf{c}_3 - \mathbf{c}_4) \\ - \dot{\mathbf{d}}(\tau)c_5,$$

and

$$\frac{d^2\mathbf{f}(\tau)}{d\tau^2} = \begin{bmatrix}\mathbf{0} \\ \frac{d^2\mathbf{f}_2(\tau)}{d\tau^2} \\ \ddot{\mathbf{d}}^T(\tau)(\mathbf{c}_3 - \mathbf{c}_4)\end{bmatrix} \quad \in \mathbb{R}^7,$$

where

$$\frac{d^2\mathbf{f}_2(\tau)}{d\tau^2} = \left[-\ddot{\mathbf{d}}(\tau)\int_{t_0}^{\tau} \mathbf{d}^T(\sigma)d\sigma - 3\dot{\mathbf{d}}(\tau)\mathbf{d}^T(\tau)\right](\mathbf{c}_3 - \mathbf{c}_4) \\ - \ddot{\mathbf{d}}(\tau)c_5.$$

Assume then that the condition in (6) is not verified, i.e. the direction vector remain constant and equal to $\mathbf{d}(t_0)$ for all $t \in \mathcal{T}$, and let $\mathbf{c}_2 = \mathbf{c}_3 = \mathbf{c}_4 = \mathbf{0}$. It results

$$\mathbf{f}_2(\tau) = \mathbf{c}_1 - \mathbf{d}(\tau)c_5.$$

Since $\mathbf{c}$ is a unit vector one can define $\mathbf{c}_1 = \frac{\sqrt{2}}{2}\mathbf{d}(t_0)$ and $c_5 = \frac{\sqrt{2}}{2}$, therefore

$$\mathbf{f}_2(\tau) = \frac{\sqrt{2}}{2}\mathbf{d}(t_0) - \mathbf{d}(\tau)\frac{\sqrt{2}}{2} = \mathbf{0} \quad \forall t \in \mathcal{T},$$

the Observability Gramian $\boldsymbol{\mathcal{W}}$ is not invertible and the LTV system is not observable. As one may conclude, the condition in (6) has to be true on $\mathcal{T}$ for the LTV system to be observable also on $\mathcal{T}$.

A different procedure is employed to show that (6) is also a sufficient condition. Start by evaluating $\mathbf{f}(\tau)$ at t_0, yielding

$$\mathbf{f}(t_0) = \begin{bmatrix}\mathbf{c}_2 \\ \mathbf{c}_1 - \mathbf{c}_2 - \mathbf{d}(t_0)c_5 \\ \mathbf{d}^T(t_0)(\mathbf{c}_3 - \mathbf{c}_4)\end{bmatrix}.$$

Firstly, two cases may be considered: i) if $\mathbf{c}_2 \neq \mathbf{0}$, then $\|\mathbf{f}(t_0)\| > 0$, and it follows from *Proposition* that $\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0,t_f)\mathbf{c} > 0$; ii) $\mathbf{c}_2 = \mathbf{0}$ and $\mathbf{c}_1 \neq \mathbf{d}(t_0)c_5$ leads again to $\|\mathbf{f}(t_0)\| > 0$, succeeding $\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0,t_f)\mathbf{c} > 0$.

Now, if $\mathbf{c}_3 \neq \mathbf{c}_4$, $\mathbf{f}_3(t_0) \neq \mathbf{0} \Rightarrow \|\mathbf{f}(t_0)\| > 0$, resulting $\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0,t_f)\mathbf{c} > 0$. For further analysis on combinations of possible values of $\mathbf{c}$, one needs the evaluation of $d\mathbf{f}(\tau)/d\tau$ at $\tau = t_0$, which is given by

$$\left.\frac{d\mathbf{f}(\tau)}{d\tau}\right|_{\tau=t_0} = \begin{bmatrix}\mathbf{c}_4 \\ \left[\mathbf{I} - \mathbf{d}(t_0)\mathbf{d}^T(t_0)\right](\mathbf{c}_3 - \mathbf{c}_4) - \dot{\mathbf{d}}(t_0)c_5 \\ \dot{\mathbf{d}}^T(t_0)(\mathbf{c}_3 - \mathbf{c}_4)\end{bmatrix}.$$

Suppose now that $\mathbf{c}_2 = \mathbf{0}$, $\mathbf{c}_1 = \mathbf{d}(t_0)c_5$, for any scalar c_5, and $\mathbf{c}_3 = \mathbf{c}_4$ so that $\mathbf{f}(t_0) = \mathbf{0}$. If $\mathbf{c}_4 \neq \mathbf{0}$, $d\mathbf{f}_1/d\tau \neq \mathbf{0} \Rightarrow \| \left. d\mathbf{f}(\tau)/d\tau\right|_{\tau=t_0} \| > 0$ and it follows from using twice the Proposition introduced in the beginning of this section that $\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0,t_f)\mathbf{c} > 0$. In opposition, if $\mathbf{c}_3 = \mathbf{0} \Rightarrow \mathbf{c}_4 = \mathbf{0}$ one obtains $\left.\frac{d\mathbf{f}(\tau)}{d\tau}\right|_{\tau=t_0} = \begin{bmatrix}\mathbf{0} & -\dot{\mathbf{d}}^T(t_0)c_5 & \mathbf{0}\end{bmatrix}^T$ and $\left.\frac{d^2\mathbf{f}(\tau)}{d^2\tau}\right|_{\tau=t_0} = \begin{bmatrix}\mathbf{0} & -\ddot{\mathbf{d}}^T(t_0)c_5 & \mathbf{0}\end{bmatrix}^T$.
Since $\|\mathbf{c}\| = 1$, and if $\mathbf{c}_1$ had been $\mathbf{0}$ before, then it must be $c_5 = 1$ to hold the fact that $\|\mathbf{c}\|$ is a unit vector, however proceed considering $\mathbf{c}_1 = \mathbf{d}(t_0)c_5$.

If $\dot{\mathbf{d}}(t_0) \neq \mathbf{0}$, $\| \left. d\mathbf{f}(\tau)/d\tau\right|_{\tau=t_0} \| > \mathbf{0}$ and, using *Proposition* twice, $\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0,t_f)\mathbf{c} > 0$. The direction is under a continuous-time function and according to (6) it is true that at least there is a moment, suppose t_1, when the direction derivative can not be zero, i.e. $\dot{\mathbf{d}}(t_1) \neq \mathbf{0}$, resulting $\| \left. d\mathbf{f}(\tau)/d\tau\right|_{\tau=t_1} \| > \mathbf{0}$. Using *Proposition* twice, $\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0,t_f)\mathbf{c} > 0$.

The proof is thus concluded as it is shown that $\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0,t_f)\mathbf{c} > 0$ for all $\|\mathbf{c}\| = 1$, which means that the observability Gramian is invertible and as such (4) is observable. ■

C. Kalman Filter

Section III-A introduced a LTV system for source localization based on direction and Doppler measurements and its observability was studied in Section III-B. In particular, it was shown that the LTV system (3) is observable if (6) is satisfied. The Kalman filter follows as the natural estimation solution and, as it is widely known, its design is omitted in this paper. The system dynamics, including additive system disturbances and sensor noise, can be written as

$$\begin{cases}\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{w}(t) \\ \mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{n}(t)\end{cases},$$

where $\mathbf{w}(t) \in \mathbb{R}^{13}$ is zero-mean white Gaussian noise, with $E[\mathbf{w}(t)\mathbf{w}^T(t-\tau)] = \boldsymbol{\Xi}\delta(\tau)$, $\boldsymbol{\Xi} \succ \mathbf{0}$, $\mathbf{n}(t) \in \mathbb{R}^7$ is zero-mean white Gaussian noise, with $E[\mathbf{n}(t)\mathbf{n}^T(t-\tau)] = \boldsymbol{\Theta}\delta(\tau)$ and $\boldsymbol{\Theta} \succ \mathbf{0}$. The noises are uncorrelated, therefore $E[\mathbf{w}(t)\mathbf{n}^T(t-\tau)] = \mathbf{0}$. Finally, the case considered here is the common one, where the noises are additive, however, that might not be the reality and thus the proposed solution is sub-optimal.

IV. SIMULATION RESULTS

This section presents the simulation results for the source localization problem in order to evaluate the performance of the proposed solutions. Since (6) must be satisfied so that the system is observable, one must add some kind of persistent excitation capable of changing the evolution of direction in the course of time. Therefore, since the behavior of the source is stochastic in most cases, the aforementioned excitation should be properly added as part of the agent movement. The resulting trajectories are depicted in Fig. 2. The initial position of the agent and the source were set

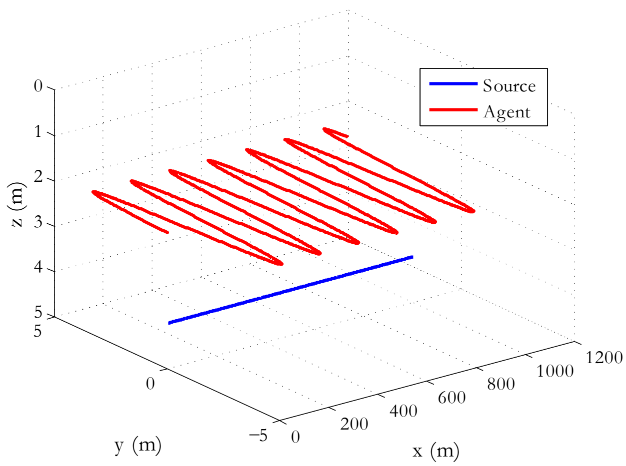


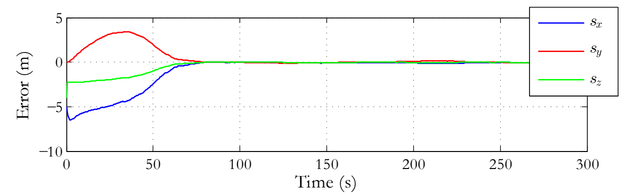
Fig. 2: Source and agent trajectories.

to $\mathbf{p}(0) = [0\ 0\ 2]^T$m and $\mathbf{s}(0) = [0\ 0\ 4]^T$m, respectively. The source velocity is assumed to be constant and the agent velocity is based on a sinusoidal evolution along the longitudinal axis. Note that despite a sinusoidal behavior, the inertial acceleration of the agent $\dot{\mathbf{v}}_p$ is indeed close to $\mathbf{0}$, whereby the assumption of $\mathbf{v}_p$ being constant is not overridden.

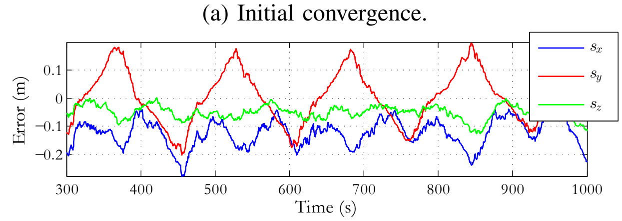
Noise was considered for both the directions measurements and the Doppler quantities. Notwithstanding, note that according to (5), since three entries of the measurements are considered to be zero, noise can be left outside in what concerns those same measurements as a consequence of a linearization manipulation. Thereby, zero-mean additive Gaussian noise was considered for the Doppler measurements, with standard deviation of 0.01m/s. The direction readings were assumed perturbed by rotations about random vectors of an angle modelled by zero-mean white Gaussian noise, with standard deviation of $1°$. Zero-mean additive Gaussian noise with standard deviation of 0.1m was considered for $\mathbf{p}(t)$.

The Kalman filter parameters were set to $\mathbf{\Xi} = \text{diag}(10^{-5}\mathbf{I}, 10^{-5}\mathbf{I}, 10^{-5}\mathbf{I}, 10^{-1}\mathbf{I}, 10^{-3})$ and $\mathbf{\Theta} = \mathbf{I}$. The initial estimates were all set to zero.

The evolution of the estimation error for the source position is depicted in Fig. 3 and for the source velocity is depicted in Fig. 4, along with their respective detailed evolutions. Fig. 5 presents the same results for the agent velocity. With the chosen filter parameters, the transients in both figures quickly fade out and the estimation error converges to zero with small deviations, proving the efficiency of the proposed solution.

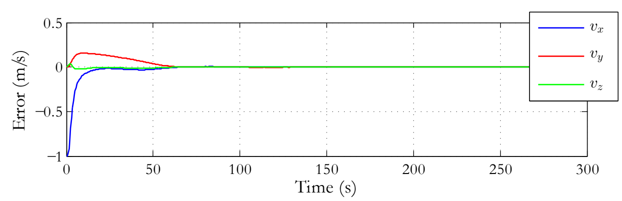


(a) Initial convergence.

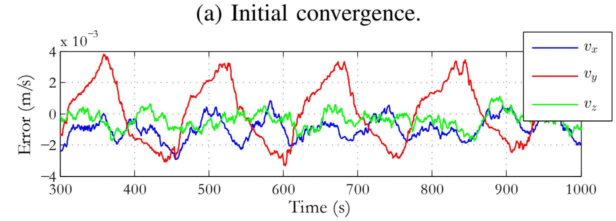


(b) Detailed evolution.

Fig. 3: Evolution of the source position estimation error.



(a) Initial convergence.



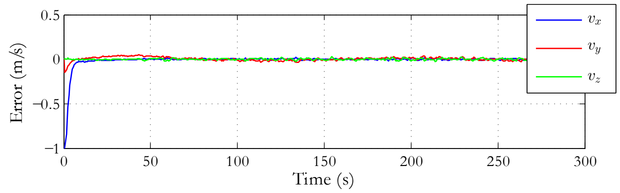
(b) Detailed evolution.

Fig. 4: Evolution of the source velocity estimation error.

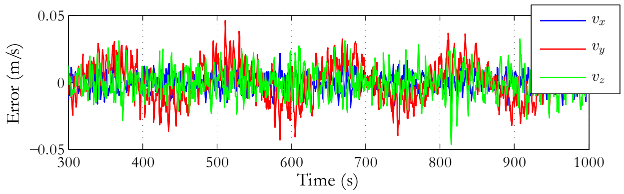
So far the source was considered to move with linear velocity. However, some marine species of reduced dimensions possess a unique way of motion characterized by constant oscillations with high frequency. When $\mathbf{v}_s$ is updated to a more realistic behavior, the estimations do not follow up to the references, which means the filter has no capability of processing such fast variations of amplitude. The solution to this problem consists in improving the dynamics model of the velocity.

The source velocity oscillations can be interpreted as an output of a second order oscillator. The latter could be in fact the output of a narrow band-pass filter when injected with white noise. Recalling the band-pass filter transfer function, it follows

$$\frac{\mathbf{V_s}(s)}{W(s)} = k\frac{2\xi\omega_s s}{s^2 + 2\xi\omega_{s,y}s + \omega_s^2}. \tag{7}$$

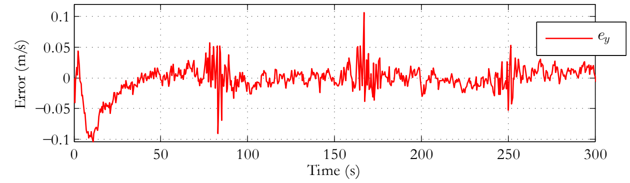


(a) Initial convergence.

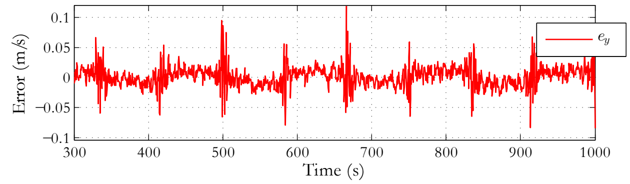


(b) Detailed evolution.

Fig. 5: Evolution of the agent velocity estimation error.



(a) Initial convergence.



(b) Detailed evolution.

Fig. 6: Evolution of the source velocity estimation error.

where ξ is the dumping factor, ω_n is the natural frequency of the oscillations, k is the filter gain and $\mathbf{V_s}(s)$ is the Laplace transform of $\mathbf{v_s}(t)$. As for the Laplace transform of the noise, $W(s)$, it was assumed additive noise in order to maintain the system with linear properties. Presupposing now that only the y-component of the source velocity is mutable, converting (7) into a time domain formalism yields

$$\ddot{v}_{\mathbf{s},y}(t) + 2\xi\omega_{\mathbf{s},y}\dot{v}_{\mathbf{s},y}(t) + \omega_{\mathbf{s},y}^2 v_{\mathbf{s},y}(t) = 2k\xi\omega_{\mathbf{s},y}\dot{w}(t). \quad (8)$$

The noise derivative is likely to be avoided. The same is valid for the derivative of the acceleration, i.e. $\ddot{v}_{\mathbf{s},y}(t)$. Integrating both sides of (8) returns

$$\dot{v}_{\mathbf{s},y}(t) + 2\xi\omega_{\mathbf{s},y}v_{\mathbf{s},y}(t) + \omega_{\mathbf{s},y}^2 s_y(t) = 2k\xi\omega_{\mathbf{s},y}w(t).$$

The dumping factor is a pseudo-indicator of how perfect the oscillations are, whereby it can be left as $\xi = 0.005$. As for the natural frequency of oscillations it was speculated a value around 0.05Hz. k is a rather ambiguous parameter and it can be interpreted as a gain adjustment.

After fine-tuning of the filter parameters, new simulation results were obtained. The evolution of the estimation error can be found in Fig. 6, while the evolution of the estimate can be seen in Fig. 7 along with the comparison with the real value evolution.

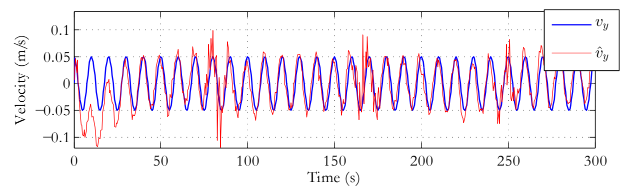


Fig. 7: Source velocity: true vs. estimated evolutions.

V. CONCLUSIONS

This paper presented a novel time-varying Kalman filter with globally asymptotically stable error dynamics for the problem of localization based on direction and Doppler measurements to a single source. The observability of the system was fully characterized, which allowed to conclude about the asymptotic stability of the filter. Simulations results were presented that illustrate the good performance achieved by the proposed solution. Future work includes sea tests under the scope of the MAST/AM project.

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