GAS Ocean Current Estimation with Limited Velocity Readings ⋆

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Abstract: This paper presents a simple setup for ocean current estimation that offers globally asymptotically stable (GAS) error dynamics. In the proposed scenario, the underwater vehicle has only access to limited relative velocity readings, along the longitudinal direction of the vehicle, in addition to position measurements. The observability of the system is analyzed and necessary and sufficient conditions, with physical meaning, are derived, hence useful for motion planning and control. A Kalman filter, with GAS error dynamics, is implemented as a solution to the estimation problem and simulation results are included that illustrate the performance of the proposed solution.

Keywords: Ocean current estimation, observability and observer design, marine systems.

1. INTRODUCTION

The knowledge of the velocity of the ocean current is vital in many underwater applications, e.g. in the design of control systems. Examples are provided in Aguiar and Pascoal (2002), Batista et al. (2009), and Batista et al. (2010a), where the guidance and control design of the vehicle must take into account, explicitly, the presence of the ocean current, which is regarded as a non-vanishing disturbance to the motion of the vehicle. For navigation systems, when it is impossible to measure the inertial velocity and only relative velocity measurements are available, it is usually required to include the ocean current velocity in the kinematic models, see e.g. Batista et al. (2010b), Batista et al. (2011b), Morgado et al. (2011), or Gadre and Stilwell (2005). For recent surveys on underwater navigation the reader is referred to Kinsey et al. (2006) and Leonard et al. (1998). In other situations, the estimation of the ocean current velocity is, itself, a goal of the mission, see Neumann, G. (1968), Wilson (1994), Kang et al. (2008), and references therein.

While there exist many sensors and methods to measure the ocean current velocity, this number is significantly reduced when these are required to be mounted on-board underwater vessels, providing in real-time estimates of the ocean current velocity. Doppler Velocity Logs (DVLs) measure both the inertial velocity and the velocity relative to the fluid when bottom lock is possible, and hence they may provide estimates of the ocean current. However, when bottom lock is not achieved, DVLs provide only the velocity relative to the fluid. Evidently, if the vehicle is stationary, they also measure the velocity of the fluid, but that is not appropriate for dynamic mission scenarios covering large regions and hence dynamic estimators, with aiding sensors, must be designed. Still, DVLs are very expensive and can also be quite bulky. Alternative sensors include Pitot tube velocity log and rotor type velocity logs, which provide good relative velocity measurements along the longitudinal direction of the vehicle. While these sensors are usually much cheaper, they usually only offer good measurements along one direction due to hydrodynamic effects.

This paper presents a novel ocean current estimation solution with limited velocity readings, considering a position sensor as an aiding device. Following previous work of the authors, Batista et al. (2010b), the kinematic equations of the vehicle, which are exact, are considered. However, as the relative velocity of the vehicle is only available along one direction, the number of differential equations that is actually available for observer design purposes is smaller. As a result, certain observability conditions must be satisfied so that the resulting system dynamics are observable. In this paper, the system dynamics are linear time-varying and as such, a simple Kalman filter, with globally asymptotically stable error dynamics under appropriate persistent excitation conditions, is proposed as an ocean current dynamic estimation solution.

The paper is organized as follows. The problems statement and the nominal system dynamics are introduced in Section 2, while the observability analysis and filter design are detailed in Section 3. Simulation results are presented in Section 4 and Section 5 summarizes the main conclusions of the paper.

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1.1 Notation

Throughout the paper the symbol $\mathbf{0}_{n\times m}$ denotes an $n \times m$ matrix of zeros, $\mathbf{I}_n$ an identity matrix with dimension $n \times n$, and $\mathrm{diag}(\mathbf{A}_1, \ldots, \mathbf{A}_\mathrm{n})$ a block diagonal matrix. When the dimensions are omitted the matrices are assumed of appropriate dimensions. If $\mathbf{x}$ and $\mathbf{y}$ are two vectors of identical dimensions, $\mathbf{x} \times \mathbf{y}$ represents the cross product.

2. PROBLEM STATEMENT

Consider an underwater vehicle moving in the presence of constant unknown ocean currents. Assume that the vehicle is equipped with a positioning system, such as a Long Baseline or an Ultra Short Baseline acoustic positioning device. In addition, suppose that the sensor suite includes a measuring device that gives the velocity of the vehicle relative to the fluid along the longitudinal direction, e.g. a Pitot tube velocity log or a rotor type velocity log. Finally, suppose that the vehicle has an Attitude and Heading Reference System (AHRS) installed on-board, which provides both the attitude and the angular velocity of the vehicle. The problem considered in the paper is that of estimating the ocean current velocity.

In order to set the problem framework, let $\{I\}$ denote an inertial reference coordinate frame and $\{B\}$ a coordinate frame attached to the vehicle, commonly denominated as the body-fixed coordinate frame. The linear motion of the vehicle is given by

$$\dot{\mathbf{p}}(t) = \mathbf{R}(t)\mathbf{v}(t),$$

where $\mathbf{p}(t) \in \mathbb{R}^3$ denotes the inertial position of the vehicle, $\mathbf{v}(t) \in \mathbb{R}^3$ is the velocity of the vehicle relative to $\{I\}$ and expressed in body-fixed coordinates, and $\mathbf{R}(t)$ is the rotation matrix from $\{B\}$ to $\{I\}$, which satisfies $\dot{\mathbf{R}}(t) = \mathbf{R}(t)\mathbf{S}(\boldsymbol{\omega}(t))$, where $\boldsymbol{\omega}(t) \in \mathbb{R}^3$ is the angular velocity of $\{B\}$, expressed in body-fixed coordinates, and $\mathbf{S}(\boldsymbol{\omega})$ is the skew-symmetric matrix such that $\mathbf{S}(\boldsymbol{\omega})\boldsymbol{x}$ is the cross product $\boldsymbol{\omega} \times \boldsymbol{x}$.

Suppose that an Ultra-Short Baseline (USBL) acoustic positioning system is installed on-board, which gives the position of a fixed landmark in the mission scenario relative to the vehicle, expressed in body-fixed coordinates, as given by

$$\mathbf{r}(t) = \mathbf{R}^T(t)\left[\mathbf{s} - \mathbf{p}(t)\right], \tag{1}$$

where $\mathbf{s} \in \mathbb{R}^3$ denotes the inertial position of the landmark and $\mathbf{r}(t) \in \mathbb{R}^3$ denotes the USBL reading. The time derivative of (1) is given by

$$\dot{\mathbf{r}}(t) = -\mathbf{S}\left[\boldsymbol{\omega}(t)\right]\mathbf{r}(t) - \mathbf{v}(t),$$

which can be rewritten as

$$\dot{\mathbf{r}}(t) = -\mathbf{S}\left[\boldsymbol{\omega}(t)\right]\mathbf{r}(t) - \mathbf{v}_r(t) - \mathbf{v}_c(t),$$

where $\mathbf{v}_r(t) \in \mathbb{R}^3$ denotes the velocity of the vehicle relative to the fluid and $\mathbf{v}_c \in \mathbb{R}^3$ denotes the fluid velocity, both expressed in body-fixed coordinates.

Assuming that the fluid velocity is constant in inertial coordinates, the time derivative of the fluid velocity expressed in inertial coordinates satisfies

$$\dot{\mathbf{v}}_c(t) = -\mathbf{S}\left[\boldsymbol{\omega}(t)\right]\mathbf{v}_c.$$

As such, the system dynamics considering a sensor suite that includes an USBL and a relative velocity sensor along the longitudinal direction of the vehicle, in addition to an AHRS, can be written as

$$\begin{cases} \dot{\mathbf{r}}(t) = -\mathbf{S}\left[\boldsymbol{\omega}(t)\right]\mathbf{r}(t) - \mathbf{v}_r(t) - \mathbf{v}_c(t) \\ \dot{\mathbf{v}}_c(t) = -\mathbf{S}\left[\boldsymbol{\omega}(t)\right]\mathbf{v}_c \end{cases}, \tag{2}$$

where measurements of $\mathbf{r}(t)$, $\boldsymbol{\omega}(t)$, and $\left[\,1\ 0\ 0\,\right]\mathbf{v}_r(t)$ are available for observer design purposes. The problem of ocean current estimation considering this sensor suite is the design of a filter for (2) assuming noisy measurements.

Consider now that, instead of an USBL, the vehicle has access to inertial position readings, as given for example by a LBL acoustic positioning system. Then, the system dynamics can be written as

$$\begin{cases} \dot{\mathbf{p}}(t) = \mathbf{R}(t)\mathbf{v}_r(t) + \mathbf{R}(t)\mathbf{v}_c(t) \\ \dot{\mathbf{v}}_c(t) = -\mathbf{S}\left[\boldsymbol{\omega}(t)\right]\mathbf{v}_c(t) \end{cases}, \tag{3}$$

where measurements of $\mathbf{p}(t)$, $\mathbf{R}(t)$, and $\left[\,1\ 0\ 0\,\right]\mathbf{v}_r(t)$ are available for observer design purposes. The problem of ocean current estimation considering this sensor suite is the design of a filter for (3) assuming noisy measurements.

Throughout the paper it is assumed that the angular velocity is bounded, which is a mild assumption that is always verified in reality as limited actuation systems induce limited velocities.

3. FILTER DESIGN

This section presents the design of ocean current estimators considering the sensor suites described in Section 2. First, a state transformation is detailed, in Section 3.1, that essentially brings together the two system dynamics (2) and (3) to a common framework. Afterwards, the observavility of the system is analyzed in Section 3.2. Finally, a Kalman filter with globally asymptotically stable error dynamics is briefly introduced in Section 3.3.

3.1 System dynamics equivalence

Let $\mathbf{T}(t) := \mathrm{diag}\left(-\mathbf{R}^\mathrm{T}(\mathrm{t}), \mathbf{I}\right) \in \mathbb{R}^{6\times 6}$ and consider the state transformation

$$\begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \end{bmatrix} := \mathbf{T}(t) \begin{bmatrix} \mathbf{p}(t) \\ \mathbf{v}_c(t) \end{bmatrix}, \tag{4}$$

which is a Lyapunov state transformation similar to one previously used by the authors, see Batista et al. (2010b). The new system dynamics are given by

$$\begin{cases} \dot{\mathbf{x}}_1(t) = -\mathbf{S}\left[\boldsymbol{\omega}(t)\right]\mathbf{x}_1(t) - \mathbf{x}_2(t) - \mathbf{v}_r(t) \\ \dot{\mathbf{x}}_2(t) = -\mathbf{S}\left[\boldsymbol{\omega}(t)\right]\mathbf{x}_2(t) \end{cases}, \tag{5}$$

Notice that, as (4) is a Lyapunov state transformation, all observability properties are preserved, see Brockett (1970). Moreover, the new system dynamics (5) resemble exactly (2). Hence, for observability and observer design purposes, it suffices to do so for (5), as all results are directly applicable to both (2) and (3).

3.2 Observability analysis

This section details the analysis of the observability of (5). First, notice that $\mathbf{x}_1(t)$ is measured, which means that the question is on the determination of $\mathbf{x}_2(t)$. Moreover, as only the first component of $\mathbf{v}_r(t)$ is available, it is impossible to use the dynamic equations for the evolution of the other two components of $\mathbf{x}_1(t)$. As such, instead of considering (5), the following system dynamics are analyzed,

$$\begin{cases} \dot{x}_{1x} = -[1\ 0\ 0]\,\mathbf{x}_2(t) + u_{x_{1x}}(t) \\ \dot{\mathbf{x}}_2(t) = -\mathbf{S}\left[\boldsymbol{\omega}(t)\right]\mathbf{x}_2(t) \end{cases}, \tag{6}$$

where

$$u_{x_{1x}}(t) := -v_{rx} + \omega_z(t)x_{1y}(t) - \omega_y(t)x_{1z}(t) \in \mathbb{R} \tag{7}$$

is considered as a deterministic input, with

$$\boldsymbol{\omega}(t) = \begin{bmatrix} \omega_x(t) \\ \omega_y(t) \\ \omega_z(t) \end{bmatrix} \in \mathbb{R}^3,$$

$$\mathbf{v}_r(t) = \begin{bmatrix} v_{rx}(t) \\ v_{ry}(t) \\ v_{rz}(t) \end{bmatrix} \in \mathbb{R}^3,$$

and

$$\mathbf{x}_1(t) = \begin{bmatrix} x_{1x}(t) \\ x_{1y}(t) \\ x_{1z}(t) \end{bmatrix} \in \mathbb{R}^3.$$

Notice that all terms of (7) are available for observer design purposes. The system dynamics are linear time-varying and the output can be considered as $y(t) = x_{1x}(t)$. In compact form, it follows that

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}u_{x_{1x}}(t) \\ y(t) = \mathbf{C}\mathbf{x}(t) \end{cases},$$

where

$$\mathbf{x}(t) = \begin{bmatrix} x_{1x}(t) \\ \mathbf{x}_2(t) \end{bmatrix} \in \mathbb{R}^4$$

is the system state, $y(t) \in \mathbb{R}$ is the system output, and

$$\mathbf{A}(t) = \begin{bmatrix} 0 & -1\ 0\ 0 \\ \mathbf{0} & -\mathbf{S}\left[\boldsymbol{\omega}(t)\right] \end{bmatrix} \in \mathbb{R}^{4\times 4},$$

$$\mathbf{B} = \begin{bmatrix} 1 \\ \mathbf{0} \end{bmatrix} \in \mathbb{R}^4,$$

and $\mathbf{C} = [1\ \mathbf{0}] \in \mathbb{R}^{1\times 4}$.

The following result [Proposition 4.2, Batista et al. (2011a)] is useful in the sequel.

Proposition 1. Let $\mathbf{f}(t) : [t_0, t_f] \subset \mathbb{R} \to \mathbb{R}^n$ be a continuous and i-times continuously differentiable function on $\mathcal{I} := [t_0, t_f]$, $T := t_f - t_0 > 0$, and such that $\mathbf{f}(t_0) = \dot{\mathbf{f}}(t_0) = \ldots = \mathbf{f}^{(i-1)}(t_0) = \mathbf{0}$. Further assume that $\left\|\mathbf{f}^{(i+1)}(t)\right\| \leq C$ for all $t \in \mathcal{I}$. If there exist $\alpha > 0$ and $t_1 \in \mathcal{I}$ such that $\left\|\mathbf{f}^{(i)}(t_1)\right\| \geq \alpha$, then there exist $0 < \delta \leq T$ and $\beta > 0$ such that $\|\mathbf{f}(t_0 + \delta)\| \geq \beta$.

The following proposition addresses the observability of the LTV system (6).

Proposition 2. The LTV system (6) is observable on $\mathcal{I} := [t_0, t_f]$, $t_0 < t_f$, if and only if for all unit vectors $\mathbf{d} \in \mathbb{R}^3$ it is possible to choose $t_i \in \mathcal{I}$ such that $[1\ 0\ 0]\,\mathbf{R}^T(t_i)\,\mathbf{R}(t_0)\,\mathbf{d} \neq 0$.

Proof. In order to proceed with the analysis of the observability of the LTV system (6), it is convenient to compute the observability Gramian associated with the pair $(\mathbf{A}(t), \mathbf{C})$ and, in order to do so, the transition matrix associated with the system matrix $\mathbf{A}(t)$, which can be easily shown to be given by

$$\boldsymbol{\phi}(t, t_0) = \begin{bmatrix} 1 & -\displaystyle\int_{t_0}^{t} [1\ 0\ 0]\,\mathbf{R}^T(\tau)\mathbf{R}(t_0)\,d\tau \\ \mathbf{0} & \mathbf{R}^T(t)\mathbf{R}(t_0) \end{bmatrix} \in \mathbb{R}^{4\times 4}.$$

The observability Gramian associated with the pair $(\mathbf{A}(t), \mathbf{C})$ is simply given by

$$\boldsymbol{\mathcal{W}}(t_0, t_f) = \int_{t_0}^{t_f} \boldsymbol{\phi}^T(t, t_0)\,\mathbf{C}^T\mathbf{C}\boldsymbol{\phi}(t, t_0)\,dt.$$

Now, let

$$\mathbf{d} = \begin{bmatrix} d_1 \\ \mathbf{d}_2 \end{bmatrix} \in \mathbb{R}^4,\ d_1 \in \mathbb{R},\ \mathbf{d}_2 \in \mathbb{R}^3,$$

be a unit vector. Then, it is straightforward to show that

$$\mathbf{d}^T\boldsymbol{\mathcal{W}}(t_0, t_f)\,\mathbf{d} = \int_{t_0}^{t_f} f^2(t, t_0)\,dt,$$

where

$$f(t, t_0) = d_1 - \int_{t_0}^{t} [1\ 0\ 0]\,\mathbf{R}^T(\tau)\mathbf{R}(t_0)\,\mathbf{d}_2 d\tau.$$

The first and second derivatives of $f(t, t_0)$ with respect to t are given by

$$\frac{d}{dt} f(t, t_0) = -[1\ 0\ 0]\,\mathbf{R}^T(t)\mathbf{R}(t_0)\,\mathbf{d}_2$$

and

$$\frac{d^2}{dt^2} f(t, t_0) = [1\ 0\ 0]\,\mathbf{S}\left[\boldsymbol{\omega}(t)\right]\mathbf{R}^T(t)\mathbf{R}(t_0)\,\mathbf{d}_2,$$

respectively. Notice that both derivatives are bounded under the assumption of bounded angular velocities.

In order to show sufficiency, suppose first that $d_1 \neq 0$. Then, $|f(t_0, t_0)| = |d_1| > 0$. Otherwise, i.e., if $d_1 = 0$, it must be $\|\mathbf{d}_2\| = 1$. Under the conditions of the proposition, it follows that for all unit vectors $\mathbf{d}_2$ it is possible to choose $t_1 \in \mathcal{I}$ such that

$$[1\ 0\ 0]^T\,\mathbf{R}^T(t_1)\,\mathbf{R}(t_0)\,\mathbf{d}_2 \neq 0$$

and hence, for $d_1 = 0$ and all unit vectors $\mathbf{d}_2$, it is possible to choose $t_1 \in \mathcal{I}$ such that

$$\left|\left.\frac{d}{dt} f(t, t_0)\right|_{t=t_1}\right| > 0.$$

Using Proposition 1 is follows that, for $d_1 = 0$ and all unit vectors $\mathbf{d}_2$, there exists $t_2 \in \mathcal{I}$ such that $|f(t_2, t_0)| > 0$. Thus, it has been shown so far that, for all unit vectors $\mathbf{d}$, there always exists $t_3 \in \mathcal{I}$ such that $|f(t_3, t_0)| > 0$. Using Proposition 1 again it follows that, under the hypothesis of this proposition, the observability Gramian $\boldsymbol{\mathcal{W}}(t_0, t_f)$ is invertible and hence the LTV system (6) is observable on $\mathcal{I}$.

To show necessity, suppose that there exists a unit vector $\mathbf{d}_u \in \mathbb{R}^3$ such that, for all $t \in \mathcal{I}$, it is true that

$$[1\ 0\ 0]\,\mathbf{R}^T(t)\,\mathbf{R}(t_0)\,\mathbf{d}_u = 0.$$

In this case, let $d_1 = 0$ and $\mathbf{d}_2 = \mathbf{d}_u$. Then, it is straightforward to realize that $f(t, t_0) = 0$ for all $t \in \mathcal{I}$ and hence $\mathbf{d}^T\boldsymbol{\mathcal{W}}(t_0, t_f)\,\mathbf{d} = 0$. Thus, the observability Gramian is not invertible and hence the LTV system (6) is not observable. Therefore, if the LTV system (6) is observable, it follows that, for all unit vectors $\mathbf{d} \in \mathbb{R}^3$ it is possible to choose $t_i \in \mathcal{I}$ such that $[1\ 0\ 0]\,\mathbf{R}^T(t_i)\,\mathbf{R}(t_0)\,\mathbf{d} \neq 0$, which concludes the proof.

□

Proposition 2 details a sufficient and necessary condition on the observability of the linear system (6). However, as the system is time-varying, stronger forms of observability are, in general, required in order to design observers with globally asymptotically stable error dynamics. This is detailed in the following proposition.

Proposition 3. The LTV system (6) is uniformly completely observable if and only if there exist positive constants $\alpha > 0$ and $\delta > 0$ such that, for all $t \geq t_0$ and all unit vectors $\mathbf{d} \in \mathbb{R}^3$, it is possible to choose $t_i \in [t, t+\delta]$ such that $\left|[1\ 0\ 0]\, \mathbf{R}^T(t_i)\, \mathbf{R}(t)\, \mathbf{d}\right| \geq \alpha$.

Proof. Let

$$\mathbf{d} = \begin{bmatrix} d_1 \\ \mathbf{d}_2 \end{bmatrix} \in \mathbb{R}^4,\ d_1 \in \mathbb{R},\ \mathbf{d}_2 \in \mathbb{R}^3,$$

be a unit vector. Then, it is straightforward to show that

$$\mathbf{d}^T \boldsymbol{\mathcal{W}}(t, t+\delta)\, \mathbf{d} = \int_t^{t+\delta} f^2(\tau, t)\, d\tau,$$

where

$$f(\tau, t) = d_1 - \int_t^{\tau} [1\ 0\ 0]\, \mathbf{R}^T(\sigma) \mathbf{R}(t)\, \mathbf{d}_2 d\sigma$$

for all $t \geq t_0$, $\tau \in [t, t+\delta]$. The LTV system (6) is uniformly completely observable if and only if there exist positive constants $c_1 > 0$ and $c_2 > 0$ such that, for all $t \geq t_0$ and all unit vectors $\mathbf{d}$, it is true that

$$c_1 \leq \mathbf{d}^T \boldsymbol{\mathcal{W}}(t, t+\delta)\, \mathbf{d} \leq c_2. \tag{8}$$

The right side of (8) is readily verified as $f(\tau, t)$ is a continuous bounded function. Therefore, only the left side of (8) requires analysis. The first two derivatives of $f(\tau, t)$ with respect to τ are given by

$$\frac{d}{d\tau} f(\tau, t) = -[1\ 0\ 0]\, \mathbf{R}^T(\tau) \mathbf{R}(t)\, \mathbf{d}_2$$

and

$$\frac{d^2}{d\tau^2} f(\tau, t) = [1\ 0\ 0]\, \mathbf{S}[\boldsymbol{\omega}(\tau)]\, \mathbf{R}^T(\tau) \mathbf{R}(t)\, \mathbf{d}_2,$$

respectively, and they are both bounded under the assumption of bounded angular velocities. The proof now follows as in Proposition 2 but considering uniform bounds.

In order to show sufficiency, suppose first that $d_1 \neq 0$. Then, notice that $|f(t, t)| = |d_1| = \alpha_1 > 0$ for all $t \geq t_0$. Otherwise, i.e., if $d_1 = 0$, it must be $\|\mathbf{d}_2\| = 1$. Under the conditions of the proposition, it follows that there exist positive constants $\alpha' > 0$ and $\delta' > 0$ such that, for all $t \geq t_0$ and all unit vectors $\mathbf{d}_2$, it is possible to choose $t_1 \in [t, t+\delta']$ such that

$$\left|[1\ 0\ 0]^T\, \mathbf{R}^T(t_1)\, \mathbf{R}(t)\, \mathbf{d}_2\right| \geq \alpha'$$

and hence, for $d_1 = 0$ and all $t \geq t_0$ and all unit vectors $\mathbf{d}_2$, it is possible to choose $t_1 \in [t, t+\delta']$ such that

$$\left| \left. \frac{d}{d\tau} f(\tau, t) \right|_{\tau = t_1} \right| = \alpha_2 > 0.$$

Using Proposition 1 is follows that there exists a positive constant $\alpha_3 > 0$ such that for all $t \geq t_0$, $d_1 = 0$, and all unit vectors $\mathbf{d}_2$, there exists $t_2 \in [t, t+\delta]$ such that $|f(t_2, t)| > \alpha_3$. Thus, it has been shown so far that there exists a positive constant $\alpha_4 := \min(\alpha_1, \alpha_3)$ such that, for all $t \geq t_0$ and all unit vectors $\mathbf{d}$, there always exists $t_3 \in [t, t+\delta]$ such that $|f(t_3, t)| > \alpha_4$. Using Proposition 1 again it follows that the left side of (8) is also verified, and hence the LTV system (6) is uniformly completely observable on $\mathcal{I}$.

To show necessity, suppose that, for all $\alpha > 0$ and $\delta > 0$, there exists $t_i \geq t_0$ and a unit vector $\mathbf{d}_u \in \mathbb{R}^3$ such that, for all $t \in [t_i, t_i + \delta]$, it is true that

$$\left|[1\ 0\ 0]\, \mathbf{R}^T(t)\, \mathbf{R}(t_i)\, \mathbf{d}_u\right| < \alpha. \tag{9}$$

Now, let $d_1 = 0$ and $\mathbf{d}_2 = \mathbf{d}_u$. Then, notice that

$$\mathbf{d}^T \boldsymbol{\mathcal{W}}(t_i, t_i+\delta)\, \mathbf{d} = \int_{t_i}^{t_i+\delta} f^2(\tau, t_i)\, d\tau$$

$$= \int_{t_i}^{t_i+\delta} \left(\int_{t_i}^{\tau} [1\ 0\ 0]\, \mathbf{R}^T(\sigma) \mathbf{R}(t_i)\, \mathbf{d}_u d\sigma \right)^2 d\tau$$

Using simple integral norm inequalities, it follows that

$$\mathbf{d}^T \boldsymbol{\mathcal{W}}(t_i, t_i+\delta)\, \mathbf{d} \leq \int_{t_i}^{t_i+\delta} \left(\int_{t_i}^{\tau} \left|[1\ 0\ 0]\, \mathbf{R}^T(\sigma) \mathbf{R}(t_i)\, \mathbf{d}_u\right| d\sigma \right)^2 d\tau. \tag{10}$$

Substituting (9) in (10) gives

$$\mathbf{d}^T \boldsymbol{\mathcal{W}}(t_i, t_i+\delta)\, \mathbf{d} < \frac{\alpha^2 \delta^2}{2}$$

and hence the LTV system (6) is not uniformly completely observable. Therefore, if the LTV system (6) is uniformly completely observable, it follows that there exist positive constants $\alpha > 0$ and $\delta > 0$ such that, for all $t \geq t_0$ and all unit vectors $\mathbf{d} \in \mathbb{R}^3$, it is possible to choose $t_i \in [t, t+\delta]$ such that $\left|[1\ 0\ 0]\, \mathbf{R}^T(t_i)\, \mathbf{R}(t)\, \mathbf{d}\right| \geq \alpha$, which concludes the proof.

□

3.3 Kalman filter

Considering that the system dynamics (6) are LTV and that necessary and sufficient conditions on uniform completely observability have been derived, the design of a state observer is now trivial. In practice, and in spite of the fact that the original system dynamics are presented in a deterministic setting, there always exists measurement noise. Therefore, a filtering solution like the Kalman filter seems appropriate, albeit other solutions could be devised, e.g., an $\mathcal{H}_\infty$ filter Sun et al. (1993).

Including system disturbances and sensor noise to tune the Kalman filter gives the final system dynamics

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}u_{x_{1x}}(t) + \mathbf{n}_x(t) \\ y(t) = \mathbf{C}\mathbf{x}(t) + n_y(t) \end{cases},$$

where it is assumed that $\mathbf{n}_x(t)$ and $n_y(t)$ are uncorrelated zero-mean Gaussian noises, with $\mathrm{E}\left[\mathbf{n}_x(t)\mathbf{n}_x^T(\tau)\right] = \mathbf{Q}_x \delta(t - \tau)$ and $\mathrm{E}\left[n_y(t)\, n_y(\tau)\right] = Q_y \delta(t - \tau)$. The design of a Kalman filter is standard, see Jazwinski (1970), A. Gelb (Ed) (1974), and therefore it is omitted.

It is important to stress, however, that this filter is not optimal. Indeed, looking into the system matrices, it is easy to see that, in the presence of noise in the angular velocity measurements, there exists multiplicative noise.

Remark 1. Even though the ocean current is assumed to be constant in the nominal models introduced in Section 2, it is possible to model it as slowly time-varying with the process noise $\mathbf{n}_x(t)$. Hence, a filtering solution should be able to track slowly time-varying ocean currents, provided that the filter is properly tuned.

4. SIMULATION RESULTS

This section presents simulations results in order to evaluate the performance of the algorithm. For the sake of simplicity of presentation, simulations are only shown in 2-D. Nevertheless, the observability analysis and filter design detailed in Section 3 apply, considering vectors belonging to $\mathbb{R}^2$, the rotation matrix $\mathbf{R}(t) \in SO(1)$, and

$$\mathbf{S}(\boldsymbol{x}) = \begin{bmatrix} 0 & -x_x \\ x_y & 0 \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} x_x \\ x_y \end{bmatrix} \in \mathbb{R}^2.$$

In the simulations, the ocean current velocity was set to $[-0.5\ 0.1]^T$ m/s, and the relative velocity of the vehicle was set to $[2\ 0]^T$ m/s. Its angular velocity is sinusoidal so that the trajectory of the vehicle results as shown in Fig. 1. Clearly, the persistent excitation condition expressed in Proposition 3 is satisfied, which means that it is possible to design a Kalman filter with globally asymptotically stable error dynamics.

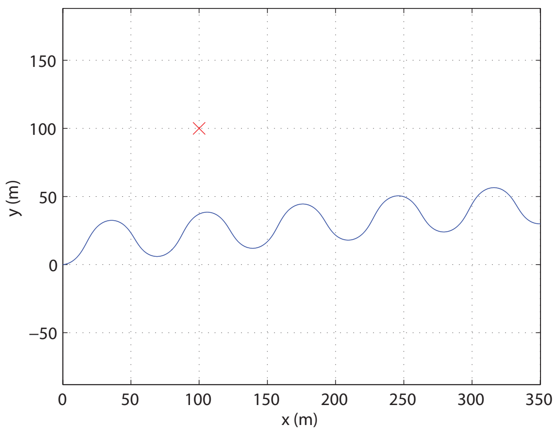

Fig. 1. Trajectory described by the vehicle. In red, the position of the external landmark, fixed in the mission scenario.

The inertial ocean current velocity is assumed to be constant. However, as it is estimated in body-fixed coordinates, it changes with the attitude of the vehicle, as given by the kinematic equation $\dot{\mathbf{v}}_c(t) = -\mathbf{S}[\boldsymbol{\omega}(t)]\,\mathbf{v}_c(t)$. The evolution of the ocean current velocity, expressed in body-fixed coordinates, is depicted in Fig. 2.

In order to estimate the ocean current velocity, the vehicle is assumed to the equipped with a USBL acoustic positioning system, installed in a so-called inverted configuration, so that it gives the position of an external feature with respect to the vehicle and expressed in body-fixed coordinates. In addition, angular velocity measurements are assumed to be available, as well as relative velocity readings along the x-axis of the vehicle. Sensor noise was considered for all sensors. In particular, the position, angular velocity, and relative velocity measurements are

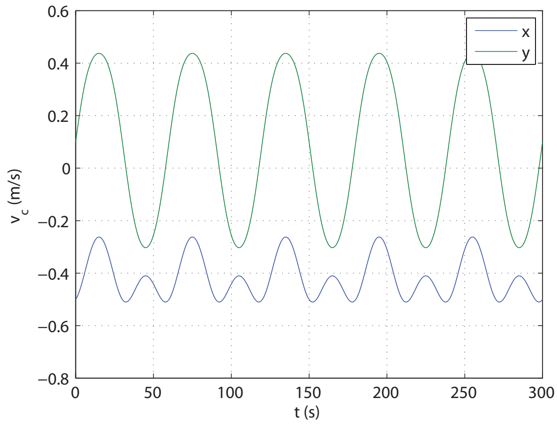

Fig. 2. Evolution of the ocean current velocity expressed in body-fixed coordinates.

assumed to be corrupted by additive uncorrelated zero-mean white Gaussian noises, with standard deviations of 1 m, 1 °/s, and 0.01 m/s, respectively. To tune the Kalman filter, the state disturbance intensity matrix was chosen as $\mathbf{Q}_x = \operatorname{diag}(0.1, 0.01, 0.01)$ and the output noise intensity matrix as $Q_y = 1$. The initial conditions were all set to zero.

The initial evolution of the position and ocean current velocity errors is depicted in Fig. 3. As it can be seen, the filter achieves steady state in less than 2 minutes. Nevertheless, it a faster convergence rate was required, it would be possible to use different gains in the beginning, switching later to different gains to achieve better steady state performance.

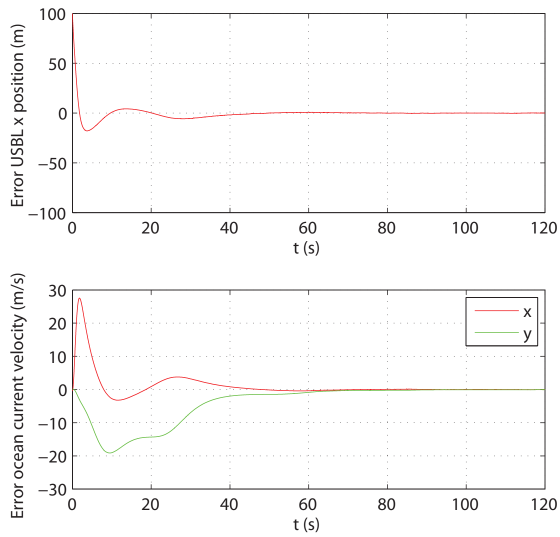

Fig. 3. Initial convergence of the position and ocean current error.

In order to better illustrate the performance achieved with the proposed solution, the steady-state errors of the position and ocean current velocity are shown in Fig. 4. Notice that the errors are confined to very tight intervals, in spite of the realistic measurement noise and low-grade sensor specifications.

In order to better evaluate the performance of the proposed solution, the Monte Carlo method was applied. In

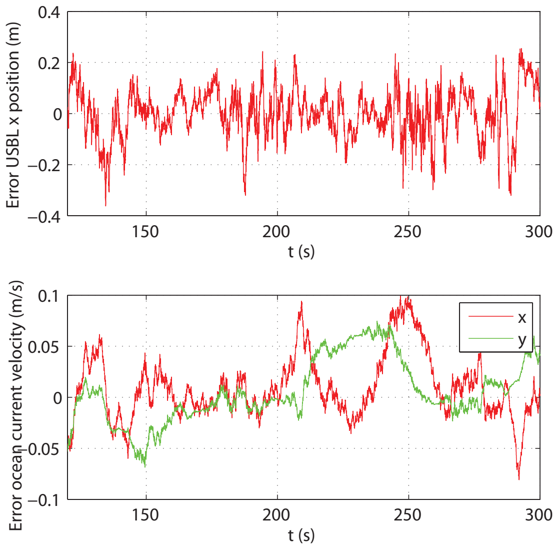

Fig. 4. Detailed evolution of the position and ocean current error.

Table 1. Standard deviation of the steady-state estimation errors of the proposed solution, averaged over 1000 runs of the simulation.

	Standard deviation
$\sigma_{\tilde{x}_{1x}}$ (m)	11.9×10^{-2}
$\sigma_{\tilde{x}_{2x}}$ (m/s)	3.68×10^{-2}
$\sigma_{\tilde{x}_{2y}}$ (m/s)	3.03×10^{-2}

particular, 1000 simulations were carried out with different, randomly generated noise signals. The mean and standard deviation of the errors were computed for each simulation and averaged over the set of simulations. The results are depicted in Table 1. As it is possible to observe, the standard deviation of the errors is very low, adequate for the sensor suite that was considered.

5. CONCLUSIONS

Whether it is for control or data acquisition purposes, the ocean current velocity is often required. While Doppler Velocity Logs measure both the inertial velocity and the relative velocity when they have bottom lock, and hence the ocean current velocity, they are expensive and bottom lock is not always available. This paper detailed a simple setup for ocean current estimation with globally asymptotically stable error dynamics. In the envisioned scenario, an underwater vehicle has only access to limited relative velocity readings, along the longitudinal direction of the vehicle, in addition to position measurements. The observability of the system was analyzed and necessary and sufficient conditions, with physical meaning, were derived, hence useful for motion planning and control. A Kalman filter, with GAS error dynamics, was proposed and simulation results show that the proposed strategy yields good performance.

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