

A time differences of arrival-based homing strategy for autonomous underwater vehicles

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SUMMARY

A new sensor-based homing integrated guidance and control law is presented to drive an underactuated autonomous underwater vehicle (AUV) toward a fixed target, in 3-D, using the information provided by an ultra-short baseline (USBL) positioning system. The guidance and control law is first derived at a kinematic level, expressed on the space of the time differences of arrival (TDOAs), as directly measured by the USBL sensor, and assuming the plane wave approximation. Afterwards, the control law is extended for the dynamics of an underactuated AUV resorting to backstepping techniques. The proposed Lyapunov-based control law yields almost global asymptotic stability (AGAS) in the absence of external disturbances and is further extended, keeping the same properties, to the case where known ocean currents affect the motion of the vehicle. Simulations are presented and discussed that illustrate the performance and behavior of the overall closed-loop system in the presence of realistic sensor measurements and actuator saturation. Copyright © 2009 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Advances in sensing devices, materials, and computational power capabilities have provided the means to develop sophisticated underwater vehicles which nowadays display the capability to perform complex tasks in challenging and uncertain operation scenarios. In the last years several sophisticated autonomous underwater vehicles (AUVs) and remotely operated vehicles (ROVs) have been developed, endowing the scientific

community with advanced research tools supported in on-board complex mission and vehicle control systems, see [1–3]. This paper presents the design and performance evaluation of a sensor-based integrated guidance and control law to drive an underactuated AUV toward a fixed target.

The topics of navigation, guidance, and control of underwater vehicles have been subject of intense research in the past decades, presenting numerous challenges that range from technical limitations, arising due to the particular nature of the surrounding oceanic environment, to theoretical problems, which exist even for fully actuated underwater vehicles. Indeed, while the control of fully actuated systems is generally fairly well understood, as evidenced by the large body of publications, see [4–6] and the references therein, for underwater vehicles there are still interesting questions springing from, e.g. the lack of coupled experimentally validated dynamic models or the inability to readily

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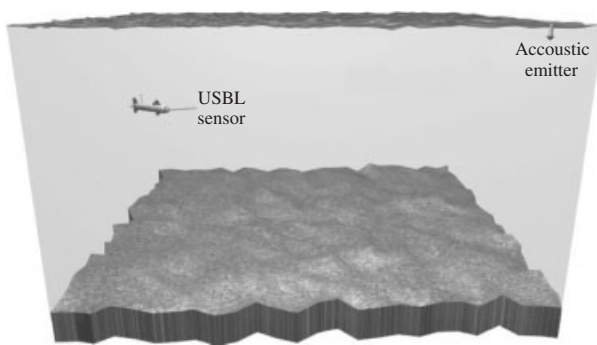


Figure 1. Mission scenario.

identify plant parameters, which exhibit, in general, strong nonlinear behaviors.

To tackle the problem of stabilization of an underactuated vehicle a variety of solutions have been proposed in the literature, e.g. [7–10]. In [11] a solution to the problem of following a straight line is presented and in [12] a way-point tracking controller for an underactuated AUV is introduced. A position and attitude tracking controller was proposed in [13], whereas trajectory tracking solutions for underactuated underwater vehicles were presented in, e.g. [14, 15]. The problem of path-following has also received much attention, see, e.g. [16, 17]. It turns out that all the aforementioned references share a common approach: the vehicle position is computed in the inertial coordinate frame and the control laws are developed in the body frame. Therefore, the computation of the linear tracking error vector is heavily affected by errors in the estimates of the attitude of the vehicle. Sensor-based control has been a hot topic in the field of computer vision where the so-called visual servoing techniques have been subject of intensive research effort during the last years, see [18, 19] for further information.

This paper addresses the design of a sensor-based integrated guidance and control law to drive an underactuated AUV to a fixed target, in 3-D. The solution for this problem, usually denominated as homing in the literature, is central to drive the vehicle to the vicinity of a base station or support vessel. This task is critical to the success of long-term autonomous operation of AUVs since it allows for the vehicle to approach

a base station or support vessel, which often offer docking capabilities and permits the AUV to sleep, recharge its batteries, transfer data, and download a new mission program. Once the vehicle is close enough to the base, different strategies are required to safely lock the AUV in the dock. This last stage, usually denominated as docking in the literature, may vary significantly depending on the vehicle itself, the location, and the type of docking station. It also usually requires extra aiding sensors, e.g. optical or electromagnetic aiding sensors, see [20–23] for further details on this subject.

In this paper it is assumed that an acoustic emitter is installed on a predefined fixed position in the mission scenario, denominated as target in the sequel, and an ultra-short baseline (USBL) sensor, composed by an array of hydrophones, is rigidly mounted on the nose of the vehicle, as depicted in Figure 1. During the homing phase the target continuously emits acoustic signals that are received by the USBL hydrophone array and the time of arrival measured by each receiver, is synchronized, detected, and recorded. In the approach followed, it is assumed, for the sake of simplicity, that the target is placed sufficiently far from the source to allow for the successful use of the planar wave approximation. This is valid when the distance between the source and the USBL array is large when compared with both the wavelength and the distance between the USBL receivers, which happens during the homing stage. A Lyapunov-based guidance and control law is first derived at a kinematic level, directly expressed in terms of the time differences of arrival (TDOAs) obtained from the USBL data. The resulting control law is then extended for the dynamics of an underactuated AUV resorting to backstepping techniques. Afterwards, this strategy is further extended to the case where known ocean currents affect the motion of the vehicle and almost global asymptotic stability (AGAS) [24] is achieved in both situations. The implementation of the control laws also requires the linear velocity of the vehicle, inertial and relative to the water, as provided by a Doppler velocity log or a Navigation System, and the vehicle attitude and angular velocity, measured by an attitude and heading reference system (AHRS). Preliminary work by the authors can be found in [25], which was expanded and improved. An alternative solution, which expressed the error

kinematics in the form of a quaternion directly obtained from the TDOAs, is detailed in [26].

The paper is organized as follows. In Section 2, the homing problem is introduced and the dynamics of the AUV are briefly described. Section 3 presents the USBL model, whereas in Section 4 a solution for the control and guidance problem in the absence of external perturbations is proposed. This control law is further extended in Section 5 to the case where ocean currents affect the motion of the vehicle. Simulation results are presented and discussed, for both cases, and considering sensor noise and actuator saturation, in Section 6, and finally Section 7 summarizes the main conclusions and results of the paper.

2. PROBLEM STATEMENT

Let $\{I\}$ be an inertial coordinate frame and $\{B\}$ the body-fixed coordinate frame, whose origin is located at the center of mass of the vehicle (see [27] for a thorough discussion of the coordinate frame conventions). Consider $\mathbf{p} = [x, y, z]^T$ as the position of the origin of $\{B\}$, described in $\{I\}$, $\mathbf{v} = [u, v, w]^T$ the linear velocity of the vehicle relative to $\{I\}$, expressed in body-fixed coordinates, and $\boldsymbol{\omega} = [p, q, r]^T$ the angular velocity, also expressed in body-fixed coordinates. The vehicle linear motion kinematics can be written as

$$\dot{\mathbf{p}} = \mathbf{R}\mathbf{v} \quad (1)$$

where $\mathbf{R}$ is the rotation matrix from $\{B\}$ to $\{I\}$ verifying

$$\dot{\mathbf{R}} = \mathbf{R}\mathbf{S}(\boldsymbol{\omega}) \quad (2)$$

where $\mathbf{S}(\mathbf{x})$ is the skew-symmetric matrix such that $\mathbf{S}(\mathbf{x})\mathbf{y} = \mathbf{x} \times \mathbf{y}$, with $\times$ denoting the cross product.

The vehicle dynamic equations of motion can be written, in a compact form, as [27]

$$\begin{aligned} \mathbf{M}\dot{\mathbf{v}} &= -\mathbf{S}(\boldsymbol{\omega})\mathbf{M}\mathbf{v} - \mathbf{D}_v(\mathbf{v})\mathbf{v} - \mathbf{g}_v(\mathbf{R}) + \mathbf{b}_v u_v \\ \mathbf{J}\dot{\boldsymbol{\omega}} &= -\mathbf{S}(\mathbf{v})\mathbf{M}\mathbf{v} - \mathbf{S}(\boldsymbol{\omega})\mathbf{J}\boldsymbol{\omega} \\ &\quad - \mathbf{D}_\omega(\boldsymbol{\omega})\boldsymbol{\omega} - \mathbf{g}_\omega(\mathbf{R}) + \mathbf{B}_\omega \mathbf{u}_\omega \end{aligned} \quad (3)$$

where $\mathbf{M} = \text{diag}\{m_u, m_v, m_w\}$ is a positive-definite diagonal mass matrix; $\mathbf{J} = \text{diag}\{J_{xx}, J_{yy}, J_{zz}\}$ is a positive-definite inertia matrix; $\mathbf{u}_v = \tau_u$ is the force

control input that acts along the x_B axis; $\mathbf{u}_\omega = [\tau_q, \tau_r]^T$ are the torque control inputs that affect the rotation of the vehicle about the y_B and z_B axes, respectively; $\mathbf{D}_v(\mathbf{v}) = \text{diag}\{X_u + X_{|u|u}|u|, Y_v + Y_{|v|v}|v|, Z_w + Z_{|w|w}|w|\}$ is the positive-definite matrix of the linear motion drag coefficients; $\mathbf{D}_\omega(\boldsymbol{\omega}) = \text{diag}\{K_p + K_{|p|p}|p|, M_q + M_{|q|q}|q|, N_r + N_{|r|r}|r|\}$ is the positive-definite matrix of the rotational motion drag coefficients; $\mathbf{b}_v = [1, 0, 0]^T$ and

$$\mathbf{B}_\omega = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$$

$\mathbf{g}_v(\mathbf{R}) = \mathbf{R}^T[0, 0, W - B]^T$ represents the gravitational and buoyancy effects, W and B , respectively, on the linear motion of the vehicle; $\mathbf{g}_\omega(\mathbf{R}) = \mathbf{S}(\mathbf{r}_B)\mathbf{R}^T[0, 0, B]^T$ accounts for the effect of the center of buoyancy displacement relatively to the center of mass, $\mathbf{r}_B$, on the vehicle rotational motion.

Matrices $\mathbf{M}$, $\mathbf{D}_v(\mathbf{v})$, and $\mathbf{D}_\omega(\boldsymbol{\omega})$ include the hydrodynamic derivatives of the underwater vehicle, which account for the interaction of the hull of the vehicle with the surrounding fluid and capture the variation of the forces and torques experienced by the vehicle. With a certain abuse of language, the hydrodynamic derivatives can be viewed as resulting from a Taylor series expansion of the forces and torques about the nominal operating condition, see [28] for a detailed introduction of these coefficients, which were rearranged in matrix form in [27].

Assume that the vehicle is neutrally buoyant, i.e. $W = B$ and therefore $\mathbf{g}_v(\mathbf{R}) = \mathbf{0}$. Further consider that the added masses associated with the sway and heave motions are similar, that is $m_v \simeq m_w$, which constitutes a reasonable assumption for most underwater vehicles and is simply a question of plane of symmetry in the geometry of the hull of the vehicle.

The homing problem considered in this paper can be stated as follows:

Problem Statement: Consider an underactuated AUV with kinematics and dynamics given by (1) and (3), respectively. Assume that there is a target placed in a fixed position, in 3-D, that emits continuously a well-known acoustic signal. Design a sensor-based integrated guidance and control law to drive the vehicle toward the target using the time differences of arrival

of the acoustic signal as measured by an USBL sensor installed on the AUV.

3. USBL MODEL

During the homing approach phase the vehicle is far away from the acoustic emitter, that is, the distance from the vehicle to the target is much larger than the distance between any pair of receivers. Therefore, the plane-wave assumption is valid. Let $\mathbf{r}_i = [x_i, y_i, z_i]^T \in \mathbb{R}^3, i = 1, 2, \dots, N$, denote the positions of the N acoustic receivers installed on the USBL sensor and consider a plane-wave traveling along the opposite direction of the unit vector $\mathbf{d} = [d_x, d_y, d_z]^T$, as shown in Figure 2. Notice that both $\mathbf{r}_i$ and $\mathbf{d}$ are expressed in the body frame. Let t_i be the instant of time of arrival of the plane-wave at i th receiver and V_p the velocity of propagation of the sound in water. Then, assuming that the medium is homogeneous and neglecting the velocity of the vehicle, which is a reasonable assumption since $\|\mathbf{v}\| \ll V_p$, the time difference of arrival between receivers i and j satisfies

$$V_p(t_i - t_j) = -[d_x(x_i - x_j) + d_y(y_i - y_j) + d_z(z_i - z_j)] \quad (4)$$

Denote by $\Delta_1 = t_1 - t_2, \Delta_2 = t_1 - t_3, \dots, \Delta_M = t_{N-1} - t_N$ all the possible combinations of differences of times of arrival between pairs of receivers, and let $\Delta = [\Delta_1, \Delta_2, \dots, \Delta_M]^T$ denote the corresponding vector

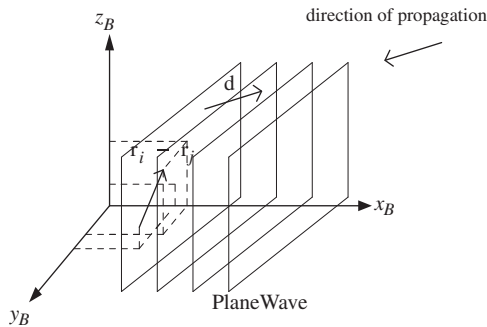


Figure 2. Plane wave and the USBL system.

of TDOAs. Define also

$$\mathbf{r}_x = [x_1 - x_2, x_1 - x_3, \dots, x_{N-1} - x_N]^T$$

$$\mathbf{r}_y = [y_1 - y_2, y_1 - y_3, \dots, y_{N-1} - y_N]^T$$

$$\mathbf{r}_z = [z_1 - z_2, z_1 - z_3, \dots, z_{N-1} - z_N]^T$$

and $\mathbf{H}_R \in \mathbb{R}^{M \times 3}$ as

$$\mathbf{H}_R = [\mathbf{r}_x, \mathbf{r}_y, \mathbf{r}_z]$$

Then, the generalization of (4) for all TDOAs yields

$$\Delta = -\frac{1}{V_p} \mathbf{H}_R \mathbf{d} \quad (5)$$

To compute the time derivative of (5) recall that the direction of propagation $\mathbf{d}$ satisfies

$$\mathbf{d} = \mathbf{R}^{T/I} \mathbf{d} \quad (6)$$

where $^I \mathbf{d}$ is the direction of propagation expressed in the inertial frame $\{I\}$. Substituting (6) in (5) and taking its time derivative gives

$$\dot{\Delta} = -\frac{1}{V_p} \mathbf{H}_R \frac{d}{dt} (\mathbf{R}^{T/I} \mathbf{d}) \quad (7)$$

Owing to the plane-wave assumption, that assumes the target at infinity, the direction of propagation expressed in the inertial frame is constant. Therefore, (7) can be simplified to give

$$\dot{\Delta} = -\frac{1}{V_p} \mathbf{H}_R \dot{\mathbf{R}}^{T/I} \mathbf{d} \quad (8)$$

Substituting (2) in (8) and simplifying yields

$$\dot{\Delta} = -\frac{1}{V_p} \mathbf{H}_R \mathbf{S}^T(\omega) \mathbf{R}^{T/I} \mathbf{d}$$

Recalling that $\mathbf{S}(\omega)$ is a skew-symmetric matrix and using (6) gives

$$\dot{\Delta} = \frac{1}{V_p} \mathbf{H}_R \mathbf{S}(\omega) \mathbf{d} \quad (9)$$

To write the time derivative of the TDOA vector Δ in closed form, define $\mathbf{H}_Q \in \mathbb{R}^{3 \times 3}$ as

$$\mathbf{H}_Q = \frac{1}{V_p} \mathbf{H}_R^T \mathbf{H}_R \quad (10)$$

which is assumed to be nonsingular. This turns out to be a weak hypothesis as it is always true if, at least, 4 receivers are mounted in concoplanar positions. In those conditions $\mathbf{H}_R$ has maximum rank and so does $\mathbf{H}_Q$. Then,

$$\mathbf{d} = -\mathbf{H}_Q^{-1} \mathbf{H}_R^T \Delta \quad (11)$$

Substituting (11) in (9) gives

$$\dot{\Delta} = -\frac{1}{V_p} \mathbf{H}_R \mathbf{S}(\omega) \mathbf{H}_Q^{-1} \mathbf{H}_R^T \Delta \quad (12)$$

which corresponds to a closed form for the dynamics of Δ .

4. CONTROLLER DESIGN

In this section an integrated nonlinear closed-loop guidance and control law is derived for the homing problem stated earlier in Section 2. Assuming that there are no ocean currents, the idea behind the control strategy proposed here is to steer the vehicle directly toward the emitter. The synthesis of the guidance and control law resorts extensively to Lyapunov's direct method and backstepping techniques.

To steer the vehicle toward the target, consider first the error variable

$$\mathbf{z}_1 := \Delta + \frac{1}{V_p} \mathbf{r}_x \quad (13)$$

As $\mathbf{z}_1$ converges to zero, the x -axis of the vehicle aligns itself in the direction of the target. However, this condition is not sufficient to ensure the desired behavior of the vehicle during the homing phase as it can still move away from the target. In order to avoid that, define a second scalar error variable

$$z_2 := [1, 0, 0] \mathbf{v} - V_d$$

where V_d is a positive constant that corresponds to the desired velocity during the homing stage. When z_2 converges to zero, the surge velocity u converges to the desired velocity V_d . Since the vehicle is correctly aligned if $\mathbf{z}_1$ is driven to zero, one could think that ensuring that both $\mathbf{z}_1$ and z_2 converge to zero, the vehicle would always approach the target. However,

this is not true as the sway and heave velocities are left free. Despite that, it will be shown that, with the control law based upon these two error variables, the sway and heave components of the velocity vector converge to zero, which completes a set of sufficient conditions that solves the problem at hand.

To synthesize the control law start by defining the Lyapunov functions

$$V_1 = \frac{1}{2} \mathbf{z}_1^T \mathbf{H}_L \mathbf{z}_1 \quad (14)$$

where

$$\mathbf{H}_L = (\mathbf{H}_Q^{-1} \mathbf{H}_R^T)^T (\mathbf{H}_Q^{-1} \mathbf{H}_R^T) \quad (15)$$

and

$$V_2 = \frac{1}{2} z_2^2 \quad (16)$$

Computing the time derivative of (16) one obtains

$$\dot{V}_2 = z_2 \dot{z}_2 = z_2 [1, 0, 0] \dot{\mathbf{v}} \quad (17)$$

Replacing the dynamics of $\mathbf{v}$ given by (3) in (17) yields

$$\begin{aligned} \dot{V}_2 &= z_2 [1, 0, 0] \mathbf{M}^{-1} \mathbf{b}_v u_v \\ &\quad - z_2 ([1, 0, 0] \mathbf{M}^{-1} [\mathbf{S}(\omega) \mathbf{M} \mathbf{v} + \mathbf{D}_v(\mathbf{v}) \mathbf{v}]) \end{aligned}$$

Choosing

$$u_v = \frac{[1, 0, 0] \mathbf{M}^{-1} [\mathbf{S}(\omega) \mathbf{M} \mathbf{v} + \mathbf{D}_v(\mathbf{v}) \mathbf{v}] - k_2 z_2}{[1, 0, 0] \mathbf{M}^{-1} \mathbf{b}_v} \quad (18)$$

where $k_2 > 0$ is a control gain, the time derivative of (16) becomes

$$\dot{V}_2 = -k_2 z_2^2$$

which yields global asymptotic stability of z_2 . Furthermore, the convergence is exponentially fast.

Using the fact that the matrix $\mathbf{H}_L$ is symmetric, the time derivative of (14) is given by

$$\dot{V}_1 = \mathbf{z}_1^T \mathbf{H}_L \dot{\mathbf{z}}_1 = \mathbf{z}_1^T \mathbf{H}_L \dot{\Delta} \quad (19)$$

Substituting (12), (13), and (15) in (19) gives

$$\begin{aligned} \dot{V}_1 &= -\frac{1}{V_p} \left(\Delta + \frac{1}{V_p} \mathbf{r}_x \right)^T (\mathbf{H}_Q^{-1} \mathbf{H}_R^T)^T \\ &\quad \mathbf{H}_Q^{-1} \mathbf{H}_R^T \mathbf{H}_R \mathbf{S}(\omega) \mathbf{H}_Q^{-1} \mathbf{H}_R^T \Delta \end{aligned} \quad (20)$$

Using (10) in (20) gives

$$\begin{aligned} \dot{V}_1 = & -(\mathbf{H}_Q^{-1}\mathbf{H}_R^T\Delta)^T\mathbf{S}(\omega)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\Delta - \left(\mathbf{H}_Q^{-1}\mathbf{H}_R^T\frac{1}{V_p}\mathbf{r}_x\right)^T \\ & \times \mathbf{S}(\omega)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\Delta \end{aligned} \quad (21)$$

Since $\mathbf{S}(\omega)$ is a skew-symmetric matrix, the first term in (21) is zero. On the other hand, it is easy to see, from (5) and (11), that

$$-\mathbf{H}_Q^{-1}\mathbf{H}_R^T\frac{1}{V_p}\mathbf{r}_x = -[1 \ 0 \ 0]^T$$

Therefore, it is possible to rewrite (21) as

$$\dot{V}_1 = -[1 \ 0 \ 0]^T\mathbf{S}(\omega)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\Delta$$

Finally, using the property

$$[1 \ 0 \ 0]^T\mathbf{S}(\omega) = -\omega^T\mathbf{S}([1 \ 0 \ 0]^T),$$

the time derivative $\dot{V}_1$ is given by

$$\dot{V}_1 = \omega^T\mathbf{S}([1 \ 0 \ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\Delta$$

Following the standard backstepping technique it is now possible to regard ω as a virtual control input that can be used to make $\dot{V}_1 \leq 0$. This is achieved by setting $\mathbf{B}_\omega^T\omega$ equal to $\mathbf{B}_\omega^T\omega_d$, where

$$\omega_d := -\mathbf{K}_1\mathbf{S}([1 \ 0 \ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\Delta$$

and $\mathbf{K}_1 = \text{diag}\{0, k_{12}, k_{13}\}$, $k_{12} > 0$, $k_{13} > 0$ is a control gain matrix. To accomplish this define a third error variable

$$\mathbf{z}_3 = \mathbf{B}_\omega^T(\omega - \omega_d)$$

and the augmented Lyapunov function

$$V_3 = V_1 + \frac{1}{2}\mathbf{z}_3^T\mathbf{z}_3 = \frac{1}{2}\mathbf{z}_1^T\mathbf{H}_L\mathbf{z}_1 + \frac{1}{2}\mathbf{z}_3^T\mathbf{z}_3$$

The time derivative of V_3 can now be written as

$$\begin{aligned} \dot{V}_3 = & -[\mathbf{S}([1 \ 0 \ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\Delta]^T \\ & \times \mathbf{K}_1[\mathbf{S}([1 \ 0 \ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\Delta] \\ & - \mathbf{z}_3^T\mathbf{B}_\omega^T\mathbf{J}^{-1}[\mathbf{S}(\mathbf{v})\mathbf{M}\mathbf{v} + \mathbf{S}(\omega)\mathbf{J}\omega \\ & + \mathbf{D}_\omega(\omega)\omega + \mathbf{g}_\omega(\mathbf{R})] \end{aligned}$$

$$\begin{aligned} & -\mathbf{z}_3^T\mathbf{B}_\omega^T(\dot{\omega}_d - \mathbf{S}([1 \ 0 \ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\Delta) \\ & + \mathbf{z}_3^T\mathbf{B}_\omega^T\mathbf{J}^{-1}\mathbf{B}_\omega\mathbf{u}_\omega \end{aligned}$$

Setting

$$\begin{aligned} \mathbf{u}_\omega = & (\mathbf{B}_\omega^T\mathbf{J}^{-1}\mathbf{B}_\omega)^{-1}[\mathbf{B}_\omega^T(\mathbf{J}^{-1}[\mathbf{S}(\mathbf{v})\mathbf{M}\mathbf{v} + \mathbf{S}(\omega)\mathbf{J}\omega \\ & + \mathbf{D}_\omega(\omega)\omega + \mathbf{g}_\omega(\mathbf{R})] + \dot{\omega}_d \\ & - \mathbf{S}([1 \ 0 \ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\Delta) - \mathbf{K}_3\mathbf{z}_3] \end{aligned} \quad (22)$$

where $\mathbf{K}_3$ is a positive-definite control gain matrix, one obtains $\dot{V}_3 \leq 0$, with $\dot{\omega}_d$ given by

$$\dot{\omega}_d = \mathbf{K}_1\mathbf{S}([1 \ 0 \ 0]^T)\mathbf{S}(\omega)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\Delta$$

The following theorem states the main result of this section.

Theorem 1

Consider a vehicle with kinematics and dynamics given by Equations (1) and (3), respectively, moving in the absence of ocean currents. Then, with the control law (18) and (22), the error variable z_2 converges globally asymptotically to zero and almost global asymptotic stability is warranted for the error variables $\mathbf{z}_1$ and $\mathbf{z}_3$. Furthermore, the sway, heave, and roll velocities converge to zero, solving the homing problem stated in Section 2.

Proof

Before going into the details a sketch of the proof is first offered. The convergence of the error variables in $\mathbf{z}_1$, $\mathbf{z}_2$, and $\mathbf{z}_3$ is a straightforward application of Lyapunov's second method. The analysis of the vehicle equations of motion, when $\mathbf{z}_1$, $\mathbf{z}_2$, and $\mathbf{z}_3$ converge to zero, allows to conclude the convergence to zero of the sway, heave, and roll velocities.

The Lyapunov function V_2 is, by construction, continuous, radially unbounded, and positive definite. With the control law (18), the time derivative $\dot{V}_2$ results negative definite. Therefore, the origin $z_2 = 0$ is a global asymptotic stable equilibrium point. Furthermore, since

$$\dot{V}_2 = -k_2 z_2^2 = -2k_2 V_2$$

z_2 converges exponentially fast to zero.

The function V_3 is, also by construction, continuous, radially unbounded, and positive definite for feasible values of $\mathbf{z}_1$. This can be easily shown as expanding V_3 gives

$$V_3 = \frac{1}{2}(\mathbf{d} - [1, 0, 0]^T)^T(\mathbf{d} - [1, 0, 0]^T) + \frac{1}{2}\mathbf{z}_3^T \mathbf{z}_3 > 0 \quad \forall \mathbf{d} \neq [1, 0, 0]^T \wedge \mathbf{z}_3 \neq 0 \Leftrightarrow \mathbf{z}_1 \neq \mathbf{0} \wedge \mathbf{z}_3 \neq 0$$

Moreover, with the control law (22), the time derivative $\dot{V}_3$ results in

$$\dot{V}_3 = -[\mathbf{S}([1 \ 0 \ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\Delta]^T\mathbf{K}_1[\mathbf{S}([1 \ 0 \ 0]^T) \times \mathbf{H}_Q^{-1}\mathbf{H}_R^T\Delta] - \mathbf{z}_3^T\mathbf{K}_3\mathbf{z}_3$$

which is negative semi-definite. The derivative $\dot{V}_3$ has two zeros, one coincident with the origin, ($\mathbf{z}_1=\mathbf{0}, \mathbf{z}_3=\mathbf{0}$), to which corresponds

$$\Delta = -\frac{1}{V_p}\mathbf{r}_x$$

and the other one at ($\mathbf{z}_1=(2/V_p)\mathbf{r}_x, \mathbf{z}_3=\mathbf{0}$), to which corresponds

$$\Delta = \frac{1}{V_p}\mathbf{r}_x$$

that is,

$$\dot{V}_3=0 \Leftrightarrow (\mathbf{z}_1=\mathbf{0}, \mathbf{z}_3=\mathbf{0}) \vee \left(\mathbf{z}_1 = \frac{2}{V_p}\mathbf{r}_x, \mathbf{z}_3=\mathbf{0} \right)$$

The equilibrium point that does not coincide with the origin corresponds to the situation where the vehicle is aligned toward the opposite direction of the target. It is now important to prove that this equilibrium point is an unstable equilibrium point. To show that, consider the function

$$V_i = \frac{1}{2}\mathbf{z}_i^T\mathbf{H}_L\mathbf{z}_i - \frac{1}{2}\mathbf{z}_3^T\mathbf{z}_3 \quad (23)$$

where

$$\mathbf{z}_i = \Delta - \frac{1}{V_p}\mathbf{r}_x$$

The time derivative of (23) can be written as

$$\dot{V}_i = [\mathbf{S}([1 \ 0 \ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\Delta]\mathbf{K}_1[\mathbf{S}([1 \ 0 \ 0]^T) \times \mathbf{H}_Q^{-1}\mathbf{H}_R^T\Delta] + \mathbf{z}_3^T\mathbf{K}_3\mathbf{z}_3$$

Since $V_i(\mathbf{0})=0$, $V_i(\mathbf{z}_i, \mathbf{z}_3)$ can assume strictly positive values arbitrarily close to the origin, and $\dot{V}_i$ is positive definite in a neighborhood of the origin, then the origin of V_i is unstable ([29], Theorem 4.4). Therefore, the only stable equilibrium point of V_3 is the origin ($\mathbf{0}, \mathbf{0}$). Thus, almost global asymptotic convergence of the error variables ($\mathbf{z}_1, \mathbf{z}_3$) to the origin is achieved.

To complete the stability analysis all that is left to do is to show that the sway, heave, and roll velocities converge to zero. The dynamics of the sway and heave velocities can be written as

$$\begin{bmatrix} \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -\frac{Y_v + Y_{|v|v}|v|}{m_v} & \frac{m_w}{m_v}p \\ -\frac{m_v}{m_w}p & -\frac{Z_w + Z_{|w|w}|w|}{m_w} \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} + \begin{bmatrix} -\frac{m_u}{m_v}ur \\ \frac{m_u}{m_w}uq \end{bmatrix}$$

From the definition of the desired angular velocity ω_d , when $\mathbf{z}_1$ converges to zero, so does ω_d . On the other hand, when $\mathbf{z}_3$ converges to zero, the pitch and yaw components of the angular velocity converge to the corresponding components of the desired angular velocity. Therefore, taking the limit of the pitch and yaw velocities when $\mathbf{z}=(\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3)$ converges to zero yields

$$\lim_{\mathbf{z} \rightarrow \mathbf{0}} \begin{bmatrix} q \\ r \end{bmatrix} = \mathbf{0}$$

On the other hand, u converges to the desired velocity V_d .

Therefore, the dynamics of the sway and heave velocities can be written as the linear time-varying system (LTVS) driven by a vanishing disturbance $\mathbf{u}_d(t)$

$$\begin{bmatrix} \dot{v} \\ \dot{w} \end{bmatrix} = \mathbf{A}(t) \begin{bmatrix} v \\ w \end{bmatrix} + \mathbf{u}_d(t) \quad (24)$$

where

$$\mathbf{A} = \begin{bmatrix} -\frac{Y_v + Y_{|v|v}|v|}{m_v} & \frac{m_w}{m_v} p \\ -\frac{m_v}{m_w} p & -\frac{Z_w + Z_{|w|w}|w|}{m_w} \end{bmatrix}$$

Let

$$\mathbf{E}(t) = \frac{1}{2}[\mathbf{A}(t) + \mathbf{A}^T(t)].$$

Notice that, using the assumption $m_v = m_w$, $\mathbf{E}(t)$ is a negative definite diagonal matrix. Consider the Lyapunov-like function

$$V_l = \frac{1}{2} \mathbf{z}_l^T \mathbf{z}_l$$

where

$$\mathbf{z}_l = \begin{bmatrix} v \\ w \end{bmatrix}$$

It is straightforward to show that

$$\begin{aligned} \dot{V}_l &= \mathbf{z}_l^T \mathbf{E}(t) \mathbf{z}_l + \mathbf{z}_l^T \mathbf{u}_d \\ &\leq -K_l \|\mathbf{z}_l\|^2 + \|\mathbf{z}_l\| \|\mathbf{u}_d\| \end{aligned}$$

where $K_l > 0$ corresponds to the absolute value of an upper bound for the maximum eigenvalue of $\mathbf{E}(t)$ for $t > t_0$. Let $0 < \theta < 1$. Then,

$$\begin{aligned} \dot{V}_l &\leq -(1-\theta)K_l \|\mathbf{z}_l\|^2 - \theta K_l \|\mathbf{z}_l\|^2 + \|\mathbf{z}_l\| \|\mathbf{u}_d\| \\ &\leq -(1-\theta)K_l \|\mathbf{z}_l\|^2 - \theta K_l \|\mathbf{z}_l\| \left(\|\mathbf{z}_l\| - \frac{1}{\theta K_l} \|\mathbf{u}_d\| \right) \\ &\leq -(1-\theta)K_l \|\mathbf{z}_l\|^2 \quad \forall \|\mathbf{z}_l\| \geq \frac{1}{\theta K_l} \|\mathbf{u}_d\| \end{aligned}$$

Therefore (24) is input-to-state stable (ISS) with $\mathbf{u}_d$ as input (see [30]). Since $\mathbf{u}_d$ converges to zero, it follows that so do the sway and heave velocities.

Finally, although the dynamic equations of the roll velocity are cumbersome, it is easy to see, from (3), that they are similar to the equations of a second-order underdamped pendulum [31] driven by a vanishing torque, as the sway, heave, pitch, and yaw velocities

converge to zero. Indeed, the dynamics of the roll may be written as

$$\begin{aligned} \dot{p}(t) &= f(t, p(t), u(t)), \\ f(t, p(t), u(t)) &= -g_1(t) \sin \left(\alpha_0 + \int_{t_0}^t p(\tau) d\tau \right) \\ &\quad - g_2(t) p(t) + u(t) \end{aligned}$$

where $g_1(t)$ and $g_2(t)$ are strictly positive bounded functions and $u(t)$ accounts for the effect of the remaining velocities on the dynamics of $p(t)$ and converges to zero. The first function, $g_1(t)$, accounts for the restoring buoyancy torque, while $g_2(t) = K_p + K_{|p|p}|p|$ represents the drag torque. Therefore, it is easy to show that

$$\begin{aligned} &-p_1 \sin \left(\alpha_0 + \int_{t_0}^t p(\tau) d\tau \right) - p_2 p(t) - U_m \\ &\leq f(t, p(t), u(t)) \leq -p_3 \sin \left(\alpha_0 + \int_{t_0}^t p(\tau) d\tau \right) \\ &\quad - p_4 p(t) + U_M \end{aligned} \quad (25)$$

for all time $t > t^*$, where α_0 , p_1 , p_2 , p_3 , p_4 , U_m , and U_M are positive constants. The left and right side of (25) correspond to the velocity dynamics of an underdamped pendulum driven by a constant torque. Since $u(t)$ converges to zero, it is possible to choose t^* such that U_m and U_M are small enough so that the velocity of these underdamped pendulums converges to zero [31]. Therefore, from (25), it follows that the roll velocity converges to zero. $\square$

5. CONTROL IN THE PRESENCE OF OCEAN CURRENTS

In this section the results from the previous sections are generalized for the case where known ocean currents are present. Consider that the vehicle is moving with water relative velocity $\mathbf{v}_r = [u_r, v_r, w_r]^T$, expressed in the body-fixed coordinate frame, as measured by a Doppler Velocity Log, and that the water is also moving with constant velocity $\mathbf{v}_c$ relatively to the inertial frame,

also expressed in body-fixed coordinates. Then, the dynamics of the vehicle can be rewritten as

$$\begin{aligned} \mathbf{M}\dot{\mathbf{v}}_r &= -\mathbf{S}(\omega)\mathbf{M}\mathbf{v}_r - \mathbf{D}_{v_r}(\mathbf{v}_r)\mathbf{v}_r + \mathbf{b}_v u_v \\ \mathbf{J}\dot{\omega} &= -\mathbf{S}(\mathbf{v}_r)\mathbf{M}\mathbf{v}_r - \mathbf{S}(\omega)\mathbf{J}\omega - \mathbf{D}_\omega(\omega)\omega \\ &\quad - \mathbf{g}_\omega(\mathbf{R}) + \mathbf{B}_\omega \mathbf{u}_\omega \end{aligned} \quad (26)$$

and the velocity of the vehicle relative to the inertial frame, expressed in body-fixed coordinates, is $\mathbf{v} = \mathbf{v}_r + \mathbf{v}_c$.

Under these conditions it is possible to conclude that the guidance and control strategy synthesized in Section 4 does not solve the current homing problem, as the new control objective is to align the velocity of the vehicle relatively to the inertial frame toward the target instead of the x -axis of the vehicle. Consider the vehicle reference relative velocity $\mathbf{v}_R$ given by

$$\mathbf{v}_R := [V_d, 0, 0]^T$$

that corresponds to a desired velocity relative to $\{I\}$ and expressed in $\{B\}$ of

$$\mathbf{v}_d = \mathbf{v}_R + \mathbf{v}_c$$

The vehicle is moving toward the target when the velocity vector $\mathbf{v}$ is aligned with the direction of the target, which corresponds to a desired TDOA vector given by

$$\Delta_d = -\frac{1}{V_p} \mathbf{H}_R \mathbf{d}_d$$

where

$$\mathbf{d}_d = \frac{\mathbf{v}_R + \mathbf{v}_c}{\|\mathbf{v}_R + \mathbf{v}_c\|}$$

Obviously the previous statement is only valid if $V_d + V_c \cos(\theta_c) > 0$, where $V_c \cos(\theta_c)$ represents the projection of the current on the vehicle's x -axis. Otherwise, it would be impossible for the vehicle to approach the target as the surge vehicle velocity is lower than the projection of the current velocity.

To solve the homing problem in the presence of currents consider the error variables

$$\mathbf{z}_1 := \Delta + \frac{1}{V_p} \mathbf{H}_R \mathbf{d}_d \quad (27)$$

and

$$z_2 := [1, 0, 0] \mathbf{v}_r - V_d$$

The convergence of the error variable z_2 to zero can be achieved following a similar procedure as the one presented in Section 4. Defining the Lyapunov function

$$V_2 = \frac{1}{2} z_2^2$$

it is straightforward to show that setting

$$u_v = \frac{[1, 0, 0] \mathbf{M}^{-1} [\mathbf{S}(\omega) \mathbf{M} \mathbf{v}_r + \mathbf{D}_{v_r}(\mathbf{v}_r) \mathbf{v}_r] - k_2 z_2}{[1, 0, 0] \mathbf{M}^{-1} \mathbf{b}_v} \quad (28)$$

the time derivative of V_2 becomes $\dot{V}_2 = -k_2 z_2^2$. Therefore, z_2 converges exponentially fast to zero.

To drive $\mathbf{z}_1$ to zero, consider the same Lyapunov function as in Section 4,

$$V_1 = \frac{1}{2} \mathbf{z}_1^T \mathbf{H}_L \mathbf{z}_1$$

Using the fact that $\mathbf{H}_L$ is symmetric the time derivative of V_1 is given by

$$\dot{V}_1 = \mathbf{z}_1^T \mathbf{H}_L \dot{\mathbf{z}}_1 \quad (29)$$

Since the velocity of the fluid expressed in the inertial coordinate frame is constant, the time derivative of $\mathbf{v}_c$ is $\dot{\mathbf{v}}_c = -\mathbf{S}(\omega) \mathbf{v}_c$. On the other hand, $\mathbf{v}_R$ is a constant vector. Therefore, the time derivative of $\mathbf{d}_d$ reads as

$$\begin{aligned} \dot{\mathbf{d}}_d &= -\frac{1}{\|\mathbf{v}_R + \mathbf{v}_c\|} \mathbf{S}(\omega) \mathbf{v}_c - \frac{\frac{d}{dt} \|\mathbf{v}_R + \mathbf{v}_c\|}{\|\mathbf{v}_R + \mathbf{v}_c\|^2} (\mathbf{v}_R + \mathbf{v}_c) \\ &= -\frac{1}{\|\mathbf{v}_R + \mathbf{v}_c\|} \mathbf{S}(\omega) \mathbf{v}_c \\ &\quad + \frac{(\mathbf{v}_R + \mathbf{v}_c)^T \mathbf{S}(\omega) \mathbf{v}_c}{\|\mathbf{v}_R + \mathbf{v}_c\|^2} \mathbf{d}_d \end{aligned} \quad (30)$$

Since $\mathbf{S}(\omega)$ is a skew-symmetric matrix, and from the definition of $\mathbf{d}_d$, it is possible to rewrite (30) as

$$\begin{aligned} \dot{\mathbf{d}}_d &= -\mathbf{S}(\omega) \mathbf{d}_d + \frac{\mathbf{v}_R^T \mathbf{S}(\omega) \mathbf{v}_c}{\|\mathbf{v}_R + \mathbf{v}_c\|^2} \mathbf{d}_d \\ &\quad + \frac{1}{\|\mathbf{v}_R + \mathbf{v}_c\|} \mathbf{S}(\omega) \mathbf{v}_R \end{aligned} \quad (31)$$

From (12) and (31), it is straightforward to conclude that the time derivative of $\mathbf{z}_1$ can be written as

$$\dot{\mathbf{z}}_1 = -\frac{1}{V_p} \mathbf{H}_R \left[\mathbf{S}(\omega) \mathbf{H}_Q^{-1} \mathbf{H}_R^T \Delta + \mathbf{S}(\omega) \mathbf{d}_d - \frac{\mathbf{v}_R^T \mathbf{S}(\omega) \mathbf{v}_c}{\|\mathbf{v}_R + \mathbf{v}_c\|^2} \mathbf{d}_d - \frac{1}{\|\mathbf{v}_R + \mathbf{v}_c\|} \mathbf{S}(\omega) \mathbf{v}_R \right] \quad (32)$$

Substituting (15), (27), and (32) in (29) gives

$$\begin{aligned} \dot{V}_1 = & - \left(\Delta + \frac{1}{V_p} \mathbf{H}_R \mathbf{d}_d \right)^T (\mathbf{H}_Q^{-1} \mathbf{H}_R^T)^T \\ & \times \mathbf{H}_Q^{-1} \mathbf{H}_R^T \frac{1}{V_p} \mathbf{H}_R \mathbf{S}(\omega) [\mathbf{H}_Q^{-1} \mathbf{H}_R^T \Delta + \mathbf{d}_d] \\ & + \mathbf{z}_1^T (\mathbf{H}_Q^{-1} \mathbf{H}_R^T)^T \mathbf{H}_Q^{-1} \mathbf{H}_R^T \frac{1}{V_p} \mathbf{H}_R \\ & \times \left[\frac{\mathbf{v}_R^T \mathbf{S}(\omega) \mathbf{v}_c}{\|\mathbf{v}_R + \mathbf{v}_c\|^2} \mathbf{d}_d + \frac{1}{\|\mathbf{v}_R + \mathbf{v}_c\|} \mathbf{S}(\omega) \mathbf{v}_R \right] \end{aligned} \quad (33)$$

Substituting (10) in (33) gives

$$\begin{aligned} \dot{V}_1 = & - \left(\mathbf{H}_Q^{-1} \mathbf{H}_R^T \Delta + \frac{1}{V_p} \mathbf{H}_Q^{-1} \mathbf{H}_R^T \mathbf{H}_R \mathbf{d}_d \right)^T \\ & \times \mathbf{S}(\omega) [\mathbf{H}_Q^{-1} \mathbf{H}_R^T \Delta + \mathbf{d}_d] + \mathbf{z}_1^T (\mathbf{H}_Q^{-1} \mathbf{H}_R^T)^T \\ & \times \left[\frac{\mathbf{v}_R^T \mathbf{S}(\omega) \mathbf{v}_c}{\|\mathbf{v}_R + \mathbf{v}_c\|^2} \mathbf{d}_d + \frac{1}{\|\mathbf{v}_R + \mathbf{v}_c\|} \mathbf{S}(\omega) \mathbf{v}_R \right] \\ = & - (\mathbf{H}_Q^{-1} \mathbf{H}_R^T \Delta + \mathbf{d}_d)^T \mathbf{S}(\omega) [\mathbf{H}_Q^{-1} \mathbf{H}_R^T \Delta + \mathbf{d}_d] \\ & + \mathbf{z}_1^T (\mathbf{H}_Q^{-1} \mathbf{H}_R^T)^T \left[\frac{\mathbf{v}_R^T \mathbf{S}(\omega) \mathbf{v}_c}{\|\mathbf{v}_R + \mathbf{v}_c\|^2} \mathbf{d}_d \right. \\ & \left. + \frac{1}{\|\mathbf{v}_R + \mathbf{v}_c\|} \mathbf{S}(\omega) \mathbf{v}_R \right] \end{aligned} \quad (34)$$

Since $\mathbf{S}(\omega)$ is skew-symmetric, it is possible to simplify (34), as given by

$$\begin{aligned} \dot{V}_1 = & (\mathbf{H}_Q^{-1} \mathbf{H}_R^T \mathbf{z}_1)^T \\ & \times \left[\frac{\mathbf{v}_R^T \mathbf{S}(\omega) \mathbf{v}_c}{\|\mathbf{v}_R + \mathbf{v}_c\|^2} \mathbf{d}_d + \frac{1}{\|\mathbf{v}_R + \mathbf{v}_c\|} \mathbf{S}(\omega) \mathbf{v}_R \right] \end{aligned} \quad (35)$$

Finally, rearranging the terms of (35), the time derivative of V_1 is given by

$$\dot{V}_1 = \frac{V_d}{\|\mathbf{v}_R + \mathbf{v}_c\|} \omega^T \mathbf{S}([1 \ 0 \ 0]^T) \omega_c$$

where

$$\omega_c = \mathbf{H}_Q^{-1} \mathbf{H}_R^T \mathbf{z}_1 - \frac{1}{\|\mathbf{v}_R + \mathbf{v}_c\|} \left[(\mathbf{H}_Q^{-1} \mathbf{H}_R^T \mathbf{z}_1)^T \mathbf{d}_d \right] \mathbf{v}_c$$

Just like in Section 4, it is now possible to regard ω as a virtual control input that one can use to make $\dot{V}_1 \leq 0$. This is achieved by setting $\mathbf{B}_\omega^T \omega$ equal to $\mathbf{B}_\omega^T \omega_d$, with

$$\omega_d := -\mathbf{K}_1 \mathbf{S}([1, 0, 0]^T) \omega_c$$

where $\mathbf{K}_1 = \text{diag}\{0, k_{12}, k_{13}\}$, $k_{12} > 0$, $k_{13} > 0$, is a control gain matrix. To accomplish this, consider a third error variable defined as

$$\mathbf{z}_3 = \mathbf{B}_\omega^T (\omega - \omega_d)$$

and the augmented Lyapunov function

$$V_3 = V_1 + \frac{1}{2} \mathbf{z}_3^T \mathbf{z}_3 = \frac{1}{2} \mathbf{z}_1^T \mathbf{H}_L \mathbf{z}_1 + \frac{1}{2} \mathbf{z}_3^T \mathbf{z}_3$$

The time derivative of V_3 can be written as

$$\begin{aligned} \dot{V}_3 = & -\frac{V_d}{\|\mathbf{v}_R + \mathbf{v}_c\|} [\mathbf{S}([1, 0, 0]^T) \omega_c]^T \\ & \times \mathbf{K}_1 \mathbf{S}([1, 0, 0]^T) \omega_c \\ & - \mathbf{z}_3^T \mathbf{B}_\omega^T (\mathbf{J}^{-1} [\mathbf{S}(\mathbf{v}) \mathbf{M} \mathbf{v} + \mathbf{S}(\omega) \mathbf{J} \omega \\ & + \mathbf{D} \omega(\omega) \omega + \mathbf{g} \omega(\mathbf{R})]) \\ & + \mathbf{z}_3^T \mathbf{B}_\omega^T \left(-\dot{\omega}_d + \frac{V_d}{\|\mathbf{v}_R + \mathbf{v}_c\|} \mathbf{S}([1, 0, 0]^T) \omega_c \right) \\ & + \mathbf{z}_3^T \mathbf{B}_\omega^T \mathbf{J}^{-1} \mathbf{B}_\omega \mathbf{u}_\omega \end{aligned}$$

For the sake of simplicity the derivative of ω_d is not presented here. Now, setting

$$\begin{aligned} \mathbf{u}_\omega = & (\mathbf{B}_\omega^T \mathbf{J}^{-1} \mathbf{B}_\omega)^{-1} \\ & \times \left[\mathbf{B}_\omega^T \left(\mathbf{J}^{-1} [\mathbf{S}(\mathbf{v}) \mathbf{M} \mathbf{v} + \mathbf{S}(\omega) \mathbf{J} \omega + \mathbf{D} \omega(\omega) \omega \right. \right. \end{aligned}$$

$$\begin{aligned} & +\mathbf{g}\omega(\mathbf{R})]+\dot{\omega}_d-\frac{V_d}{\|\mathbf{v}_R+\mathbf{v}_c\|}\mathbf{S}([1, 0, 0]^T\omega_c) \\ & -\mathbf{K}_3\mathbf{z}_3 \end{aligned} \quad (36)$$

where $\mathbf{K}_3 \in \mathbb{R}^{2 \times 2}$ is a positive-definite control gain matrix, $\dot{V}_3$ becomes

$$\begin{aligned} \dot{V}_3 = & -\frac{V_d}{\|\mathbf{v}_R+\mathbf{v}_c\|}[\mathbf{S}([1, 0, 0]^T\omega_c)]^T \\ & \times \mathbf{K}_1[\mathbf{S}([1, 0, 0]^T\omega_c)]-\mathbf{z}_3^T\mathbf{K}_3\mathbf{z}_3 \end{aligned} \quad (37)$$

which is negative semi-definite.

The following theorem is the main result of this section.

Theorem 2

Consider a vehicle with kinematics and dynamics given by Equations (1) and (26), respectively, moving in the presence of ocean currents. Then, with the control law (28) and (36), the error variable z_2 converges globally asymptotically to zero and almost global asymptotic stability is warranted for the error variables in $\mathbf{z}_1$ and $\mathbf{z}_3$. Furthermore, the sway, heave, and roll velocities converge to zero, solving the homing problem stated in Section 2 in the presence of constant ocean currents.

Proof

The proof of convergence of the error variables $\mathbf{z}_1$, z_2 , and $\mathbf{z}_3$ is similar to the one presented in Theorem 1. When $\mathbf{z}_1$, z_2 , and $\mathbf{z}_3$ converge to zero so do the heave and sway velocities. Using similar arguments as in Theorem 1, the resulting roll motion converges to zero, which completes a set of sufficient conditions that ensures that the proposed control law solves the homing problem.

The convergence analysis of the error variable z_2 is the same as the one presented in Theorem 1 and therefore it is omitted.

The Lyapunov function V_3 is, by construction, continuous, radially unbounded, and positive definite for feasible values of $\mathbf{z}_1$. It can be easily shown that V_3 satisfies

$$\begin{aligned} V_3 = & \frac{1}{2}(\mathbf{d}-\mathbf{d}_d^T)^T(\mathbf{d}-\mathbf{d}_d^T)+\frac{1}{2}\mathbf{z}_3^T\mathbf{z}_3 \\ > 0 \forall & \mathbf{d} \neq \mathbf{d}_d^T \wedge \mathbf{z}_3 \neq 0 \Leftrightarrow \mathbf{z}_1 \neq 0 \wedge \mathbf{z}_3 \neq 0 \end{aligned}$$

Moreover, using the control law (36), its time derivative is given by (37), which is negative semi-definite. In order to proceed the solution of $\dot{V}_3=0$ must be computed. Recalling that $\mathbf{K}_1=\text{diag}\{0, k_{12}, k_{13}\}$, it is straightforward to conclude that

$$\dot{V}_3=0 \Leftrightarrow \omega_c=[\alpha, 0, 0]^T \wedge \mathbf{z}_3=0$$

After a few computations, it can be shown that

$$\dot{V}_3=0 \Leftrightarrow (\mathbf{z}_1=0, \mathbf{z}_3=0) \vee \left(\mathbf{z}_1=\frac{2}{V_p}\mathbf{H}_R\mathbf{d}_d, \mathbf{z}_3=0 \right)$$

from which can be concluded that the time derivative of V_3 has two zeros, one coincident with the origin and another that corresponds to the situation where the direction of the final desired velocity $\mathbf{d}_d$ points in the opposite direction of the target. However, this last zero turns out to correspond to an unstable equilibrium point of V_3 . To show this, consider the function

$$V_i=\frac{1}{2}\mathbf{z}_i^T\mathbf{H}_L\mathbf{z}_i-\frac{1}{2}\mathbf{z}_3^T\mathbf{z}_3$$

where

$$\mathbf{z}_i=\mathbf{\Lambda}-\frac{1}{V_p}\mathbf{H}_R\mathbf{d}_d$$

and the time derivative of V_i can be written as

$$\begin{aligned} \dot{V}_i = & \frac{V_d}{\|\mathbf{v}_R+\mathbf{v}_c\|}[\mathbf{S}([1, 0, 0]^T\omega_c)]^T \\ & \mathbf{K}_1\mathbf{S}([1, 0, 0]^T\omega_c)+\mathbf{z}_3^T\mathbf{K}_3\mathbf{z}_3 \end{aligned}$$

Since $V_i(\mathbf{0})=0$, $V_i(\mathbf{z}_i, \mathbf{z}_3)$ assumes strictly positive values near the origin, and $\dot{V}_i$ is positive definite in a neighborhood of the origin, then the origin of V_i is unstable ([29], Theorem 4.4). Therefore, the only stable equilibrium point of V_3 is the origin $(\mathbf{0}, \mathbf{0})$, thus achieving almost global asymptotic convergence of the error variables $(\mathbf{z}_1, \mathbf{z}_3)$.

The completion of the proof follows the proof of Theorem 1. $\square$

6. SIMULATION RESULTS

To illustrate the performance of the proposed integrated guidance and control laws three computer simulations

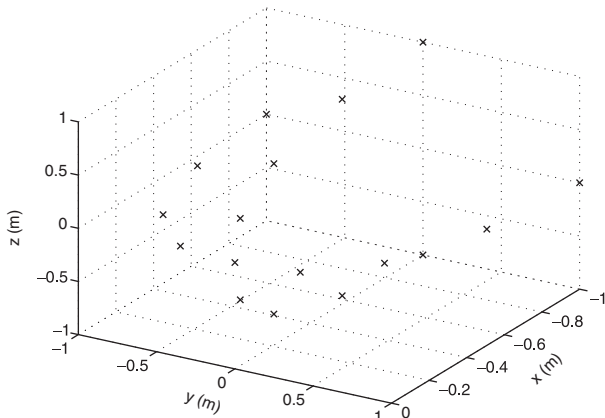


Figure 3. USBL semi-spherical geometry.

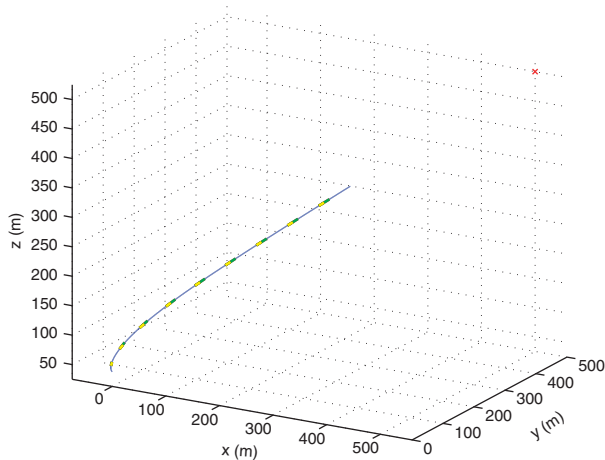


Figure 4. Trajectory described by the vehicle.

are presented in this section. In the simulations a simplified model of the SIRENE vehicle was used, assuming the vehicle is directly actuated in force and torque [3].

In the first simulation there are no external disturbances. The vehicle starts at position $[0, 0, 50]^T$ m and the acoustic pinger is located at position $[500, 500, 500]^T$ m. The control parameters were chosen as follows: $\mathbf{K}_1 = \text{diag}(0, 10^{-4}, 10^{-4})$, $k_2 = 0.025$ and $\mathbf{K}_3 = \text{diag}(40, 40)$. The magnitude of the gains was chosen in simulation by taking into account the resulting performance of the closed-loop system

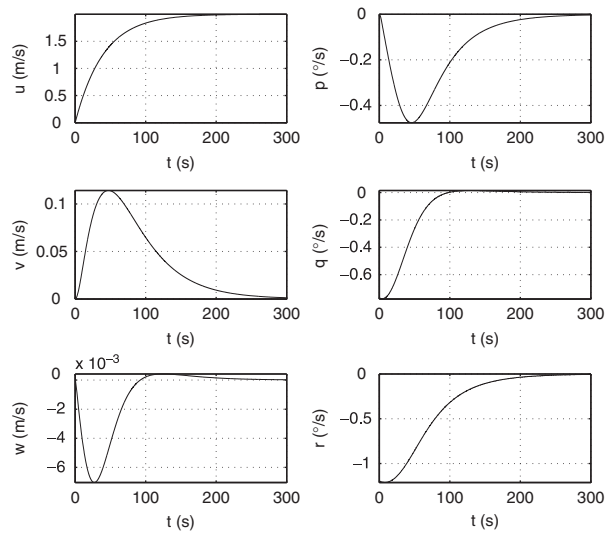


Figure 5. Time evolution of body-fixed velocities of the vehicle.

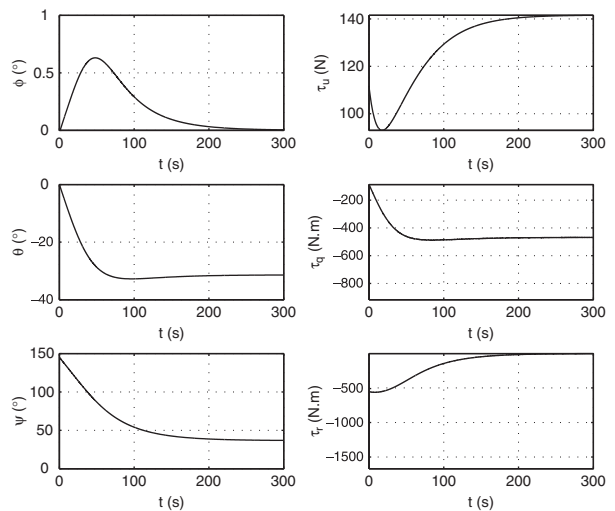


Figure 6. Time evolution of Euler angles and control inputs.

and the actuation signal characteristics. The desired velocity was set to $V_d = 2$ m/s, and a semi-spherical symmetric USBL sensor, with seventeen receivers, was placed on the vehicle's nose, as shown in Figure 3. Figure 4 shows the trajectory described by the vehicle, whereas Figure 5 depicts the evolution of the linear and

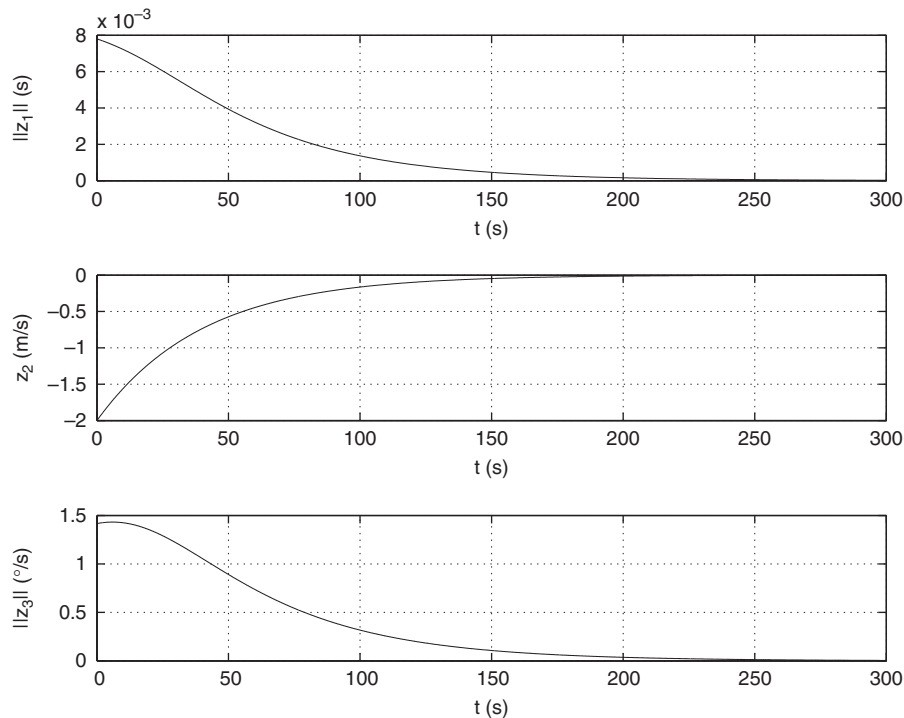


Figure 7. Time evolution of the error variables $\|z_1\|$, z_2 , $\|z_3\|$.

angular velocities. The evolution of the control inputs, as well as the roll, pitch, and yaw Euler angles, is shown in Figure 6. From the figures it can be concluded that the vehicle is driven toward the target describing a smooth trajectory. The control inputs are smooth and the resulting angular and lateral velocities converge to zero, as expected. For the sake of completeness, the evolution of the error variables is shown in Figure 7, more specifically, the evolution of z_2 and the norms of z_1 and z_3 . This plot clearly shows the convergence to zero of the error variables.

In the second simulation the vehicle has to counteract an ocean current with velocity $[0, -1, 0]^T$ m/s, expressed in the inertial frame. The control parameters are the same as in the previous simulation. Figure 8 shows the trajectory described by the vehicle, whereas the linear and angular velocities are depicted in Figure 9. The evolution of the control inputs and the roll, pitch, and yaw Euler angles is depicted in

Figure 10. As expected the trajectory and control inputs are smooth and the angular, sway, and heave velocities also converge to zero.

The third simulation is similar to the second but realistic sensor noise was considered, as well as actuators saturation. In particular, the measurements of the vehicle velocity relative to the water were assumed to be corrupted by zero-mean white Gaussian noise with standard deviation of 0.01 m/s. The AHRS was assumed to provide the roll, pitch, and yaw Euler angles, also corrupted by white Gaussian noises with standard deviation of 0.03° for the roll and pitch and 1° for the yaw, and the angular velocity corrupted with Gaussian noise with standard deviation of $0.1^\circ/\text{s}$. The noise of the USBL sensor is also zero-mean white Gaussian noise, and the standard deviation of the error on the TDOAs was set to $1\ \mu\text{s}$. Figure 11 shows the trajectory described by the vehicle, whereas the evolution of the linear and angular velocities is depicted in Figure 12. Finally, the

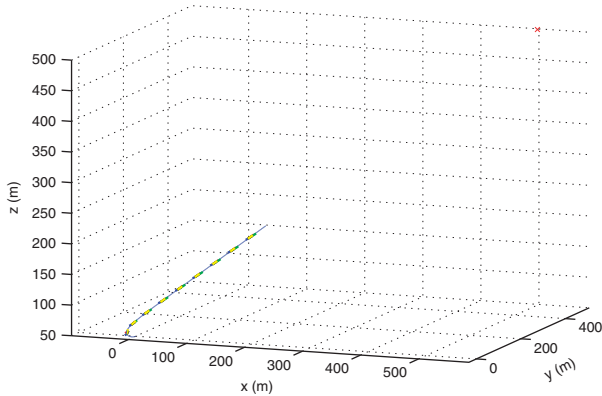


Figure 8. Trajectory described by the vehicle in the presence of currents.

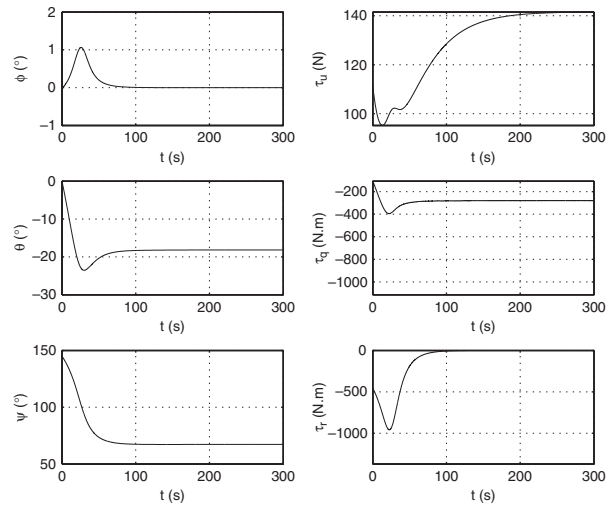


Figure 10. Time evolution of Euler angles and control inputs in the presence of currents.

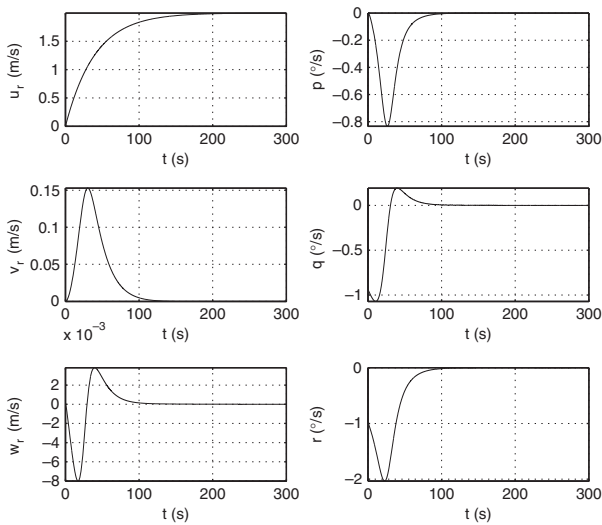


Figure 9. Time evolution of body-fixed velocities of the vehicle in the presence of currents.

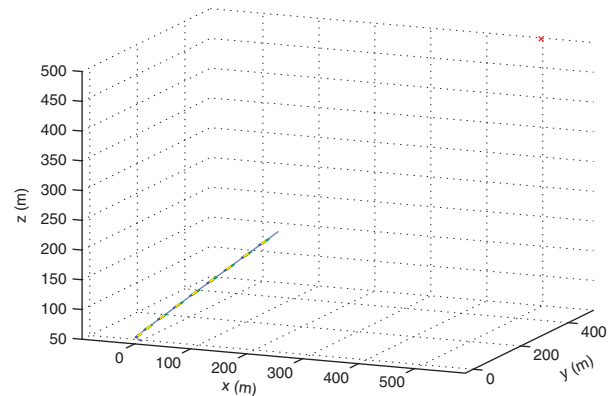


Figure 11. Trajectory described by the vehicle in the presence of ocean currents, sensors noise, and actuator saturation.

evolution of the control inputs and roll, pitch, and yaw Euler angles is depicted in Figure 13. The effect of the measurement noise is visible in the evolution of the control signals but it should be noted that the trajectory described by the vehicle is not significantly affected, in spite of the presence of realistic measurement noise. The saturation effect is also noticeable during the first few seconds of the simulation, when the torques τ_q and

τ_r saturate. However, the attitude quickly converges to the desired one and the actuation enters the linear zone. It should be noticed that, if the thrust force τ_u is not enough to achieve the desired steady-state velocity V_d , the proposed solution may fail to achieve its purpose in the presence of strong ocean currents.

While the configuration of the USBL sensor does not affect the results of the first two simulations since these

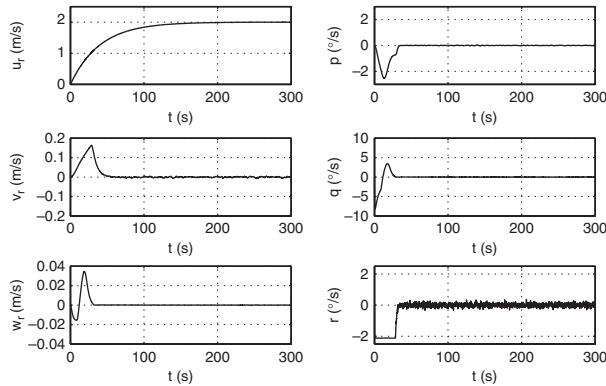


Figure 12. Time evolution of body-fixed velocities of the vehicle in the presence of currents, sensors noise, and actuator saturation.

are carried out in the absence of measurement noise, it does impact on the results of the third simulation since the measurements of the USBL are corrupted with noise. The semi-spherical configuration that was chosen does not favor any particular direction of the target. In the absence of strong currents the attitude of the vehicle quickly converges to a situation where the target is placed in a direction closer to the direction of the x -axis of the vehicle. A sharp-pointed configuration of the hydrophones of the USBL would reduce the effect of the measurement noise on the overall close-loop system since this configuration privileges the directions close to the x -axis of the vehicle. It would, however, increase the sensitivity of the system if the initial attitude of the vehicle was such that the target was placed in a direction far away from the x -axis of the vehicle.

7. CONCLUSIONS

The paper presented a new homing sensor-based integrated guidance and control law to drive an under-actuated autonomous underwater vehicle (AUV) to a fixed target, in 3-D, using the information provided by an ultra-short baseline (USBL) positioning system. The guidance and control laws were first derived at a kinematic level, expressed as Time Differences Of Arrivals measured by the USBL sensor, and then extended to the dynamics of an AUV resorting to backstepping

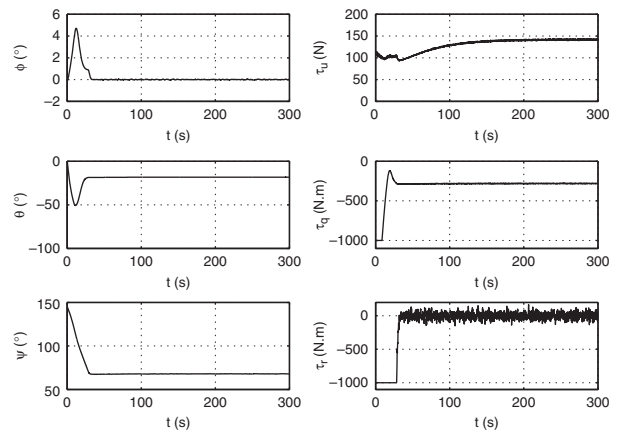


Figure 13. Time evolution of Euler angles and control inputs in the presence of currents, sensors noise, and actuator saturation.

techniques. Almost global asymptotic stability was achieved for the guidance and control law in the presence (and absence) of known ocean currents. Realistic simulation results, in the presence of sensor noise and actuators saturation, were presented and discussed. These simulations show that good performance is achieved with the proposed solutions.

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