Complementary Filter Design with Three Frequency Bands: Robot Attitude Estimation

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***Abstract*—This paper extents the by now classic sensor fusion complementary filter (CF) design, involving two sensors, to the case where three sensors that provide measurements in different bands are available. This paper shows that the use of classical CF techniques to tackle a generic three sensors fusion problem, based solely on their frequency domain characteristics, leads to a minimal realization, stable, sub-optimal solution, denoted as Complementary Filters^3 (CF^3). Then, a new approach for the estimation problem at hand is used, based on optimal linear Kalman filtering techniques. Moreover, the solution is shown to preserve the complementary property, i.e. the sum of the three transfer functions of the respective sensors add up to one, both in continuous and discrete time domains. This new class of filters are denoted as Complementary Kalman Filters^3 (CKF^3). The attitude estimation of a mobile robot is addressed, based on data from a rate gyroscope, a digital compass, and odometry. The experimental results obtained are reported.**

I. INTRODUCTION

The design of navigation systems, based on information from sensors installed on board is still a topic of current interest. One of the topics where researchers have invested their effort is the localization system, with special focus on the mobile robot attitude estimation. Nowadays, with the increasing development of MicroElectroMechanical Systems (MEMS), several sensors are available to estimate the robot attitude (e.g. compass, rate-gyros, accelerometers, etc.), beyond the odometry measure provided by encoders, usually connected to the drive wheel motors [11], [1], [14]. However, each sensor has different characteristics, namely in terms of dynamics and bandwidth response. Thus, to obtain a more accurate and robust estimate of a variable, sensor fusion techniques have been developed, in engineering and in robotics in particular, allowing to obtain estimates based on the measurements provided by more than one sensor [6], [5], [13].

The classical sensor fusion technique, for linear systems corrupted by stochastic white noise uncertainties with measurements also corrupted by white noise, is the celebrated Kalman filter (KF). This method solves a Minimum Mean Square Error (MMSE) estimation problem, that provides optimal estimates, based on a set of Kalman gains. However, KF requires a complete characterization of the process and observed noises, a task that may be difficult, or not suited to specific problems. The main difficulty associated with this method is the complexity in the identification of a good model for sensors and the robotic system. Another drawback is the high computational complexity of the solution, which involves intensive matrix manipulation. Alternatively to the use of KF for the data fusion, works based on other methodologies can also be found in the literature. One such methodology is Complementary Filtering, being a class of estimators based on the Wiener theory for linear systems corrupted by stationary noise, which allows the fusion of trustworthy signals with different frequency bandwidths to produce more precise signals in the time domain. The main advantages of Complementary Filters (CF) are the low computational cost, the faster dynamic response, and the more intuitive parameter tuning.

CF estimates variables considering signals provided by two sensors in distinct and complementary frequency bands, without the need of characterizing the stationary white noise present [9]. In [2], one such CF estimator is used for navigation. It has been further used integrated in several navigation systems [1], [10], [12], [8].

Optimal results can be obtained for CFs based on measurements from two sensors, in the case where the noise is stationary. Moreover, the estimators design based on sensors working in different bands is often useful to develop redundant measurement systems to fuse signals provided by sensors with different dynamics. Complementary filtering allows the estimator fine-tune for the frequency band where the sensors provide better performance. For instance, to estimate the tilt angle relative to gravity, an inclinometer and a rate gyro can be used. The rate gyro provides a measure of the angular velocity with a nice flat frequency response to about 50 Hz. However, the angular position, if obtained from integration, is highly affected by the bias error, quickly giving an unacceptable, ever increasing error drift on the position signal. The inclinometer measures tilt angle relative to gravity, does not suffer from a drift problem, but has a low bandwidth (0.5 Hz to 6Hz) which is too slow for many robotic applications. A CF may hence be used for estimating both angular position and angular velocity over a larger bandwidth with negligible drift [7]. The use of CF has been successfully applied to estimate mobile robot attitude, merging the signals provided by two sensors, where frequency bands complementarity is performed by a low-pass and a high-pass filter, both with the same cut-off frequency obtained from the frequency response of each sensor [8], [12].

Nowadays there is a large number of commercialy available sensors that can provide measurements related to attitute. Thus, the existence of more than two sensors in a sensor fusion approach allows a more accurate estimate or to avoid erroneous estimation results, thus providing redundancy and a more robust or reliable estimate.

The contributions of this paper are threefold: i) the design of complementary filters with three inputs, hereby denoted as CF^3, with the purpose of merging signals provided by three different sensors, leading to minimal realizations, stable and with strong frquency domain interpretation; ii) to enphasize that the optimality property of two sensors complementary filters is lost when extending the approach for three sensors

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fusion, as thus filters are related to a non-convex estimation problem, and iii) to design optimal linear Kalman filters for three sensors, preserving the complementary filter property, with optimal performance, both in continuous and discrete time domains, denoted as CKF^3.

This paper is organized as follows: Section II presents the synthesis and analysis of CF^3, aiming to fuse data from threee sensors. The optimal linear Kalman filter estimator CKF^3 design is detailed in section III, for the continuous time domain. Section IV presents the discrete time optimal complementary filter design and shows that the complementary property is preserved. Section V illustrates the experimental results obtained for the estimation of the attitude of a mobile robot, when resorting to the filters introduced in this work. Finally, Section VI presents some conclusions and unveils future work.

II. COMPLEMENTARY FILTER DESIGN WITH THREE FREQUENCY BANDS - CF^3

Complementary filters are estimators commonly used in mobile robot navigation, allowing the fusion of two sensors in order to estimate one unknown variable [8], [12]. However, a different approach will be presented in this section, aiming at the development of a robot attitude estimator inspired on a CF, merging signals provided by three different sensors, considering only the most reliable frequency range of each sensor response. Thus, from each sensor only part of frequency spectrum is used. All sensors together cover the whole spectrum, thus providing distortionless estimates of the unknown quantities. These filters will be denoted as CF^3. This means that one sensor complements others in frequency domain, thus the name complementary. The CF^3 estimator considers three inputs merged in different, yet complementary frequency bands. The frequency spectrum is split in three bands. The CF^3 is composed by a low-pass, a band-pass and a high-pass filter. Complementarity is achieved if the estimator output has a unitary magnitude gain over whole frequency spectrum.

To focus the reader's attention, lets adress the problem of attitude estimation for a mobile robot. Moreover, $\psi(s)$ denots the Laplace transform (LT) of the signal $\psi(t)$, abreviated in the sequel as ψ. Assuming that the sensors provide measurements related to the attitude, the filters are complementary if the following equality holds:

$$T_1(s) + T_2(s) + T_3(s) = 1 \tag{1}$$

where $T_1(s)$, $T_2(s)$ and $T_3(s)$ are the transfer functions from each of the sensors $i = 1, ..., 3$ to the robot attitude ψ, i.e. the CF^3 output.

Furthermore, being k_1 and k_2 positive parameters, then, one possible transfer function that relates the LT of the measured attitude $\psi(s)$ with its estimate $\hat{\psi}(s)$, is:

$$\hat{\psi}(s) = \psi(s) = \frac{s^2 + k_1 s + k_2}{s^2 + k_1 s + k_2}\psi(s) \tag{2}$$

This second order transfer function can be decomposed into three transfer functions, ensuring the equality given by equation (1), for the purpose of obtaining the transfer function that relates each input with the CF^3 output:

$$\hat{\psi}(s) = T_1(s)\psi(s) + T_2(s)\psi(s) + T_3(s)\psi(s) \tag{3}$$

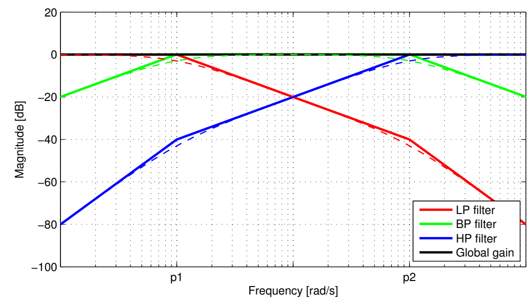

Fig. 1. CF^3 Bode diagram.

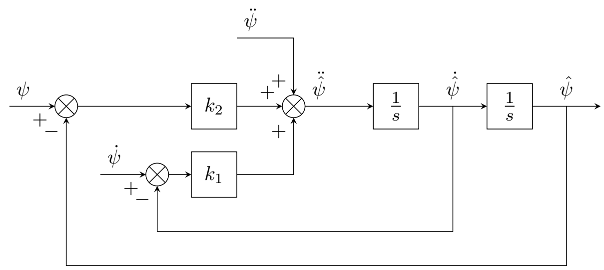

Fig. 2. CF^3 block diagram

where $T_1(s)$ is a high-pass filter, $T_2(s)$ a band-pass filter, and $T_3(s)$ a low-pass filter, characterized respectively by the following transfer functions:

$$T_1(s) = \frac{s^2}{s^2 + k_1 s + k_2} \tag{4}$$

$$T_2(s) = \frac{k_1 s}{s^2 + k_1 s + k_2} \tag{5}$$

$$T_3(s) = \frac{k_2}{s^2 + k_1 s + k_2}. \tag{6}$$

Figure 1 shows the Bode diagrams corresponding to the transfer functions given by equations (4)–(6). Unitary gain along the whole frequency spectrum is thus obtained. All filters have the same two eigenvalues, i.e. the roots of the characteristic equation, given by:

$$p_{T_{1,2,3}} = -\frac{k_1 \pm \sqrt{k_1^2 - 4k_2}}{2} \tag{7}$$

and the zeros for each filter can be obtained as follows,

$$z_{T_1} = \begin{cases} 0 \\ 0 \end{cases}, z_{T_2} = 0 \quad , z_{T_3} = \{\} \tag{8}$$

Reorganizing the transfer function given by equation (2), the model of a CF^3 could be analytically represented as the differential equation

$$\ddot{\hat{\psi}} = \ddot{\psi} + k_1(\dot{\psi} - \dot{\hat{\psi}}) + k_2(\psi - \hat{\psi}), \tag{9}$$

with the corresponding block diagram depicted in Fig. 2. Notice that the CF^3 has three inputs, allowing to merge the measurements provided by three sensors, each one corresponding to a different physical quatity, that can be denominated generically as (attitude) position (ψ), velocity ($\dot{\psi}$), and acceleration ($\ddot{\psi}$). The position signal is filtered by the low-pass filter, the velocity by the band-pass filter, and the acceleration by the high-pass filter. The filters gains can be computed based on filters parameters without the need to know the stochastic noise and any system physical model. Hence, to implement the CF^3, it is only needed to take into account the complementary filter property (as far as the frequencies spectrum is concerned) along with the characteristic of the individual sensors used, allocating the sensor signals to the corresponding filter input.

To analyze if the obtained CF^3 is optimal, the model given by equation (9) has been written with a space-state representation, considering the state and output vectors $\hat{\mathbf{x}} = \left[\begin{array}{cc} \hat{\psi} & \dot{\hat{\psi}} \end{array} \right]^T$ and $\mathbf{z} = \hat{\mathbf{x}}$, respectively:

$$\dot{\hat{\mathbf{x}}} = \mathbf{F}\hat{\mathbf{x}} + \mathbf{B}\ddot{\psi} + \mathbf{K}(\mathbf{z} - \mathbf{H}\hat{\mathbf{x}}) \quad (10)$$

$$\mathbf{z} = \mathbf{H}\mathbf{x} \quad (11)$$

with:

$$\mathbf{F} = \left[\begin{array}{cc} 0 & 1 \\ 0 & 0 \end{array} \right], \mathbf{B} = \left[\begin{array}{c} 0 \\ 1 \end{array} \right], \mathbf{H} = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right] \quad (12)$$

and the CF^3 gain matrix is given by,

$$\mathbf{K} = \left[\begin{array}{cc} 0 & 0 \\ k_2 & k_1 \end{array} \right]. \quad (13)$$

Notice that the gain matrix $\mathbf{K}$ is sparse, with a corresponding non-convex estimation problem. The most important consequence is that optimality is lost in general, i.e. tunning the gains k_1 and k_2 does not render the CF^3 optimal. To attain optimality, a linear Kalman filter must be designed for the problem at hand. In that case the gain matrix has the same dimensions but the $\mathbf{K}$ matrix is full. The objective of the next section is to detail the design of such optimal filter.

III. OPTIMAL COMPLEMENTARY KALMAN FILTER DESIGN - CKF^3

In this section, the relationship between the CF^3 and the equivalent CKF^3 is analyzed. Such approach allows the design of a fusion solution that best suits the sensors characteristics, preserving the the corresponding filter frequency band, while providing optimal estimates.

Pursuing the classical Kalman filter design approach, a full Kalman gain matrix $\mathbf{K}$ is obtained, with four gains to be determined, i.e.

$$\mathbf{K} = \left[\begin{array}{cc} k_{11} & k_{12} \\ k_{21} & k_{22} \end{array} \right]. \quad (14)$$

Using (14) in the model (10)–(12) leads to the CKF^3 filter shown in Fig. 3. Notice that two more gains are now present in this model, when compared with the suboptimal model depicted in Fig. 2. Analyzing the CKF^3 structure presented in Fig. 3, it can be concluded that the transfer functions relating the signal from each sensor with the corresponding filter output are as follows:

$$F_1(s) = \frac{\hat{\psi}(s)}{\ddot{\psi}(s)} = \frac{1}{(1+k_{12})s^2 + (k_{11}+k_{22})s + k_{21}} \quad (15)$$

$$F_2(s) = \frac{\hat{\psi}(s)}{\dot{\psi}(s)} = \frac{k_{12}s + k_{22}}{(1+k_{12})s^2 + (k_{11}+k_{22})s + k_{21}} \quad (16)$$

$$F_3(s) = \frac{\hat{\psi}(s)}{\psi(s)} = \frac{k_{11}s + k_{21}}{(1+k_{12})s^2 + (k_{11}+k_{22})s + k_{21}} \quad (17)$$

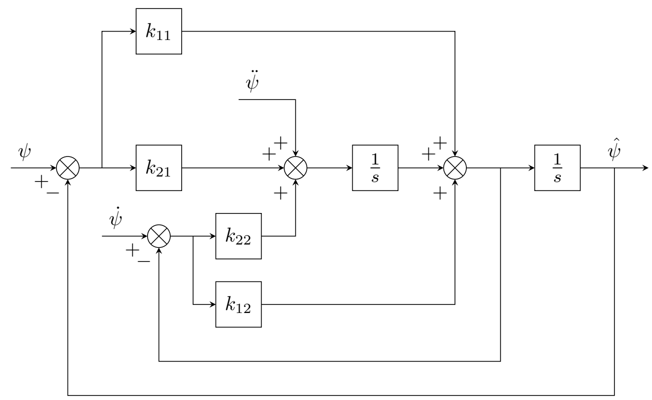

Fig. 3. CKF^3 block diagram.

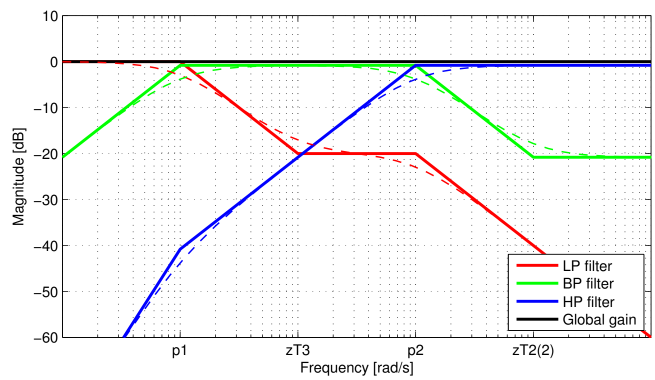

Fig. 4. CKF^3 Bode diagram.

and the global transfer function will be given by the following equation,

$$\hat{\psi}(s) = F_1(s)\ddot{\psi}(s) + F_2(s)\dot{\psi}(s) + F_3(s)\psi(s) \quad (18)$$

$$= F_1(s)s^2\psi(s) + F_2(s)s\psi(s) + F_3(s)\psi(s) \quad (19)$$

where the characteristics of the sensors where used.

Finally, equation (19) can be written as equation (3) being $T_1(s) = F_1(s)s^2$, $T_2(s) = F_2(s)s$ and $T_3(s) = F_3(s)$. Moreover, analyzing the transfer functions of each filter (15)–(17) it can be concluded that the condition expressed by equation (1) is also satisfied resulting a global transfer function as follows:

$$\hat{\psi}(s) = \frac{(1+k_{12})s^2 + (k_{11}+k_{22})s + k_{21}}{(1+k_{12})s^2 + (k_{11}+k_{22})s + k_{21}}\psi(s) \quad (20)$$

In conclusion, the use of the structure associated to the linear Kalman filter lead also to a complementary filter, with the advantage of attaining optimal performance, preserving the minimal representation and the stability.

Figure 4 shows the Bode diagram for the CKF^3. Once again, three filters working in complementary frequency bands and with unitary global gain along the whole frequency spectrum are observed. The Bode diagrams are designed according to the gain matrix , since there is a relationship between the filter gains and the corresponding system eigenvalues and zeros, which can be obtained from the transfer functions of each filter. Thus, all filters have the same two eigenvalues, given by:

$$p_{T_{1,2,3}} = -\frac{k_{11} + k_{22} \pm \sqrt{(k_{11} + k_{22})^2 - 4(1 + k_{12})k_{21}}}{2(1 + k_{12})} \quad (21)$$

and the zeros for each filter can be obtained as follows,

$$z_{T_1} = \begin{cases} 0 \\ 0 \end{cases}, z_{T_2} = \begin{cases} 0 \\ -\frac{k_{22}}{k_{12}} \end{cases}, z_{T_3} = -\frac{k_{21}}{k_{11}}. \quad (22)$$

The CKF^3 to provide optimal estimations, requires the computation of the optimal gains for the matrix $\mathbf{K}$, resorting to a classical Kalman filter design. This can be achieved through the stochastic characterization of the uncertainty present in the sensors measurements. Being $\mathbf{R}$ the covariance of the white Gaussian noise associated with the low-pass and band-pass filters (measurement noise), and $\mathbf{Q}$ the covariance of the white Gaussian noise associated with the high-pass filter (process noise), the CKF^3 optimal gains can be obtained as usual, i.e.

$$\mathbf{K} = \mathbf{PH^TR^{-1}} \quad (23)$$

where P is the covariance of the estimation error, solution of the algebraic Riccati equation,

$$\mathbf{FP} + \mathbf{PF^T} - \mathbf{PH^TR^{-1}HP} + \mathbf{Q} = \mathbf{0}. \quad (24)$$

The sensor fusion approach based on the CKF^3 depicted in Fig. 3 considers that each sensor measures a different physical quantities, related with the same variable. In this case position, velocity and acceleration of the attitude angle of a mobile robot. Furthermore, it is possible to obtain alternative CKF^3 estimator structures tacking into account alternative sensors to physical be merged. The CKF3 structure previously detailed fuses position, velocity, and acceleration measurements. In practice, for attitude estimation only "position" and "velocity" sensors are available. Thus, alternative structures can be obtained to merge several combinations of the sensors available.

With the purpose of implementing these new CKF^3 filtering solutions in a real application, discrete time models must be obtained. The new discrete time models must also be analyzed in order to study if the complementary property is preserved. This is the purpose of the next Section.

IV. OPTIMAL DISCRETE TIME COMPLEMENTARY KALMAN FILTER DESIGN - $DCKF^3$

Lets consider the purpose of implementing the CKF^3 in a digital processor, to estimate the mobile robot attitude, where signals are assumed to be constant between two sampling times (zero order hold assumption). To that purpose, a discrete complementary Kalman filter with three input signals ($DCKF^3$) will be presented in this section. The $DCKF^3$ design is developed considering the sensors physical measurements that will be merged and keeps the transfer function complementarity, providing optimal estimates. Consider the continuous model given by equations (10) and (11).

The step invariant discrete time linear model, for a sampling time T, is given by:

$$\hat{\mathbf{x}}(k+1) = \mathbf{F}\hat{\mathbf{x}}(k) + \mathbf{B}\omega(k) + \mathbf{K}(\mathbf{z}(k) - \mathbf{H}\hat{\mathbf{x}}(k)) \quad (25)$$

$$\mathbf{z}(k) = \mathbf{H}\mathbf{x}(k) \quad (26)$$

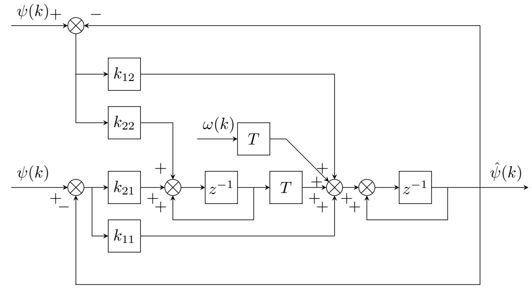

Fig. 5. Discrete time optimal $DCKF^3$ block diagram (LP: ψ, BP:ψ, HP:ω).

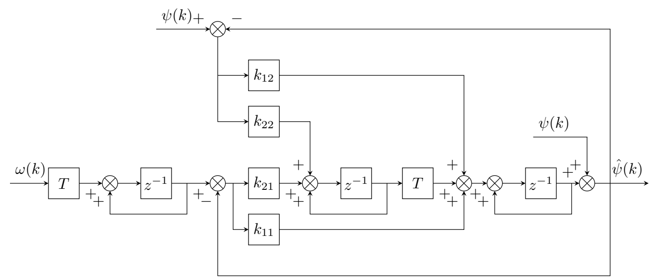

Fig. 6. Discrete time optimal $DCKF^3$ block diagram (LP: ω, BP:ψ, HP:ψ).

where $\omega(k)$ is the sampled angle rate, denoted previously as "velocity" and the system matrices are:

$$\mathbf{F} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} T \\ 0 \end{bmatrix}, \mathbf{H} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad (27)$$

corresponding to the block diagram depicted in Fig. 5. The discrete Kalman gain $\mathbf{K}$ is a full matrix, with a structure similar to that represented in equation (14), computed based on the discrete Kalman methodology, represented briefly in the sequel as

$$\mathbf{K} = \mathbf{PH^T}(\mathbf{HPH^T} + \mathbf{R})^{-1} \quad (28)$$

where $\mathbf{P}$ will be computed by solving the discrete time algebraic Riccati equation [3].

Analyzing equations (25)–(27) it can be observed that the $DCKF^3$ model was obtained from an estimator that receives $[\ \psi(k) \quad \psi(k) \quad \omega(k)\ ]^T$ signals in the low, band and high-pass filter inputs, respectively. However, as it will be shown ahead in a specific real application, the best solution to estimate the mobile robot attitude merging signals provided by gyros, compass and odometry sensors corresponding to have a $DCKF^3$ structure that receives $[\ \omega(k) \quad \psi(k) \quad \psi(k)\ ]^T$ as inputs. Formaly, two of these quantities are measurements in the Kalman filter and one is an input. Hence, an equivalent structure has been designed to cope with the connection of the rate-gyros, compass and odometry sensors, respectively in the correspondent $DCKF^3$ input being the corresponding block diagram depicted in Fig. 6.

Analyzing this filter, the transfer functions that relate the signals from each sensor with the filter output are as follows:

$$F_1(z) = \frac{\hat{\psi}(z)}{\psi(z)} = \frac{z^{-2} - 2z^{-1} + 1}{den(z)} \quad (29)$$

$$F_2(z) = \frac{\hat{\psi}(z)}{\psi(z)} = \frac{(-k_{12} + Tk_{22})z^{-2} + k_{12}z^{-1}}{den(z)} \quad (30)$$

$$F_3(z) = \frac{\hat{\psi}(z)}{\omega(z)} = \frac{(-Tk11 + T^2k_{21})z^{-3} + Tk_{11}z^{-2}}{(1 - z^{-1})den(z)} \quad (31)$$

where

$$den(z) = (1 - k_{12} - k_{11} + Tk_{21} + Tk_{22})z^{-2} + + (k_{11} + k_{12} - 2)z^{-1} + 1. \quad (32)$$

The global transfer function is thus given by:

$$\hat{\psi}(z) = F_1(z)\psi(z) + F_2(z)\psi(z) + F_3(z)\omega(s) \quad (33)$$

Establishing $T_1(z) = F_1(z)$, $T_2(z) = F_2(z)$ and $T_3(z) = F_3(z)\frac{1-z^{-1}}{Tz^{-1}}$, the complementary property in the discrete-time is once again satisfied, i.e.

$$T_1(z) + T_2(z) + T_3(z) = 1. \quad (34)$$

Thus, the complementary property is preserved for the discrete time filters $DCKF^3$.

V. EXPERIMENTAL VALIDATION

A. Simulations Results

The proposed optimal CF^3 design was tested and simulated for the attitude estimation of a mobile robot. The estimated attitude was obtained from the measures provided by different combinations of three sensors with different characteristics, namely a rate gyro, a compass, and odometry. The robot trajectory simulated was a round corners figure square. The robot kept straight during 600 s and turned smoothly 90° to the left during 10 s. After four times, the robot was back to the initial position having performed the round corners square trajectory. In the performed simulations, the following noises associated with the sensors were added to the model:

- Rate-gyros - Gaussian noise $\mu_g \sim N(0, 2 \times 10^{-3}$ and a bias of 0.8 °$/s$. However, the bias can be compensated, without disturbing the CF^3 input signal, because it has been observed in a real experiment that after the temperature stabilization that occurs in 20 min the rate-gyros bias remains approximately constant.
- Compass - Gaussian noise $\mu_c \sim N(0, 2 \times 10^{-3})$ and a magnetic perturbation of $20\sin(\psi_c)$;
- Odometry - Gaussian noise $\mu_o \sim N(0, 4 \times 10^{-6})$ and a bias of 0.5 °$/s$. Such a value could change in a real control loop implementation.

Once known the sensors stochastic characteristics, the attitude estimation was simulated considering several fusion alternatives, with the corresponding optimal gains. For the two sensor fusion case (CF), the attitude is estimated by the classical discrete Kalman Filter with two inputs, thus optimal but with only a subset of the sensors available. For the three sensor fusion case, the CKF^3 solutions were exploited. In both cases the estimators parameters were tuned, resulting in optimal gains. Tables I and II show the Mean Square

TABLE I. MSE FOR DIFFERENT CF - SIMULATION RESULTS

Low-pass	High-pass	MSE [rad^2]
$\dot{\psi}_{rate-gyros}$	$\psi_{odometry}$	2.01×10^{-3}
$\dot{\psi}_{rate-gyros}$	$\psi_{compass}$	2.02×10^{-3}
$\psi_{compass}$	$\psi_{odometry}$	2.62×10^{-2}
$\psi_{compass}$	$\dot{\psi}_{rate-gyros}$	2.53×10^{-2}
$\psi_{odometry}$	$\psi_{compass}$	1.41×10^{2}
$\psi_{odometry}$	$\dot{\psi}_{rate-gyros}$	1.41×10^{2}

TABLE II. MSE FOR DIFFERENT CF^3 - SIMULATION RESULTS

Low-pass	Band-pass	High-pass	MSE [rad^2]
$\dot{\psi}_{rate-gyros}$	$\psi_{compass}$	$\psi_{odometry}$	1.91×10^{-3}
$\dot{\psi}_{rate-gyros}$	$\psi_{odometry}$	$\psi_{compass}$	1.91×10^{-3}
$\psi_{compass}$	$\dot{\psi}_{rate-gyros}$	$\psi_{odometry}$	2.70×10^{-2}
$\psi_{compass}$	$\psi_{odometry}$	$\dot{\psi}_{rate-gyros}$	2.61×10^{-2}
$\psi_{odometry}$	$\dot{\psi}_{rate-gyros}$	$\psi_{compass}$	1.46×10^{2}
$\psi_{odometry}$	$\psi_{compass}$	$\dot{\psi}_{rate-gyros}$	1.46×10^{2}

Error (MSE) for the different combinations, considering the classical CF structure (with two inputs) and the proposed CF^3 structure with three inputs, respectively. Analyzing Table I it can be observed that the CF inputs combination that provides estimations with less MSE is the one that has the rate-gyro signal as input of the low-pass filter and the odometry as input of the high-pass filter. The same combination adding now the compass as input of the band-pass filter, thus resulting in a CF^3, is the best solution. See Table II) where a slighly better performance is achieved. Furthermore, the addition of more one sensor increases the estimator robustness when in presence of erroneous signals or sensor faults.

B. Experimental Results

Aiming to validate the proposed methodology in a real application, the $DCKF^3$ has been implemented using a low cost mobile robotic platform [4], with the configuration of a Dubins car. Such a platform is equipped with two encoders coupled to the motors, a digital compass located on the extension arm (robot rear part) to avoid the magnetic interference from the motors, and a rate-gyro over the platform (Fig. 7). To test the mobile robot attitude estimation the proposed $DCKF^3$ was tested considering a trajectory combining both straight lines (constant attitude) with semi-circumferences (linear attitude changing) allowing to assess the localization system operation under different experimental conditions. The tests were performed with a 0.1 $\mathrm{m} \cdot \mathrm{s}^{-1}$ robot velocity and 5 Hz of sampling frequency. During the robot motion the real mobile robot trajectory is measured allowing the comparison of the

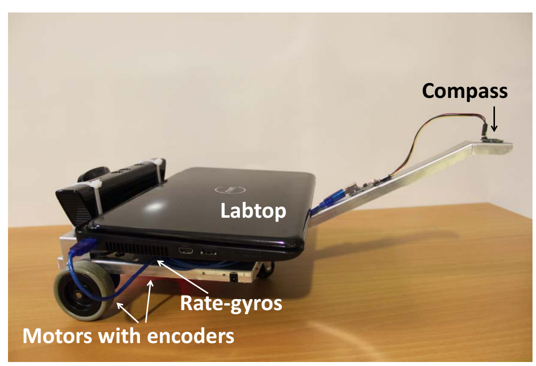

Fig. 7. Mobile robot platform

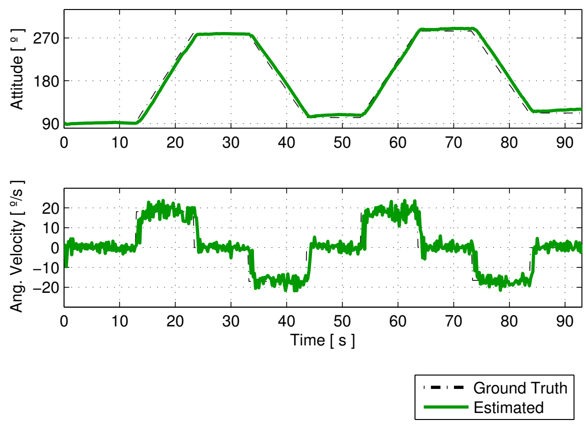

Fig. 8. Attitude and angular velocity estimated - experimental results

TABLE III. CF ESTIMATION ERROR ANALYSIS - EXPERIMENTAL RESULTS

quantity	average	variance
$\tilde{\psi}$ $[rad]$	-3.73×10^{-2}	1.9×10^{-2}
$\tilde{\omega}$ $[rad/s]$	-1.7×10^{-3}	5.3×10^{-3}

estimated position with the real one (ground truth test) and the corresponding error is analyzed. As it is possible to see in Fig. 8 the attitude and the angular velocity estimated by the $DCKF^3$ are very close to the ground truth (Table III).

Figure 9 shows the dead-reckoning position computed using the Linear Parametric Varying (LPV) model proposed in [5] considering now the angular velocity computed by the estimated attitude numerical differentiation obtained from $DCKF^3$. Analyzing the results depicted in Fig. 9 it can be observed that the dead-reckoning results are close to the ground truth trajectory.

VI. CONCLUSIONS

This paper introduced a new class of complementary filters extenting the by now classic sensor fusion complementary filter (CF) design. In the cases addressed in this work, three sensors are usedt providing measurements in different bands. First, an extension (CF^3) is proposed, but leads to a sub-optimal solution. Then, the problem at hand was reformulated and casted as a optimal linear Kalman filtering (CKF^3). Moreover, this solution is shown to preserve the complementary property, i.e. the sum of the three transfer functions of the respective sensors add up to one. The same synthesis and analyzes was done for the discrete time case, thus leading to $DCKF^3$ filters domains, that were shown to preserve the complementary property.

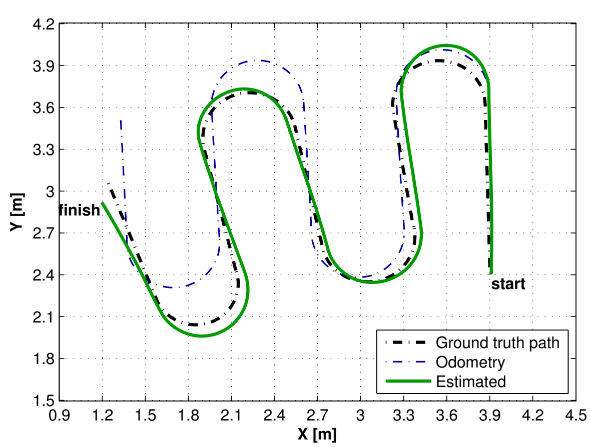

Fig. 9. Estimated dead-reckoning - experimental results

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