PCA Positioning Sensor Characterization for Terrain Based Navigation of UVs

Paulo Oliveira*

IST/DEEC and ISR, Torre Norte 8^o andar
Av. Rovisco Pais, 1, 1049 – 001 Lisbon, Portugal
pjcro@isr.ist.utl.pt
http://www.isr.ist.utl.pt/~pjcro/indexi.html

Abstract. Principal Component Analysis has been recently proposed as a nonlinear positioning sensor in the development of tools for Terrain Based Navigation of Underwater Vehicles [10]. In this work the error sources affecting the proposed unsupervised methodology will be enumerated, the stochastic characterization will be studied, and the attainable performance will be discussed. Based on a series of Monte Carlo experiments for a large set of synthesized terrains, conclusions will be drawn on the adequacy of the proposed nonlinear approach.

1 Introduction

Navigation systems design for long range missions of underwater vehicles (UVs) in unstructured environments, without resorting to external sensors, and with bounded error estimates, has been a major challenge in underwater robotics [6]. Unmodelled dynamics, time-varying phenomena, and the noise present in the sensor measurements continuously degrades the navigation system accuracy along time, precluding its use in a number of interesting applications. To overcome this limitation, external positioning systems have been proposed and successfully operated in the past, as extensively enumerated in [13], and integrated in navigation systems for underwater applications, as reported in the design examples found in [1, 14]. Unfortunately, all those positioning systems only locally provide accurate measurements (a few square kilometers), take long time to deploy, and are hard to calibrate, strongly constraining the area where the missions can take place, and ultimately the use of UVs.

One alternative central to this work has been exploited in the past: in the case where the missions take place in areas where detailed bathymetric data are available, the terrain information can aid to bound the error estimates of the navigation systems leading to Terrain Based, Terrain Reference, or Terrain Aided Navigation Systems. Applications with relative success have been reported in the past for air [2, 4], land [3] and underwater [5, 12] robotic platforms.

Extended Kalman Filtering has been the most common synthesis technique to tackle the terrain based navigation system design, as reported in [4, 12] and

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the references therein. However, examples of performance degradation (including instability) on the proposed solutions have been reported by the same authors, precluding their use in general. Correlation techniques [2, 8], image matching techniques [12], and particle filters [5, 11] have also been proposed requiring high computational load, with limited performance and robustness.

This paper tries to elucidate on the adequacy of using unsupervised optimal processing techniques of random signals, namely Principal Component Analysis (PCA) (based on the Karhunen-Loève Transform) [7], to obtain a nonlinear positioning sensor instead of using a nonlinear estimator. The performance of the proposed sensor will be studied in a large set of terrains carefully chosen, providing bounds on the expected performance for the problem at hand.

The paper is organized as follows: in section 2 the sensor package installed onboard is introduced and the underlying estimator structure is briefly described. Section 3 reviews the background on the Karhunen-Loève transform, basis for the principal component analysis of stochastic signals and details on the approach for the bathymetric data decomposition. In section 4 relevant terrains are discussed and the impact of PCA design parameters on the overall performance are enumerated and studied in detail. A stochastic characterization, resorting to a series of Monte Carlo experiments, is presented. Finally, in section 5 some conclusions are drawn on the adequacy of the proposed nonlinear approach and future work is unveiled.

2 UV Sensor Package and Navigation System

2.1 Notation and Sensor Package

Let $\{\mathcal{I}\}$ be an inertial reference frame located at the pre-specified mission scenario origin with North, East, and Down axes (without loss of generality at mean sea level), as depicted in Fig. 1 and let $\{\mathcal{B}\}$ denote a body-fixed frame that moves with an Underwater Vehicle (UV). The vehicle will be equipped with an Attitude and Heading Reference System (AHRS) that provides measurements of the attitude $\lambda = [\phi \;\; \theta \;\; \psi]^T$, i. e. the vector of roll, pitch, and yaw angles that parameterize locally the orientation of frame $\{\mathcal{B}\}$ with respect to $\{\mathcal{I}\}$, ${}^{\mathcal{I}}_{\mathcal{B}}\mathcal{R}(\lambda)$ and of the angular velocities expressed in body frame ${}^{\mathcal{B}}({}^{\mathcal{I}}\omega_{\mathcal{B}})$, i.e. body-fixed angular velocity. Note that since $\mathcal{R}$ is a rotation matrix, it satisfies the orthogonality condition $\mathcal{R}^T = \mathcal{R}^{-1}$ that is, $\mathcal{R}^T\mathcal{R} = I$. To complement the information available onboard the UV, a Doppler velocity log and a depth cell, providing measurements of ${}^{\mathcal{B}}({}^{\mathcal{I}}\mathbf{v}_{\mathcal{B}})$ and the vertical coordinate z, respectively, are used.

To provide measurements for the PCA based positioning system a sonar ranging sensor is required, with a linear array of beams, where a bearing angle ϵ associated with each of the beams is used. See Fig. 1 in detail, where the seafloor points sensed in several ranging measurements - $d(i)$ - are depicted in red. Assuming, without loss of generality, that the sonar is installed at the origin of the reference frame $\mathcal{B}$ pointing down, and the bearing angle lies in the transversal plane (containing the (y_B, z_B) axes), the i^{th} measurement can be geo-referenced in the inertial reference frame $\mathcal{I}$ using

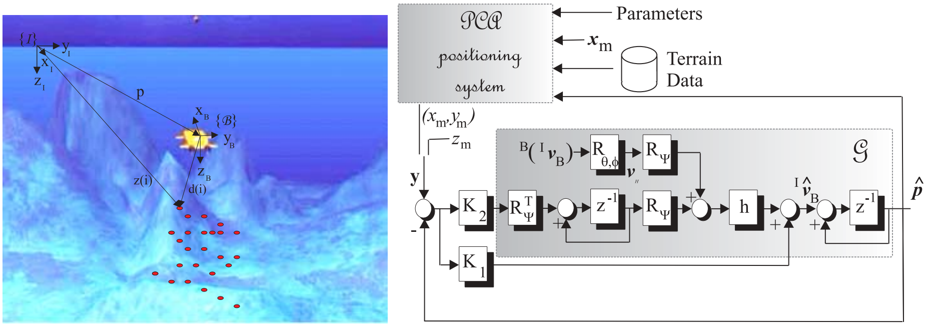

Fig. 1. Left: UV inertial and local coordinate frames. Mechanical scanning sonar range measurements. Right: Block diagram of the nonlinear estimator.

$$z(i) = \mathbf{p} +_{\mathcal{B}}^{\mathcal{I}} \mathcal{R}(\lambda)\mathcal{R}_X(\epsilon)[0\ \ 0\ \ d(i)]^T, \tag{1}$$

where $\mathcal{R}_X(\epsilon)$ is the rotation matrix, relative to the x_S axis, from the instantaneous sonar bearing to the UV reference frame $\mathcal{B}$ and $\mathbf{p} := [x\ \ y\ \ z]^T$ is the UV position relative to the inertial frame $\mathcal{I}$. It is important to remark that no support from other external systems/devices will be required.

2.2 Navigation System

An estimator for the state estimate $\hat{\mathbf{z}}(k) = [\hat{\mathbf{p}}^T(k)\ \hat{\mathbf{b}}^T(k)]^T$, corresponding to the vector obtained stacking the position estimate $\hat{\mathbf{p}}$ and the bias estimates $\hat{\mathbf{b}}$, due to velocity sensor installation and calibration mismatches assumed constant or slowly varying, can be written resorting to the usual recursions for the Kalman filter:

$$\begin{cases} \hat{\mathbf{z}}^-(k+1) = \mathbf{A}(k)\hat{\mathbf{z}}^+(k) + \mathbf{B}_1(k)\mathbf{v}_H(k) \\ \mathbf{P}^-(k+1) = \mathbf{A}(k)\mathbf{P}^+(k)\mathbf{A}^T(k) + \mathbf{Q}(k), \end{cases} \tag{2}$$

where h is the sampling period, k describes in compact form the time instant $t_k = kh$, for $k = 0, 1, \ldots, T$ (the final mission time), $\hat{\mathbf{z}}^-(k+1)$ is the predicted state variable estimate, and $\mathbf{P}^-(k)$ is the covariance of the prediction estimation error, as detailed in [10]. Given a PCA position measurement, the state and error covariance updates, $\hat{\mathbf{z}}^+(k)$ and $P^+(k)$, respectively, will be given by

$$\begin{cases} \hat{\mathbf{z}}^+(k) = \hat{\mathbf{z}}^-(k) + \mathbf{K}(k)(y - \mathbf{C}(k)\hat{\mathbf{z}}^-(k)) \\ \mathbf{P}^+(k) = \mathbf{P}^-(k) - \mathbf{P}^-(k)\mathbf{C}^T(k)(\mathbf{C}(k)\mathbf{P}^-(k)\mathbf{C}^T(k) + \mathbf{R}(k))^{-1}\mathbf{C}^T(k)\mathbf{P}^-(k) \end{cases} \tag{3}$$

where $\mathbf{K}(k) = \mathbf{P}^-(k)\mathbf{C}^T(k)(\mathbf{C}(k)\mathbf{P}^-(k)\mathbf{C}^T(k) + \mathbf{R}(k))^{-1} = [\mathbf{K}_p^T\ \mathbf{K}_b^T]^T$ is the Kalman filter gain, separable in two diagonal blocks and

$$\mathbf{R}(k) = \mathbf{f}\, r_{PCA}(k) \tag{4}$$

is the covariance of the observation noise. The factor $\mathbf{f}$, relating the PCA decomposition covariance and the sensor noise covariance is central for the problem at end and will be the subject of a detailed stochastic characterization study. The resulting nonlinear estimator is represented in Fig. 1 on the right, with some abuse of notation.

3 Principal Component Analysis

Considering all linear transformations, the Karhunen-Loève (KL) transform allows for the optimal approximation to a stochastic signal, in the least squares sense. Furthermore, it is a well known signal expansion technique with uncorrelated coefficients for dimensionality reduction. These features make the KL transform interesting for many signal processing applications such as data compression, image and voice processing, data mining, exploratory data analysis, pattern recognition and time series prediction.

3.1 PCA Background

Consider a set of M stochastic signals $\mathbf{x}_i \in \mathcal{R}^N, i = 1, \ldots, M$, each represented as a column vector, with mean $m_x = 1/M \sum_{i=1}^{M} \mathbf{x}_i$. The purpose of the KL transform is to find an orthogonal basis to decompose a stochastic signal $\mathbf{x}$, from the same original space, to be computed as $\mathbf{x} = \mathbf{U}\mathbf{v} + m_x$, where the vector $\mathbf{v} \in \mathcal{R}^N$ is the projection of $\mathbf{x}$ in the basis, i.e., $\mathbf{v} = \mathbf{U}^T(\mathbf{x} - m_x)$. The matrix $\mathbf{U} = [\mathbf{u}_1 \ \mathbf{u}_2 \ \ldots \ \mathbf{u}_N \ ]$ should be composed by the N orthogonal column vectors of the basis, verifying the eigenvalue problem

$$\mathbf{R}_{xx}\mathbf{u}_j = \lambda_j \mathbf{u}_j, \ \ j = 1, ..., N, \tag{5}$$

where $\mathbf{R}_{xx}$ is the ensemble covariance matrix, computed from the set of M experiments

$$\mathbf{R}_{xx} = \frac{1}{M-1} \sum_{i=1}^{M} (\mathbf{x}_i - m_x)(\mathbf{x}_i - m_x)^T. \tag{6}$$

Assuming that the eigenvalues are ordered, i.e. $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N$, the choice of the first $n << N$ principal components, leads to an approximation to the stochastic signals given by the ratio on the covariances associated with the components, i.e. $\sum_n \lambda_n / \sum_N \lambda_N$. In many applications, where stochastic multidimensional signals are the key to overcome the problem at hand, this approximation can constitute a large dimensional reduction and thus a computational complexity reduction. The advantages of PCA are threefold: i) it is an optimal (in terms of mean squared error) linear scheme for compressing a set of high dimensional vectors into a set of lower dimensional vectors; ii) the model parameters can be computed directly from the data (by diagonalizing the ensemble covariance); iii) given the model parameters, projection into and from the bases are computationally inexpensive operations of complexity $\mathcal{O}(nN)$.

3.2 PCA Based Positioning System

Assume a mission scenario where bathymetric data are available and that a terrain based navigation system should be designed. The steps to implement a PCA based positioning sensor using this bathymetric data will be outlined next.

Prior to the mission, the bathymetric data of the area under consideration is partitioned in *mosaics* with fixed dimensions N_x by N_y. After reorganizing these two-dimensional data in vector form, e.g. stacking the columns, a set of M stochastic signals $\mathbf{x}_i \in \mathcal{R}^N$, $N = N_x N_y$, results. The number of signals M to be considered depends on the mission scenario and on the *mosaic* overlapping. The KL transform is computed *a priori*, using (5) and (6), the eigenvalues are ordered, and the number n of the principal components to be used are selected, according with the required level of approximation. The following data should be recorded for latter use: *i*) the data ensemble mean m_x; *ii*) the matrix transformation with n eigenvectors $\mathbf{U}_n = [\mathbf{u}_1 \ \ldots \ \mathbf{u}_n]$; *iii*) the projection on the selected basis of all the *mosaics*,computed using $\mathbf{v}_i = \mathbf{U}_n^T(x_i - m_x), i = 1, \ldots, M$; and *iv*) the coordinates of the center of the *mosaics*, (x_i, y_i), $i = 1, \ldots, M$. During the mission, the last geo-referenced range measurements are packed and will constitute the input signal $\mathbf{x}$ to the PCA positioning system. The following tasks should then be performed:

i) compute the projection of the signal $\mathbf{x}$ into the basis, using $\mathbf{v} = \mathbf{U}_n^T(\mathbf{x} - m_x)$;
ii) given an estimate on the actual horizontal coordinates of the UV position $\hat{x}$ and $\hat{y}$, provided by the navigation system, search on a given neighborhood δ the *mosaic* that verifies

$$\forall_i \|[\hat{x}\ \hat{y}]^T - [x_i\ y_i]^T\|_2 < \delta, \ r_{PCA} = \min_i \ \|\mathbf{v} - \mathbf{v}_i\|_2; \tag{7}$$

iii) given the *mosaic* i that is the closest to the present input, its center coordinates (x_i, y_i) will be selected as the position measurement.

Special attention should be taken next to two well known cases of poor performance: i) if the data correspond to white noise, the decomposition will result in equal eigenvalues, thus the use of $n << N$ principal components will only explain the data covariance fraction n/N; ii) in the case where data is spatially homogeneous (*flatland*) the decomposition is not unique, as any eigenvalue with null components associated, explains the (information empty) data.

4 PCA Positioning Sensor Performance

To study the performance of PCA as a positioning sensor a series of Monte Carlo tests were carried out using synthesized stochastic terrains

$$z(x, y) = \sum_{m=1}^{100} A(m) sin(\Omega_x(m)x + \Phi_x(m)) sin(\Omega_y(m)y + \Phi_y(m)),$$

where the spatial amplitude $A(m) \approx \mathcal{N}(0, 1)$, i.e. a white noise random variable with zero expected value and unitary variance, spatial frequency $\Omega_i(m) = 2\pi f(m)$, $i = \{x, y\}$, $f(m) \approx \mathcal{U}(0, \overline{f})$ [1], where $\overline{f}$ is the maximum terrain bandwidth, and random spatial phase offsets $\Phi_x(m)$ and $\Phi_y(m)$. The height is known

[1] Compact form to describe an uniformly distributed stochastic variable, in the interval expressed by the first and second arguments, respectively.

in a lattice $x \in [\underline{x}, \overline{x}]$ and $y \in [\underline{y}, \overline{y}]$, at half meter intervals. Note that selecting $\overline{f} = 1\,m$, the Nyquist spatial frequency for the terrain representation in the present study, corresponds to white noise terrain and the *flatland* case is recovered in the limit where $\overline{f} \to 0$. Each experiment consists of randomly select a position (x, y) and bathymetric data (or some percentage of it) of the mosaic size according to (1), where each SONAR measurement is corrupted by zero mean white noise with $\sigma_{SONAR} = 0.1\,m$. The search in (7) is then performed over all the PCA data and the average and covariance of the position error are updated accordingly.

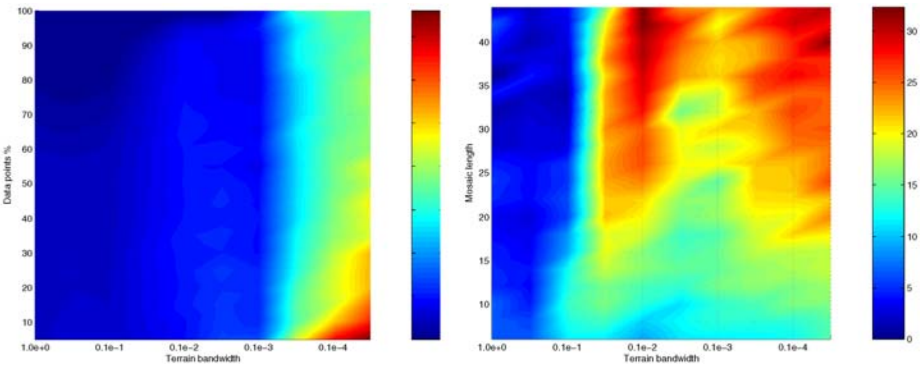

Fig. 2. Covariance relation **f** from (4), for 1000 Monte Carlo experiments in each parameter combination, where $M = 50 * 50$ mosaics were considered. Left: variations on the percentage of scanned points in the mosaic with dimensions $N = 20 * 20$ versus terrain bandwidth. Right: variations on the mosaic leght versus terrain bandwidth.

A number of parameters impact on the PCA positioning sensor performance. Next, some of the more relevant parameters are enumerated and the impact is discussed both based on Monte Carlo results and on the properties of the bathymetric terrain model:

1. Number of Principal Components - the increase on the number of components n increases the covariance accuracy explained (according to the ratio $\sum_n \lambda_n / \sum_N \lambda_N$) [7]. Monte Carlo simulations revealed that a small number of components, are enough to explain in the excess of 95% of data covariance, thus validating the use of PCA as a low complexity positioning sensor.

2. Number of Mosaics - In the case where the neighborhood $\delta \to \infty$ in (7), as considered in all this study, the performance of the overall PCA positioning system degrades linearly with the number of mosaics, due to the linear increase on the number of elements to be searched. In real applications, a careful choice of this parameter can improve the PCA performance, given an estimated position available from the estimator briefly introduced in section 2, as depicted in figure 1, on the right, thus bounding the positioning sensor error.

3. Percentage of Points Scanned in a Mosaic - Due to the velocity of propagation of sound in the water, only a fraction of the total *mosaic* area can be scanned with a sonar. A graceful degradation on the performance is confirmed from the results, along any vertical line on the left of figure 2.

4. Terrain Bandwidth - This parameter is of utmost importance on the performance of the PCA positioning system, confirmed by the results depicted in Fig. 2. Note that on both left and right pictures, the white noise and the "flatland" cases were considered. The variation on f is nonlinear in both cases, thus prior to be used in real applications a multi model adaptive estimation strategy should be considered, specially when missions taking place on large areas are considered.

5. Mosaic Dimension - The mosaic dimension represents a compromise: small mosaic sizes increase the accuracy of the PCA positioning system, with an increase on the total number of mosaics. Large mosaic sizes diminish the accuracy and augments the correlation stored in each mosaic requiring an increase on the number of components. On the right part of Fig. 2, for a fixed number of components n, the performance degradation is evident, with a more severe increase in large mosaics. The impact on the performance is also nonlinear, thus providing an insight on strategies to tune the mosaic length selection.

The results obtained reinforce the usefulness of the proposed method as a basic positioning sensor, allowing the design of bounded accuracy underwater navigation and guidance systems. For a design example on the development of navigation tools, based on a PCA positioning sensor see [10].

5 Conclusions and Future Work

After performing a large number of Monte Carlo experiments with the proposed PCA positioning sensor, some conclusions can be drawn: the sensor is non-biased however it presents nonlinear characteristics for different terrains and PCA parameters'. In general, equal covariance in both dimensions were found, given the homogeneous definition of the set of terrains used. Thus, the results obtained pave the way to the use of the proposed sensor in real positioning applications for underwater robotics.

Future work will be carried out on the implementation of multi model adaptive estimator design and analysis tools for underwater navigation systems, where Doppler log/PCA and INS/PCA systems are of interest. It is important to remark that the design of navigation systems based on other geophysical sensors, such as magnetometers and gradiometers, is obvious.

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