Underwater Source Localization Based on USBL Measurements*

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Abstract. This paper presents a new sensor fusion technique for 3-D tracking of underwater targets based on direction measurements and a single range measurement. Applications relying on this solution include, above all, marine animal studies, consisting of two Ultra-Short Baseline (USBL) aided INS navigation systems and an acoustic transducer of reduced dimensions attached to the body of the target. A Linear Time-Varying (LTV) system is designed, which presents a globally asymptotically stable (GAS) error dynamics. The conditions to ensure observability are derived and the performance of the proposed solution is assessed through proper simulations.

Keywords: Marine Robotics, Localization Filter, Estimation Theory.

1 Introduction

The problem of underwater source localization has been a challenge for the scientific community [1],[2]. Where humans can not go, Autonomous Underwater Vehicles (AUV) go further. Interestingly, the cooperative aspect of navigation has deserved much of the emphasis, as seen in [3] and references therein. In addition, the question of local or absolute localization of marine vehicles has also been a growing field of study: the authors in [4] consider a simultaneous estimation of both localizations, while Antonelli et. al., in their works in [5], achieve a solution of relative localization for AUVs and they also study the observability of the problem. As for simultaneous localization and navigation, in the presence of an autonomous vehicle, the works in [6] presented a set of filters with globally exponentially stable error dynamics. However, the total absence of

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AUVs, in exchange of elements with random and unpredicted behavior, such as marine animals, reconfigures the mission scenario, especially in the mathematical point of view. Moreover, the inclusion of humans sets aside convenient properties like control inputs to fix a desired velocity. In this paper, the authors propose a novel approach to the problem of relative localization of a tagged underwater target, using exclusively the spatial information obtained from the reception of acoustic signals emitted by the target, complemented by information from an USBL installed at the surface.

This paper is organized as follows. Section 2 describes the framework of the problem and outlines the system dynamics. The filter design and the observability are presented in Section 3. Section 4 includes simulation results along with discussions. A brief set of conclusions and references to future work are reported in Section 5.

1.1 Notation

Throughout the paper, a bold symbol stands for a multi-dimensional variable, the symbol $\mathbf{0}$ denotes a matrix of zeros and $\mathbf{I}$ an identity matrix, both of appropriate dimensions. A block diagonal matrix is represented as $\text{diag}(\mathbf{A}_1,...,\mathbf{A}_n)$ and the set of unit vectors on $\mathbb{R}^3$ is denoted by $S(2)$. $\delta(t)$ represents the Dirac delta function.

2 Problem Statement

2.1 Motivation and Framework

The objective of this work consists in determining the position of a moving target (source) in an underwater environment through the use of acoustic signals. The target is equipped with an acoustic pinger which produces a known signal. In order to determine the position of that source, two receivers are available: i) a portable underwater robotic tool (PURT) carried by a diver (the whole set will henceforth be designated as the agent); and ii) a surface robotic tool (SRT), which may or may not be adrift and can be optionally used for precise target positioning and diver localization in an inertial frame $\mathcal{I}$. Both the PURT and the SRT are equipped with hydrophone arrays in an inverted USBL configuration, which are used to obtain directions of arrival (DOA), and acoustic transponders to interrogate each other. The assumed mission scenario is depicted in Fig. 1.

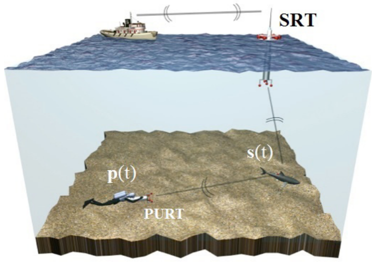


Fig. 1. Graphical representation of the mission scenario

2.2 System Dynamics

The mission scenario, in its most simplistic approach, dwells in a moving plane defined by three parties: an agent; a randomly moving source with an acoustic pinger attached to its body, commonly a fish under a study routine, periodically emitting a known signal; and lastly, a SRT on the surface, henceforth referred to as a transponder. By interrogating the transponder, the agent learns both the distance and the direction between them, thus determining a relative position. The distance between these last two elements can be resolved resorting to a query-response scheme, with the emission of appropriate signals, by measuring the round-trip time, assuming the response involves a fixed and known delay to resolve ambiguity problems and a constant known speed of sound in the medium. Further, the transponder and the agent determine the direction of the source relative to each one of them.

Consolidating the aforementioned localization paradigm, let $\mathbf{q}(t), \mathbf{s}(t) \in \mathbb{R}^3$ be the positions of the transponder and the source, respectively, w.r.t. the agent body frame $\mathcal{B}$. In accordance with this last assignment, let $\mathbf{v_q}(t) = \dot{\mathbf{q}}(t)$, $\mathbf{v_s}(t) = \dot{\mathbf{s}}(t) \in \mathbb{R}^3$ be the velocities of the transponder and the source, respectively, expressed in $\mathcal{B}$. Given the nature of the problem, one can mildly (and locally) assume $\dot{\mathbf{v}}_\mathbf{q}(t) = \mathbf{0}$ and $\dot{\mathbf{v}}_\mathbf{s}(t) = \mathbf{0}$, i.e. that both velocities are constant. In practice, one can adjust the gains so that it is possible to slowly track time-varying velocities with relatively small error.

From the set of measured directions, two of them are conveniently expressed in $\mathcal{B}$: the directions of the source and of the transponder, hereinafter labeled, respectively, as $\mathbf{d_s}(t), \mathbf{d_q}(t) \in S(2)$. The third direction, which relates the source to the transponder, is firstly expressed in the body of the latter, and it can be written as $\mathbf{d}^{\mathcal{Q}}_{\mathbf{s|q}}(t) \in S(2)$, where $\mathcal{Q}$ is the transponder body frame. However, to fulfill the problem requirements, the direction needs to go through two rotations in order to be expressed in $\mathcal{B}$. Hence, it follows $\mathbf{d_{s|q}}(t) = {}^{\mathcal{B}}_{\mathcal{I}}\mathcal{R}\, {}^{\mathcal{I}}_{\mathcal{Q}}\mathcal{R}\, \mathbf{d}^{\mathcal{Q}}_{\mathbf{s|q}}(t) \in S(2)$, where ${}^{\mathcal{B}}_{\mathcal{A}}\mathcal{R} \in SO(3)$ is the rotation matrix from frame $\mathcal{A}$ to frame $\mathcal{B}$.

The aim of the problem is thus to estimate the position and the velocity of the source. If the SRT that houses the transponder is equipped with a GPS antenna, the overall scheme of underwater localization can be expressed in absolute terms. In order to devise such paradigm, it suffices that an inertial point of reference be known.

3 Localization Filter Design

This section presents a filter design methodology for the problem stated in Section 2. First, a linear time-varying system (LTV) is introduced in Section 3.1. Afterwards, the observability of this system is studied in Section 3.2. The filter design is then discussed in 3.3.

3.1 System States

In the vast majority of localization problems, the main difficulty is to draw an estimator with global asymptotic stability guarantees. In this paper, the authors propose a strategy for linear filter design, thus guaranteeing a GAS evolution of the estimation errors. In the situation described in Section 2.2, the overall system is devoid of any inputs, whereby the dynamics can be written as a LTV expressed by

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) \\ \mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) \end{cases}. \tag{1}$$

Contrary to what is usually done, consider first the following measurements vector:

$$\mathbf{y} = \begin{bmatrix} \mathbf{d}_{\mathbf{s}}^T & \mathbf{d}_{\mathbf{q}}^T & \mathbf{d}_{\mathbf{s}|\mathbf{q}}^T & \|\mathbf{q}\| \end{bmatrix}^T \quad \in \mathbb{R}^{10}. \tag{2}$$

Presented therein are three directions and a single range measurement, all w.r.t. $\mathcal{B}$. However, knowing that $\mathbf{d}_{\mathbf{q}}(t)\|\mathbf{q}(t)\| = \mathbf{q}(t)$, one could condense the number of measurements involved by simply considering $\mathbf{q}(t)$ instead of the pair $\{\mathbf{d}_{\mathbf{q}}(t), \|\mathbf{q}(t)\|\}$. But doing so, it could be obliterated the fact that the noise affects each element of the last mentioned pair of measurements, and not $\mathbf{q}(t)$ on its own.

Nevertheless, directions sustain a non-linear relation between the vector numerator and its norm. To circumvent this problem, one can assume that instead of directions, the corresponding measurements are zero, although it remains implicit in the observations matrix that an association between states exists. The new measurements vector, $\mathbf{y} \in \mathbb{R}^{10}$ thus results in

$$\begin{aligned} \mathbf{y} &= [\mathbf{0} \;\; \mathbf{0} \;\; \mathbf{0} \;\; \|\mathbf{q}\|]^T \\ &= \begin{bmatrix} \{\mathbf{d}_{\mathbf{s}}^T\|\mathbf{s}\| - \mathbf{s}^T\} & \{\mathbf{d}_{\mathbf{q}}^T\|\mathbf{q}\| - \mathbf{q}^T\} & \left\{\mathbf{d}_{\mathbf{s}|\mathbf{q}}^T\|\mathbf{s}-\mathbf{q}\| - (\mathbf{s}-\mathbf{q})^T\right\} & \|\mathbf{q}\| \end{bmatrix}^T. \end{aligned} \tag{3}$$

Therefore, to continue to deal with a linear problem, the states vector $\mathbf{x}(t) \in \mathbb{R}^{15}$ and its time derivative $\dot{\mathbf{x}}(t) \in \mathbb{R}^{15}$ follow as

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{q}^T(t) & \mathbf{v}_{\mathbf{q}}^T(t) & \|\mathbf{q}(t)\| & \mathbf{s}^T(t) & \mathbf{v}_{\mathbf{s}}^T(t) & \|\mathbf{s}(t)\| & \|\mathbf{s}(t) - \mathbf{q}(t)\| \end{bmatrix}^T \tag{4}$$

and

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \mathbf{v}_{\mathbf{q}}^T(t) & \mathbf{0} & \mathbf{d}_{\mathbf{q}}^T(t)\mathbf{v}_{\mathbf{q}}(t) & \mathbf{v}_{\mathbf{s}}^T(t) & \mathbf{0} & \mathbf{d}_{\mathbf{s}}^T(t)\mathbf{v}_{\mathbf{s}}(t) & \mathbf{d}_{\mathbf{s}|\mathbf{q}}^T(t)(\mathbf{v}_{\mathbf{s}}^T(t) - \mathbf{v}_{\mathbf{q}}^T(t)) \end{bmatrix}^T. \tag{5}$$

Finally, the dynamics matrix $\mathbf{A}(t) \in \mathbb{R}^{15\times15}$ and the observations matrix $\mathbf{C}(t) \in \mathbb{R}^{10\times15}$ can be easily derived from (5) and (3), respectively.

3.2 Observability Analysis

Throughout this section, the observability of the problem is analyzed, taking into consideration the directions and range readings. The following proposition [Proposition 4.2, [7]] is useful in the sequel.

Proposition: Let $\mathbf{f}(t) : [t_0, t_f] \subset \mathbb{R} \to \mathbb{R}^n$ be a continuous and i-times continuously differentiable function on $\mathcal{T} := [t_0, t_f], T := t_f - t_0 > 0$, and such that

$$\mathbf{f}(t_0) = \dot{\mathbf{f}}(t_0) = ... = \mathbf{f}^{i-1}(t_0) = \mathbf{0}. \tag{6}$$

Further assume that there exists a nonnegative constant C such that $\|\mathbf{f}^{(i+1)}(t)\| \leq C$ for all $t \in \mathcal{T}$. If there exist $\alpha > 0$ and $t_1 \in \mathcal{T}$ such that $\|\mathbf{f}^{(i)}(t_1)\| \geq \alpha$ then there exist $0 < \delta \leq T$ and $\beta > 0$ such that $\|\mathbf{f}(t_0 + \delta\| \geq \beta$.

From the Peano-Baker series it follows the transition matrix associated to $\mathbf{A}(t)$, denoted by $\boldsymbol{\phi}(t,t_0) \in \mathbb{R}^{15\times 15}$.

The LTV system (1) is said to be observable only under the condition that the Observability Gramian is invertible, the latter being defined as

$$\boldsymbol{\mathcal{W}}(t_0, t_f) = \int_{t_0}^{t_f} \boldsymbol{\phi}^T(\tau, t_0)\mathbf{C}^T(\tau)\mathbf{C}(\tau)\boldsymbol{\phi}(\tau, t_0)d\tau. \tag{7}$$

Theorem: The LTV system (1) is observable on $\mathcal{T}$ if and only if the transponder, the agent and the source do not have collinear positions in two successive moments. Mathematically speaking,

$$\exists_{t_1 \in \mathcal{T}}: \quad \forall\ i,j = \{\mathbf{s}, \mathbf{q}, \mathbf{s}|\mathbf{q}\}, i \neq j, \quad \left|\mathbf{d}_i^T(t_0)\mathbf{d}_j(t_1)\right| < 1. \tag{8}$$

Proof. Let $\mathbf{c} = [\mathbf{c}_1^T \ \ \mathbf{c}_2^T \ \ c_3 \ \ \mathbf{c}_4^T \ \ \mathbf{c}_5^T \ \ c_6 \ \ c_7]^T \in \mathbb{R}^{15}$, with $\mathbf{c}_i \in \mathbb{R}^3$, for $i = 1,2,4,5$, and $c_3, c_6, c_7 \in \mathbb{R}$, be a unit vector. Since it must be $\|\mathbf{c}\| = 1$ for all $\mathbf{c}$,

$$\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0, t_f)\mathbf{c} \neq 0 \tag{9}$$

in order for the LTV system to be observable. Then,

$$\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0, t_f)\mathbf{c} = \int_{t_0}^{t_f} \|\mathbf{f}(\tau)\|^2 d\tau, \tag{10}$$

where $\mathbf{f}(\tau) \in \mathbb{R}^{10}, \tau \in \mathcal{T}$ is given by

$$\mathbf{f}(\tau) = \begin{bmatrix} -\mathbf{c}_4 + \left[\mathbf{d}_\mathbf{s}(\tau)\int_{t_0}^{\tau}\mathbf{d}_\mathbf{s}^T(\sigma)d\sigma - (\tau - t_0)\mathbf{I}\right]\mathbf{c}_5 + \mathbf{d}_\mathbf{s}(\tau)c_6 \\ -\mathbf{c}_1 + \left[\mathbf{d}_\mathbf{q}(\tau)\int_{t_0}^{\tau}\mathbf{d}_\mathbf{q}^T(\sigma)d\sigma - (\tau - t_0)\mathbf{I}\right]\mathbf{c}_2 + \mathbf{d}_\mathbf{q}(\tau)c_3 \\ \left\{\mathbf{c}_1 + \left[(\tau - t_0)\mathbf{I} - \mathbf{d}_{\mathbf{s}|\mathbf{q}}(\tau)\int_{t_0}^{\tau}\mathbf{d}_{\mathbf{s}|\mathbf{q}}^T(\sigma)d\sigma\right]\mathbf{c}_2 + \ldots\right. \\ \left.\ldots - \mathbf{c}_4 + \left[(\tau - t_0)\mathbf{I} + \mathbf{d}_{\mathbf{s}|\mathbf{q}}(\tau)\int_{t_0}^{\tau}\mathbf{d}_{\mathbf{s}|\mathbf{q}}^T(\sigma)d\sigma\right]\mathbf{c}_5 + \mathbf{d}_{\mathbf{s}|\mathbf{q}}(\tau)c_7\right\} \\ \int_{t_0}^{\tau}\mathbf{d}_\mathbf{q}(\sigma)d\sigma\mathbf{c}_2 + c_3 \in \mathbb{R} \end{bmatrix}. \tag{11}$$

It can be easily shown that the condition (8) is necessary. For instance, suppose $\mathbf{c} = \begin{bmatrix}\mathbf{0} & \mathbf{0} & 0 & \mathbf{c}_4^T & \mathbf{0} & c_6 & c_7\end{bmatrix}^T$. It results

$$\mathbf{f}(\tau) = \left[\left\{-\mathbf{c}_4^T + \mathbf{d}_\mathbf{s}^T(\tau)c_6\right\} \ \ \mathbf{0} \ \ \left\{-\mathbf{c}_4^T + \mathbf{d}_{\mathbf{s}|\mathbf{q}}^T(\tau)c_7\right\} \ \ 0\right]^T. \tag{12}$$

Taking into account that $\mathbf{c}$ is a unit vector, and defining $c_6 = c_7 = \frac{\sqrt{3}}{3}$ and $\mathbf{c}_4 = k\frac{\sqrt{3}}{3}\mathbf{d}_\mathbf{q}(t_0)$, with $k = \{-1, +1\}$, then under the supposition that the condition in (8) is not verified, it is true that (with implicit collinearity)

$$\mathbf{f}(\tau) = \left[\left\{k\frac{\sqrt{3}}{3}\mathbf{d}_\mathbf{q}^T(t_0) + \mathbf{d}_\mathbf{s}^T(\tau)\frac{\sqrt{3}}{3}\right\} \ \mathbf{0} \ \left\{k\frac{\sqrt{3}}{3}\mathbf{d}_\mathbf{q}^T(t_0) + \mathbf{d}_{\mathbf{s}|\mathbf{q}}^T(\tau)\frac{\sqrt{3}}{3}\right\} \ 0\right]^T = \mathbf{0}$$

if the transponder is in between the source and the agent or if it is the agent who is in the middle, obviously considering the right choice for k. Therefore, the Observability Gramian $\boldsymbol{\mathcal{W}}$ is not invertible and the system is not observable. Notwithstanding, if $c_6/c_7 = -1$, by adjusting the value of k one obtains again $\mathbf{f}(\tau) = \mathbf{0}$, reaching to the same conclusions, proving that if the source is in between the agent and the transponder, and the condition (8) is not verified, the LTV system (1) is not observable.

At this point, (8) was shown to be a necessary condition; another procedure is contemplated to make a statement about its sufficiency: to prove that whatever the value of $\mathbf{c}$ might be, if (8) is verified, then the system is observable. Starting with the evaluation of $\mathbf{f}(\tau)$ at $\tau = t_0$, which is given by

$$\mathbf{f}(t_0) = \Big[\{-\mathbf{c}_4 + \mathbf{d_s}(t_0)c_6\}^T \quad \{-\mathbf{c}_1 + \mathbf{d_q}(t_0)c_3\}^T \quad \dots \\ \dots \quad \left\{\mathbf{c}_1 - \mathbf{c}_4 + \mathbf{d}_{\mathbf{s}|\mathbf{q}}(t_0)c_7\right\}^T \quad c_3\Big], \tag{13}$$

if $c_3 \neq 0$ then $\|\mathbf{f}(t_0)\| > 0$, and it follows from *Proposition* that $\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0, t_f)\mathbf{c} > 0$. In a similar way, evaluating the derivative of $\mathbf{f}(\tau)$ at $\tau = t_0$ yields for its last entry (the scalar one) the term $\mathbf{d}_\mathbf{q}^T(t_0)\mathbf{c}_2$. Clearly, if $\mathbf{c}_2 \neq \mathbf{0}$, it is implicit that $\| \, d\mathbf{f}(\tau)/d\tau|_{\tau=t_0} \| > \mathbf{0}$, whereby using *Proposition* twice it follows that $\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0, t_f)\mathbf{c} > 0$. Proceed now assuming that $\mathbf{c}_2 = \mathbf{0}$ and $c_3 = 0$. In case $\mathbf{c}_4 \neq \mathbf{d_s}(t_0)c_6$ or $\mathbf{c}_1 \neq \mathbf{0}$ it follows $\|\mathbf{f}(t_0)\| > 0$, therefore $\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0, t_f)\mathbf{c} > 0$. Next, by doing $\mathbf{c}_1 = \mathbf{c}_4 = \mathbf{0}$, if $c_7 \neq 0$ $\|\mathbf{f}(t_0)\| > 0$, and again the use of the *Proposition* leads to $\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0, t_f)\mathbf{c} > 0$. However, to reach the same conclusion when $c_7 = 0$ it implies that $c_6 \neq 0$. On the other hand, $c_6 = 0 \Rightarrow \mathbf{f}(t_0) = \mathbf{0}$; consequently, the time derivative of $\mathbf{f}(t_0)$ becomes

$$\left.\frac{d\mathbf{f}(\tau)}{d\tau}\right|_{\tau=t_0} = \Big[\left\{\left[\mathbf{d_s}(t_0)\mathbf{d_s}^T(t_0) - \mathbf{I}\right]\mathbf{c}_5\right\}^T \quad \mathbf{0} \quad \left\{\left[\mathbf{d_s}(t_0)\mathbf{d_s}^T(t_0) + \mathbf{I}\right]\mathbf{c}_5\right\}^T \quad \mathbf{0}\Big]^T. \tag{14}$$

$\mathbf{c}_5$ cannot be zero since that assumption would violate the fact that $\mathbf{c}$ is a unit vector. The first entry of $d\mathbf{f}(\tau)/d\tau|_{\tau=t_0}$ determines the projection of $\mathbf{c}_5$ over the plan orthogonal to $\mathbf{d_s}(t_0)$. The term is $\mathbf{0}$ only when $\mathbf{c}_5||\mathbf{d_s}(t_0)$. Nonetheless, if such situation occurs, the third entry of $d\mathbf{f}(\tau)/d\tau|_{\tau=t_0}$ would be different from $\mathbf{0}$. Hence, $\| \, d\mathbf{f}(\tau)/d\tau|_{\tau=t_0} \| > \mathbf{0}$ and using the *Proposition* twice allows to conclude that $\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0, t_f)\mathbf{c} > 0$. The proof is thus concluded, evidencing that $\mathbf{c}^T\boldsymbol{\mathcal{W}}(t_0, t_f)\mathbf{c} > 0$ for all $\|\mathbf{c}\| = 1$, which means that the observability Gramian is invertible and as such (1) is observable. ■

3.3 Kalman Filter

Section 3.1 introduced a LTV system for source localization based on direction and range measurements and its observability was studied in Section 3.2. It was proven that the LTV system (1) is observable if (8) is satisfied. Taking advantage of the fact that (1) is linear, the Kalman filter follows as the natural estimation solution, with GAS error dynamics. As it is widely known, the Kalman filter design is omitted in this paper. The system dynamics, including additive system disturbances and sensor noise, can be written as

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{w}(t) \\ \mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{n}(t) \end{cases}, \tag{15}$$

where $\mathbf{w}(t) \in \mathbb{R}^{15}$ is zero-mean white Gaussian noise, with $E[\mathbf{w}(t)\mathbf{w}^T(t-\tau)] = \boldsymbol{\Xi}\delta(\tau)$, $\boldsymbol{\Xi} \succ \mathbf{0}$, $\mathbf{n}(t) \in \mathbb{R}^{10}$ is zero-mean white Gaussian noise, with $E[\mathbf{n}(t)\mathbf{n}^T(t-\tau)] = \boldsymbol{\Theta}\delta(\tau)$ and $\boldsymbol{\Theta} \succ \mathbf{0}$. The noises are uncorrelated, therefore $E[\mathbf{w}(t)\mathbf{n}^T(t-\tau)] = \mathbf{0}$. Notwithstanding, noises are here considered to be additive, which might not be the reality, and thus the proposed solution is sub-optimal.

4 Simulation Results

To assess the performance of the localization filter, a simulated mission scenario was built that represents a typical spatial arrangement in which most underwater missions fit. Hence, first consider a SRT at the surface moving due to ocean currents, with small variations on its vertical speed. Since the behavior of marine animals is quite unpredictable, the chosen curved and descending source path is an educated guess. The agent (in terms of depth) is placed in between the source and the SRT, featuring a sinusoidal motion along its longitudinal and transversal axes to ensure the condition in (8) is verified. The resulting trajectories are depicted in Fig. 2.

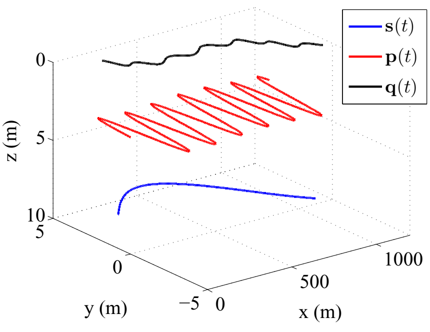
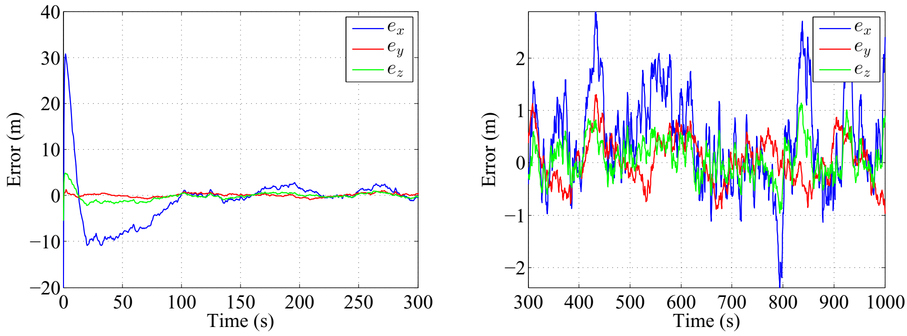


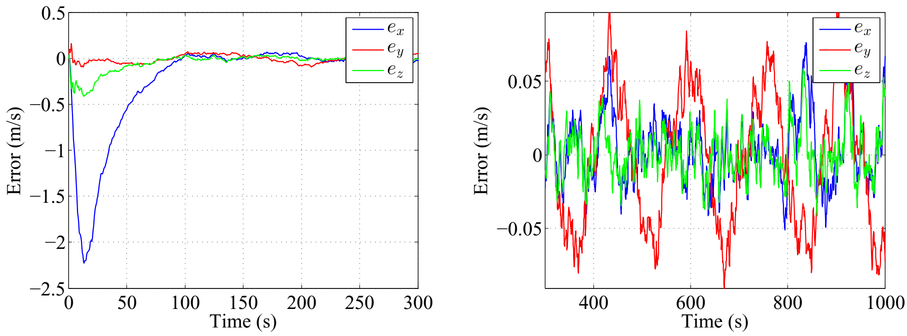
Fig. 2. Simulated Trajectories

Noise was considered for both the directions and single range measurements [8]. According to (3), the null entries are virtual measurements, therefore no noise is to be added there. Direction readings were assumed perturbed by standard deviations of 0.5° on the elevation and azimuth angles. Zero-mean additive Gaussian noise with unitary standard deviation was considered for range measurements. The Kalman filter parameters were tuned to $\boldsymbol{\Xi} = \text{diag}(10^{-5}\mathbf{I},\ 10^{-5}\mathbf{I},\ 10^{-1},\ 10^{-5}\mathbf{I},\ 10^{-5}\mathbf{I},\ 10^{-1},\ 10^{-1})$ and $\boldsymbol{\Theta} = \mathbf{I}$. The initial estimates were all set to zero. Results for the source position and velocity estimation errors are presented in Fig. 3 and Fig. 4, respectively. With the chosen filter parameters, the transients in both figures quickly fade out and the estimation error converges to zero with small deviations, proving the efficiency of the proposed solution.



(a) Initial transient of $e_{\mathbf{s}}$. (b) Detailed convergence of $e_{\mathbf{s}}$.

Fig. 3. Evolution of the source position estimation error



(a) Initial transient of $e_{\mathbf{v_s}}$. (b) Detailed convergence of $e_{\mathbf{v_s}}$.

Fig. 4. Evolution of the source velocity estimation error

5 Conclusions

This paper presented a novel time-varying Kalman filter with globally asymptotically stable error dynamics for the problem of single source localization based on direction and range measurements. The observability of the system was studied, which allowed to conclude about the asymptotic stability of the Kalman filters. Simulations results were presented that illustrate the good performance achieved by the proposed solution. Future work includes real sea tests to evaluate the overall physical system.

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