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# Online Model Identification for Set-valued State Estimators With Discrete-Time Measurements

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**Abstract:** Although Kalman Filters provide an optimal solution to the state estimation problem for linear systems, they require knowledge of an accurate description of the model of the system. More robust approaches to Kalman Filtering for systems with uncertain models have been developed, such as set-valued state estimators, which identify the set of possible states that the system may be in, given a description of its uncertainties. The theory behind this class of filters contemplates the possibility of determining if a given system model is feasible with respect to its measurements, but does not explore the possibility of explicitly identifying the system online. This work provides insight into the advantages of performing simultaneous model identification for the class of set-valued estimators, by introducing an Adaptive Set-Valued Estimator, and presenting a practical example of its usage. The results of this robust estimator are then compared to those obtained when using a classical set-valued estimator.

*Keywords:* Kalman filters, Robust estimation, Parameter identification, Adaptive control, Nonlinear systems.

## 1. INTRODUCTION

Kalman Filters are a very significant and well-established development in the field of systems and control theory. Given a linear system affected by stochastic noise, the Kalman Filter is able to provide the optimal estimates of the state of the system based on gathered noisy observations. However, this class of filters is not robust to large unmodeled variations in the model of the system. In this case, optimality is lost, stability is not guaranteed, and the performance of the filter can degrade considerably. Since, in practice, there are situations where it is difficult to estimate the model of the system reliably, or it may possess parameters which are impossible to fully specify beforehand, there is a need for estimators which are robust to this type of variations, even if they are suboptimal. Examples of previously proposed robust filters include Guaranteed Cost State Estimators (Savkin and Petersen (1999)), Set-Valued State Estimators (Savkin and Petersen (1999, 1995); A. Savkin and Moheimani (1999)) , robust $H^{\infty}$ filters (Rangan and Poolla (1995); Smith and Dullerud (1996); Gao et al. (2005)), adaptive filters such as the Multiple Model Adaptive Estimator (MMAE) (Magill (1965); Anderson and Moore (1979); Athans and Chang (1976)), and filters over alternative metrics such as the squared residual norm (Sayed (2001)). The class of set-valued state estimators, for instance, provides a wide range of functionalities, since it allows robust estimation under dynamic, non-linear uncertainties in the plant and sensor models, by defining a general constraint (Linear Quadratic Constraint) over these uncertainties, based on an uncertainty description introduced in the work of Yakubovich (see Yakubovich (1988)). This class of estimators also allows for on-line model validation, and is suitable for applications involving mixed continuous/discrete measurements (or discrete at different rates), even in the case of missing data.

This work studies the application of a set-valued state estimator in a simulated realistic scenario, and presents results that demonstrate that it is possible to obtain reliable state estimates in the presence of model uncertainty. Although the authors of this class of estimators explicitly considered the problem of model validation, i.e. to determine if a given model is compatible with the controls applied to the system and the resulting measurements produced by it, its application is restricted to the problem of obtaining a suitable set of possible system models. In practice, it is advantageous to also produce an on-line estimate of the true system model, since this will evidently reduce the amount of uncertainty present in the state estimate. This issue is also explored in this work, by inspecting the behavior of the estimator if the uncertainty of the system is re-evaluated online. This is accomplished by resorting to a bank of parallel set-valued estimators, and evaluating the probability of correctness of each of the associated models, using the resulting information to infer updated bounds on the uncertainty of the system. The results with respect to state estimation of such an adaptive set-valued estimator are then compared to those of a regular set-

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valued estimator, noting that while the latter uses solely a deterministic description of system uncertainty, the former also requires a stochastic one. The impact of this fact on filter usability is then discussed.

## 2. HYBRID SET-VALUED STATE ESTIMATION

This section introduces the basic concepts and techniques behind set-valued state estimators (see Savkin and Petersen (1999) for more details). Consider the linear uncertain system described by equations (1)-(5), where $x(t)$ is the state of the system, $u(t)$ is a deterministic control input, $w(t)$, $v_c(t)$, $v_d(t)$ are "uncertainty inputs", $y_c(t)$, $y_d(t_j)$ are the measured outputs of the system, $z_c(t)$, $z_d(t_j)$ are the "uncertainty outputs" of the system, $A(t)$, $B_1(t)$, $B_2(t)$, $K_c(t)$, $G_c(t)$, $C_c(t)$ are bounded piecewise continuous matrix functions, and $K_d(t_j)$, $G_d(t_j)$, $C_d(t_j)$ are matrix sequences.

$$\dot{x}(t) = A(t)x(t) + B_1(t)w(t) + B_2(t)u(t) \tag{1}$$
$$z_c(t) = K_c(t)x(t) + G_c(t)u(t) \tag{2}$$
$$z_d(t_j) = K_d(t_j)x(t_j) + G_d(t_j)u(t_j) \quad \forall j = 1, ..., k \tag{3}$$
$$y_c(t) = C_c(t)x(t) + v_c(t) \tag{4}$$
$$y_d(t_j) = C_d(t_j)x(t_j) + v_d(t_j) \quad \forall j = 1, ..., k \tag{5}$$

Note that the system definition is general enough to include the possibility of both continuous and discrete-time measurements, and could be easily extended to include multiple assynchronous discrete-time sources of information. The uncertainty in this system is assumed to verify that, within a time interval $]0, s]$, for given initial conditions $x_0$, and for some constant $d$:

$$\begin{aligned}(x(0) - x_0)^T P_0^{-1}(x(0) - x_0) + \int_0^s (w(t)^T Q(t)w(t) + \\ + v_c(t)^T R_c(t)v_c(t))dt + \sum_{t_j \leq s} v_d(t_j)^T R_d(t_j)v_d(t_j) \leq \\ \leq d + \int_0^s ||z_c(t)||^2 dt + \sum_{t_j \leq s} ||z_d(t_j)||^2\end{aligned} \tag{6}$$

Where $P_0$ is an appropriate positive definite matrix, and $Q(t), R_c(t), R_d(t_j)$ are positive semi-definite. The above is known as an Integral Quadratic Constraint. Intuitively, it imposes an upper bound on the uncertainty introduced by $w(t), v_c(t), v_d(t)$, as defined by linear functions $z_c(t), z_d(t_j)$, which converge if the overall system is stable. This allows the uncertainty inputs to be defined as possibly time-varying and non-linear. It also implies that a suitable description for this type of uncertainty is by establishing norm bounds upon each of the uncertain terms of the model. It is easy to remark that, assuming that the uncertainty inputs satify the above constraints for every possible state, then the set of possible states at the initial instant is an ellipsoid whose shape is defined by $P_0$ and $d$. If this set is bounded for every possible $x_0$, $d$, and every history of inputs $u(t)|_0^s$ and measurements $y_c(t)|_0^s, y_d(t)|_0^s$, then the system is said to be "verifiable" (Savkin and Petersen (1999)) . If such is the case, the theory behind set-valued state estimators allows for the possible state ellipsoid to be propagated into any given time instant, and it can be shown that the set remains an ellipsoid under these circumstances (see Savkin and Petersen (1999)).

The inclusion of both continous and discrete time components in the model of the system requires the following auxiliary definition:

$$f(t_j^-) = \lim_{t \to t_j, t < t_j} f(t) \tag{7}$$

With this in mind, and following Savkin and Petersen (1999), the design of a set-valued state estimator relates to the following equations:

$$\begin{aligned}\dot{P}(t) = A(t)P(t) + P(t)A(t)^T + B_1(t)Q(t)^{-1}B_1(t)^T + \\ + P(t)\left(K_c(t)^T K_c(t) - C_c(t)^T R_c(t)C_c(t)\right)P(t)\end{aligned} \tag{8}$$

$$\begin{aligned}P(t_j) = \big(P(t_j^-)^{-1} + C_d(t_j)^T R_d(t_j)C_d(t_j) - \\ - K_d(t_j)^T K_d(t_j)\big)^{-1} \quad \forall j = 1, ..., k\end{aligned} \tag{9}$$

These equations are known as *jump Ricatti equations*, owing to their possible discontinuities at the sampling instants $t_j$. It is shown that if equations (8),(9) have a positive definite solution with initial condition $P(0) = P_0$, then the system is verifiable. The estimate of the system's state may then be taken as the center of the ellipsoid of possible states, which has the following update equations:

$$\begin{aligned}\dot{\hat{x}}(t) = \left(A(t) + P(t)K_c(t)^T K_c(t)\right)\hat{x}(t) + \\ + P(t)C_c(t)^T R_c(t)r_c(t) + \\ + \left(B_2(t) + P(t)K_c(t)^T G_c(t)\right)u(t)\end{aligned} \tag{10}$$

$$\begin{aligned}\hat{x}(t_j) = \left(I + P(t_j^-)K_d(t_j)^T K_d(t_j)\right)\hat{x}(t_j^-) + \\ + P(t_j^-)C_d(t_j)^T R_d(t_j)r_d(t_j) + \\ + P(t_j^-)K_d(t_j)^T G_d(t_j)u(t_j) \quad \forall j = 1, ..., k\end{aligned} \tag{11}$$

where $r_c(t) = y_c(t) - C_c(t)\hat{x}(t)$ and $r_d(t_j) = y_d(t_j) - C_d(t_j)\hat{x}(t_j^-)$ are the residual errors between predicted and gathered observations.

If the system is verifiable, then another quantity may be defined, which can be seen as a generalized error between the uncertainty outputs of the system and the uncertainty effectively generated when receiving observations $y_d(t_j)$,$y_c(t)$:

$$\begin{aligned}&\rho_s(u, y_d, y_c) = \\ &\int_0^s \left\{||K_c(t)\hat{x}(t) + G_c(t)u(t)||^2 - r_c(t)^T R_c(t)r_c(t)\right\}dt + \\ &+ \sum_{t_j \leq s} \big\{||K_d(t_j)\hat{x}(t_j) + G_d(t_j)u(t_j)||^2 - \\ &- r_d(t_j)^T R_d(t_j)r_d(t_j)\big\}\end{aligned} \tag{12}$$

It can then be shown that the set of possible states that the system may be in, at any time instant $s$, is given by:

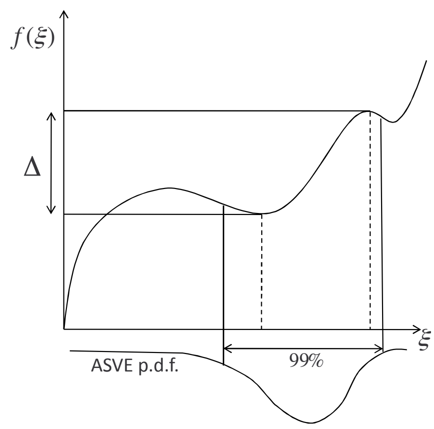


Fig. 1. Estimation of the norm bounds of the uncertain parameters from the ASVE user weight estimate.

$$X(s, x_0, d, u(t)|_0^s, y_c(t)|_0^s, y_d(t)|_0^2) = \{x \in \Re^n : (x - x_0)^T P^{-1}(s)(x - x_0) \leq d + \rho_s(u, y_d, y_c)\} \quad (13)$$

It is also evident that this set is only non-empty if $\rho_s(u, y_d, y_c) \geq -d$, in which case the system is said to be "realizable" (Savkin and Petersen (1999)), which intuitively means that, for the given parameter uncertainties, this set of states is reachable through $u(t)|_0^s$, $y_c(t)|_0^s$, $y_d(t)|_0^s$, from the initial conditions $x_0$. The process of model validation refers to using this property to ascertain if a given system model is realizable. In contrast, state estimation in this class of filters requires obtaining the set (13) from the premise that the system is in fact realizable.

It is clear from equations (2),(3), that this type of estimators allows for dynamic re-evaluation of the uncertainty over the system's parameters, by adjusting matrices $K_c(t), K_d(t_j), G_c(t), G_d(t_j)$ accordingly. If these are such that the norm of the uncertainty outputs is minimized, then the set of possible states will also be miminized through (13). To the best knowledge of the authors, no methodology has been proposed to tackle the system identification problem for this type of set-valued observers. The next section proposes a novel joint system identification and state estimation framework that allows for an estimate of the parameters of the system to be obtained online.

## 3. ADAPTIVE SET-VALUED ESTIMATION

Given the theory so far described, it is simple to determine if a given model is realizable or not by checking if the resulting set of possible states is non-empty. However, in general it is not straightforward to determine the set of parameters that generate realizable models. If a realizable model is known, it is advantageous to be able to refine it, by estimating the uncertainty of the system online.

The technique for system identification here introduced, Adaptive Set-Valued Estimation (ASVE), is deeply rooted on the Multiple Model Adaptive Estimation (Magill (1965); Anderson and Moore (1979); Athans and Chang (1976)), but applied to the framework of set-valued estimation. In this technique, a bank of set-valued estimators is implemented, and to each of these parallel estimators a hypothetic value of the unknown parameters is assigned.

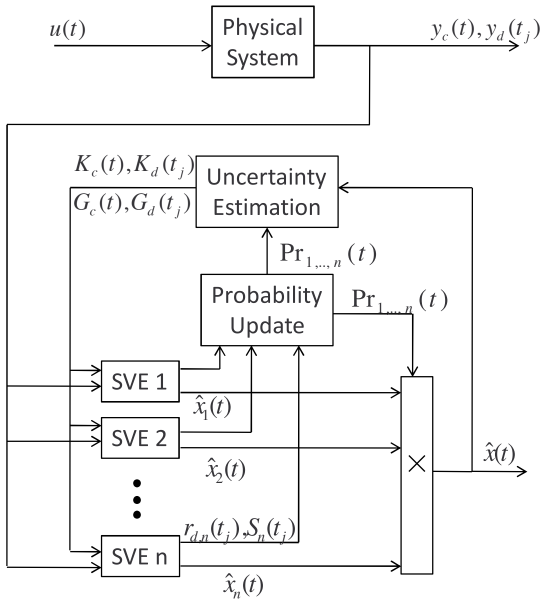


Fig. 2. Architecture of an Adaptive Set-Valued Estimator.

Given enough filters to cover a region of interest in the unknown parameter space, an a posteriori probability measure is then assigned to each of these estimators, by inferring over the collected data and respective state estimates, which represents the likelihood of each of the observers being the correct one, in the sense that they more accurately describe the true system model. The discussion on the number and on the assignment values for the unknown parameters is out of the scope of this paper, although it is an important problem for this kind of multiple-model estimators Fekri et al. (2006). The interested reader is referred to Baram (1976), for one possible systematic procedure to tackle this problem. The estimate of the state of the system is then taken as the expected value of this probability density function (p.d.f.), from which the state covariance can also be obtained.

The *a posteriori* probability over the bank of $N$ filters is updated, for each filter $i$, by:

$$\Pr_i(t_{j+1}) = \frac{\beta_i(t_{j+1}) e^{-\frac{1}{2}\mathrm{w}_i(t_{j+1})}}{\sum_{k=0}^{N} \beta_k(t_{j+1}) e^{-\frac{1}{2}\mathrm{w}_k(t_{j+1})} \Pr_k(t_j)} \Pr_i(t_j) \quad (14)$$

where, given the residual $r_i(t_{j+1}) \equiv y(t_{j+1}) - \hat{y}_i(t_{j+1}|t_j)$ of each filter, and the covariance over this residual, $S_i(t_{j+1}) \equiv cov\{r_i(t_{j+1}), r_i(t_{j+1})\}$, the relevant weighting terms are defined as:

$$\beta_i(t_{j+1}) = \frac{1}{2\pi\sqrt{\det S_i(t_{j+1})}} \quad (15)$$

$$\mathrm{w}_i(t_{j+1}) = r_i(t_{j+1})^T S_i(t_{j+1})^{-1} r_i(t_{j+1}) \quad (16)$$

Note that, in order to obtain the residual covariance for each of the estimators, it is necessary to maintain an estimate of the state covariance as well. This implies that a stochastic description of the system must also be provided, assuming, for each estimator, that the only sources of uncertainty are sensor and process noises (i.e. that their local parameters are exact). The uncertainty

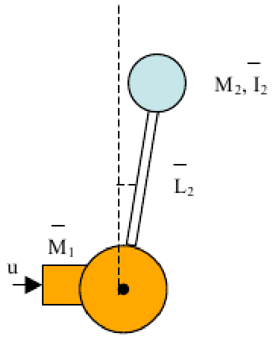


Fig. 3. Representation of the Human Transportation System.

inputs of the system are then dynamically re-calculated by approximating the probability density function over its filter bank as a Normal distribution, and taking the norm bound of the dependent parameters by establishing an appropriate confidence interval over them. This process is depicted in Fig. 1. These updated norm bounds are then fed back into the ASVE, influencing the values of $P(t)$ and $\rho(t)$, and therefore the set of possible states for the system.

## 4. RESULTS

The application domain here considered is a simulation of the dynamics of a Human Transportation System (HTS), for which the control problem is basically the stabilization of an inverted pendulum. Consider the configuration of the HTS, present in fig. 3, in which its main physical characteristics are presented. A user riding the HTS is represented by a point mass of $M_2$ at a height $\bar{L}_2$, resulting in an inertia $\bar{I}_2$. The HTS itself possesses mass $\bar{M}_1$. A force $u(t)$ is applied by its wheels to keep the system stable. It is assumed that there are sensors installed onboard able to measure its angular position, $\theta(t)$ (inclinometer), and angular velocity, $\dot{\theta}(t)$ (rate-gyro).

The dynamics of the HTS can be modelled as the nonlinear system:

$$\ddot{\theta} = \frac{1}{m(M_2, \theta)} \left( u - \left( M_2 + \bar{M}_1 \right) g \tan(\theta) + M_2 \bar{L}_2 \sin(\theta) \dot{\theta}^2 \right) \tag{17}$$

where $g = 9.81ms^{-2}$, and

$$m(M_2, \theta) = M_2 \bar{L}_2 \cos(\theta(t)) - \frac{\left( M_2 + \bar{M}_1 \right) \left( M_2 \bar{L}_2^2 + \bar{I}_2 \right)}{M_2 \bar{L}_2 \cos(\theta(t))} \tag{18}$$

However, for small angles, these dynamics can be approximated by the linear system:

$$\begin{bmatrix} \dot{\theta}(t) \\ \ddot{\theta}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \alpha(M_2) & 0 \end{bmatrix} \begin{bmatrix} \theta(t) \\ \dot{\theta}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \beta(M_2) \end{bmatrix} u(t) \tag{19}$$

where $\alpha(M_2)$, $\beta(M_2)$ are nonlinear functions of parameter $M_2$ (their dependence on $M_2$ will be ommited, for the sake of compactness). The latter is, therefore, the source of uncertainty in the model of the system. It is assumed that the admissible user weights lie in the $M_2 \in [50, 100]kg$ range. As an uncertain system, the HTS can then be approximated by:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ \alpha & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u(t) + w(t) \tag{20}$$

$$z_c(t) = \begin{bmatrix} 0 & 0 \\ k_\alpha(t) & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ k_\beta(t) \end{bmatrix} u(t) \tag{21}$$

$$y_d(t_j) = x(t_j) + v_d(t_j) \tag{22}$$

with the following additional norm constraint for $w(t)$:

$$w(t) = \Delta z_c(t) \qquad ||\Delta|| \leq 1 \tag{23}$$

The observation noise $v_d(t_j)$ is assumed to be white, Gaussian with zero mean and uncorrelated with the other sources of uncertainty. The initial conditions are assumed to be such that:

$$N = P(0)^{-1} = \begin{bmatrix} (\pi/180)^2 & 0 \\ 0 & (\pi/180)^2 \end{bmatrix} \tag{24}$$

$$x(0) = [\, 0.3 \ \ 0 \,]^T \tag{25}$$

$$d = 1 \tag{26}$$

The matrix weighting functions are assumed to be $Q(t) = I$,$R_d(t_j) = 5I$. The sampling period of the sensors is assumed to be $T = 0.01s$.

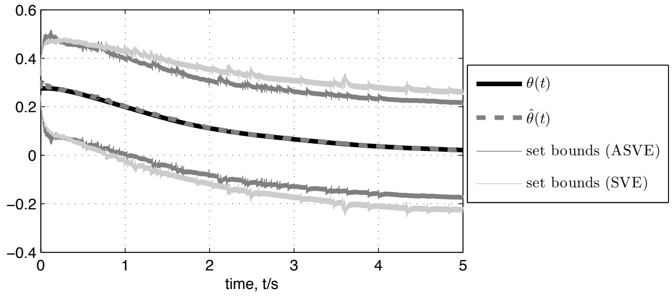


Fig. 4. Evolution of the angular position of the system and respective ASVE output, for a user of $M_2 = 90$ Kg. The bounds denote projections of the limit of the set of possible states, for the ASVE, and also for a regular SVE without system identification.

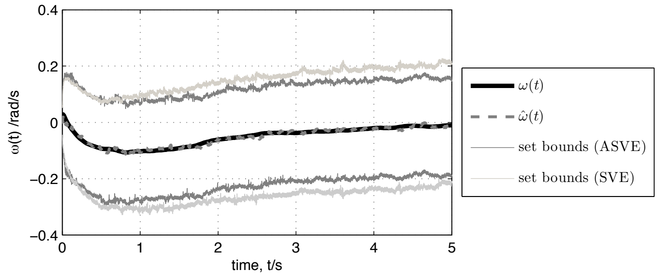


Fig. 5. Evolution of the angular velocity of the system and respective ASVE output, for a user of $M_2 = 90$ Kg. The bounds denote projections of the limit of the set of possible states, for the ASVE, and also for a regular SVE without system identification.

Results were taken from the proposed experimental setup by using MATLAB/Simulink to implement the system. In figs. 4 and 5, the evolution of the state variables is shown, along with the estimate obtained through the ASVE and the set bounds, for a situation where there are no external disturbances. For comparison, the bounds returned by a classical, independent set-valued estimator are also displayed. From these, it can be seen that the ASVE is able to obtain smaller bounds on the possible state of the system, and that it is able to correctly track the true value of the state.

In fig. 6, a representation of the convergence of the probability density function over the filter bank to the correct user weight is shown. From these, it is possible to establish that the parameters $k_\alpha$ and $k_\beta$ of the uncertainty output may also be dynamically estimated by the ASVE. These results are shown in fig 7. These estimates eventually converge, since the p.d.f over the filter bank also stabilizes towards a final form. The final estimate for the unknown parameters depends intrinsically on the weighing matrices $R_d$ and $Q_d$. A careful balance must exist in such an application domain regarding these weighing terms. This is due to the fact that the open-loop plant dynamics are unstable within the region of interest of the unknown parameters. Since the system only performs measurements at discrete time instants, the amount of information retrieved from these measurements (influenced by $R_d$) must be such that the system is able to compensate its unstable dynamics between two measurements. In the absence of continuous-time measurements, a larger weight $R_d$ also means that the eigenvalues of $P(t)$ will be able to converge to smaller values, which result in a smaller set of possible states for the system to be in. However, increasing the relative weight of these observations will mean that the residuals produced by each of the individual set-valued estimators in the filter bank will tend to become similar, thereby affecting the speed at which the p.d.f. over the unknown parameters converges, as well as its final shape, and limiting the improvement over the estimate of the uncertainty of the system.

Note that there is also lower saturation on these estimates, which corresponds to the resolution of the ASVE with respect to the weight of the user. Given a aribitrarily large filter bank, if the uncertainty affecting the model is assumed constant, then these parameters would decay to 0, eliminating all associated uncertainty since the exact model of the system would be identified. Even though, in this particular example, the unknown parameter (the weight of the user) is held constant, it should be pointed out that time-varying uncertainties could also be dealt with, which in fact constitutes one of the most relevant qualities of set-valued estimation.

To make the test more realistic, a sinusoidal disturbance was then added to the physical output of the system. The results in Fig. 8 show that the discrepancy between the set bounds of the ASVE and those of the regular set-valued estimator are more evident. This is due to the fact that, since the estimate of the state of the system is lagging with respect to its true value, the system produces larger residuals, and in this case the p.d.f. associated with the filter bank of the ASVE is able to converge more efficiently, and reduce the norm bounds placed upon the uncertainty

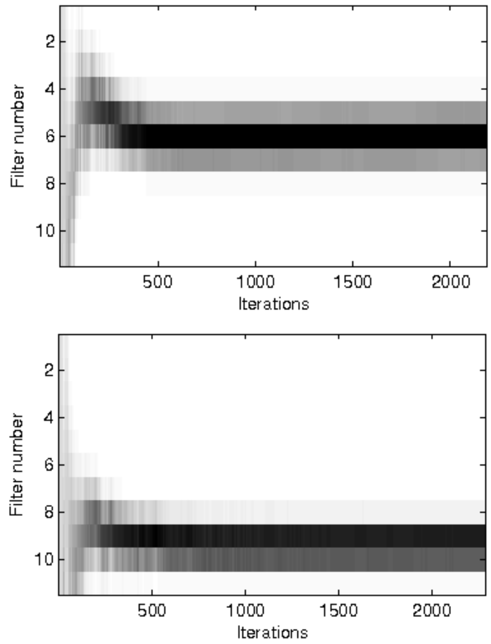


Fig. 6. Convergence of the ASVE probability density function. Each vertical line in the images represents the p.d.f. over the bank of filters for a given iteration. Darker colors are associated with higher probability. Top: Real user weight is 75 Kg (class 6). Bottom: Real user weight is 90 Kg (class 9).

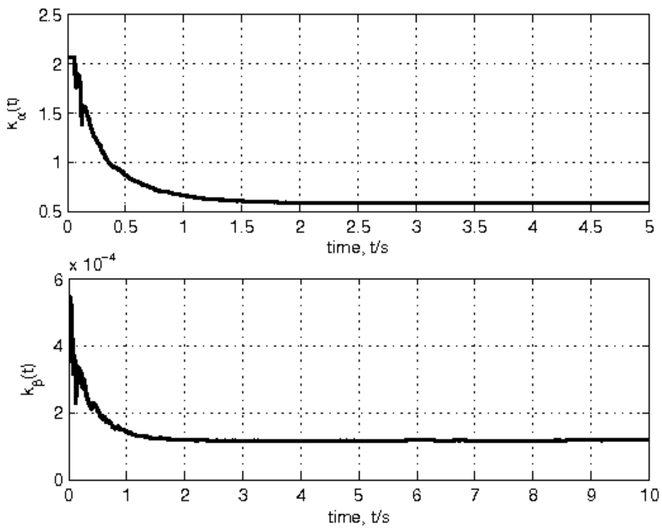


Fig. 7. Evolution of the estimate of the uncertain parameters $k_\alpha$ (top) and $k_\beta$ (bottom), during system identification.

of the system to a further extent. This improvement on the identification results is not surprising and can be interpreted as the well known advantages of persistent excitation techniques used commonly in adaptive system identification methods. It should be noted that, as seen in the beginning of this section, the main limitation to the minimum size of the possible state bounds lies in the fact that this system is open-loop unstable, and only provides measurements at discrete-time instants. In situations where the plant of the system is stable, or where there are hybrid discrete-continuous devices supplying

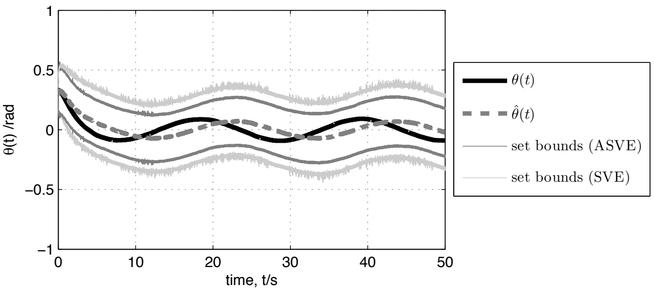


Fig. 8. Evolution of the angular position of the system and respective ASVE output, for a user of $M_2 = 90$ Kg, and forcing an external sinusoidal disturbance upon the output with amplitude 0.1 rad and frequency 0.3 rad/s. The bounds denote projections of the limit of the set of possible states, for the ASVE, and also for a regular SVE without system identification.

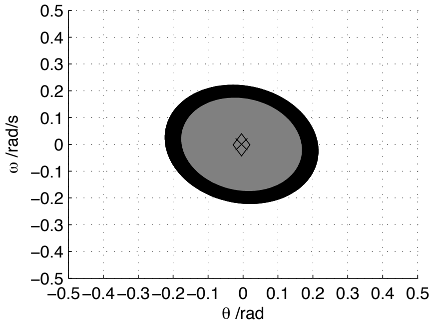


Fig. 9. Representation, for a single time instant $t = 10$ s, and for a user weight $M_2 = 90$ Kg, of the shape of the possible states ellipsoid, when using an ASVE (grey) and a classical SVE without system identification (black).

information about the state of the system, the estimator would be able to provide less conservative bounds.

The effect of performing online model identification upon the set of possible states at each instant is graphically exemplified in Fig. 9. It can be seen that, for a given time instant, the set of possible states is considerably reduced (reducing semi-axis length by approximately 20%) by dynamically reducing the uncertainty norm bounds, as expected.

## 5. CONCLUSION

The main advantage of set-valued estimation lies in its deterministic description of uncertainty, since its associated Integral Quadratic Constraint allows for a large class of uncertainties to be modeled, while maintaining the possibility of using a format similar to a Kalman Filter. This allows the set-valued estimator to be more general than when compared to, for example, the MMAE, which would not handle well time-varying uncertain parameters without at least a proper model to describe its evolution. In this work, an extension to the framework of set-valued estimation was proposed, the Adaptive Set-Valued Estimator, which draws upon the main concepts of multiple-model adaptive estimation, while retaining the advantage of being able to describe parameter uncertainty in a deterministic manner. Although a stochastic description of system noise is also necessary, this does not impact the functionality of the ASVE with respect to uncertainty introduced by unknown parameters, the main difficulty of robust estimation.

The ASVE was shown to be able to dynamically reduce the norm bounds placed upon the unknown parameters, which in turn result in smaller sets of possible states for the system. This was seen to be valid even for a system which is unstable with respect to its open-loop dynamics, and which is only able to provide measurements at discrete time instants. In this respect, the ASVE was shown to be superior to a classical set-valued estimator, since the uncertainty being introduced into the system is effectively reduced online. This prompts the suggestion that robust estimation should be accompanied by model identification whenever possible.

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