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The Nonlinear Piezoelectric Tuned Vibration Absorber

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Summary: *This paper proposes a piezoelectric vibration absorber, termed the nonlinear piezoelectric tuned vibration absorber (NPTVA), for the mitigation of nonlinear resonances of mechanical systems. The new feature of the NPTVA is that its nonlinear restoring force is designed according to a principle of similarity, i.e., the NPTVA should be an electric analog of the primary system. Doing so, a nonlinear generalization of Den Hartog's equal-peak tuning rule is developed. The performance of the NPTVA will be illustrated using a nonlinear system possessing different types of nonlinearities.*

1. INTRODUCTION

Piezoelectric tuned vibration absorbers (PTVAs) represent interesting alternatives to mechanical tuned vibration absorbers in that they have no moving parts and they can be fine-tuned online to compensate any modeling errors [1]. PTVAs are implemented with a piezoelectric transducer (PZT) bonded to, or embedded in the structure and shunted with an electrical impedance. Resonant circuit shunting is considered where the inherent capacitance of the PZT is shunted with a resistor R and an inductor L. Different approximate tuning rules for RL shunts were proposed in the literatures [2, 3, 4] while the exact closed-form solution for the design of PTVAs was established in reference [5].

With continual interest in expanding the performance envelope of engineering systems, nonlinear components are increasingly utilized in real-world applications. Mitigating the resonant vibrations of nonlinear structures is therefore becoming a problem of great practical significance.

In this context, the objective of this study is to introduce a new piezoelectric vibration absorber, termed the nonlinear piezoelectric tuned vibration absorber (NPTVA). The nonlinear restoring force of the NPTVA is tuned according to the nonlinear restoring force of the host structure. Specifically, we extend the principle of similarity to nonlinear systems and demonstrate that the NPTVA should be an electrical analog of the nonlinear host system for effective vibration mitigation.

2. The linear piezoelectric tuned vibration absorber (LPTVA)

A LPTVA is coupled to an undamped one-degree-of-freedom modal model of the host structure that represents the resonance of interest (Figure 1). The PZT transducer is considered as a one-dimensional PZT rod in which both the expansion and polarization directions coincide with the central axis of the rod (conventionally called '3-direction'). The capacitance of the PZT rod with no external forces c_{PZT} and its stiffness with short-circuited electrodes k_{PZT} are defined as:

$$c_{PZT} = \epsilon_3^T \frac{s_0}{l_0}, \quad k_{PZT} = \frac{1}{s_{33}^E}\frac{s_0}{l_0}, \tag{1}$$

where s_0 and l_0 are the cross section area and length of the PZT rod, respectively. ϵ_3^T and s_{33}^E are the permittivity under constant strain and the compliance under constant electric field of the PZT rod in 3-direction, respectively [6]. The governing equations of the coupled system are

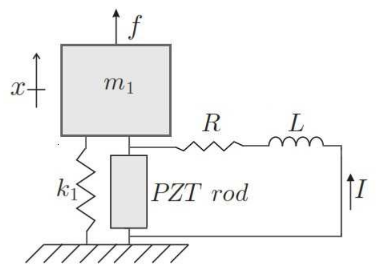

Figure 1. Piezoelectric vibration absorber with a series RL shunt.

written as:

$$\begin{aligned} m_1\ddot{x} + \left(k_1 + \overline{k}_{PZT}\right)x - \theta q &= f\sin\omega t, \\ L\ddot{q} + R\dot{q} + \overline{c}_{PZT}^{-1}\, q - \theta x &= 0, \end{aligned} \tag{2}$$

where

$$\overline{c}_{PZT} = c_{PZT}(1-k_0^2), \quad \overline{k}_{PZT} = \frac{k_{PZT}}{1-k_0^2}, \quad \theta = \frac{k_0}{1-k_0^2}\sqrt{\frac{k_{PZT}}{c_{PZT}}}, \tag{3}$$

are the capacitance of the PZT rod under constant strain, the stiffness of the PZT rod with open electrodes, and the electromechanical coupling factor θ, respectively. k_0 is defined as the electromechanical coupling coefficient in d_{33}-mode:

$$k_0 = d_{33}\sqrt{\frac{k_{PZT}}{c_{PZT}}} = d_{33}\frac{1}{\sqrt{s_{33}^E\epsilon_3^T}}. \tag{4}$$

The governing equations (2) are recast into:

$$\begin{aligned} \widetilde{x}'' + \widetilde{x} - \delta\alpha\widetilde{q} &= f_0 \sin\gamma\tau \\ \widetilde{q}'' + r\delta^2\widetilde{q}' - \delta\alpha\widetilde{x} + \delta^2\widetilde{q} &= 0, \end{aligned} \tag{5}$$

where prime denotes differentiation respect to the dimensionless time $\tau = \omega_1 t$ and the other parameters are defined as in [5] :

$$
\begin{aligned}
&\omega_1 = \sqrt{\frac{k_1 + \overline{k}_{PZT}}{m_1}}, \quad \omega_e = \frac{1}{\sqrt{L\,\overline{c}_{PZT}}}, \quad \gamma = \frac{\omega}{\omega_1}, \quad \delta = \frac{\omega_e}{\omega_1}, \\
&\widetilde{x} = \sqrt{m_1}\,x, \quad \widetilde{q} = \sqrt{L}\,q, \quad r = R\,\overline{c}_{PZT}\,\omega_1, \\
&\kappa = k_1/\overline{k}_{PZT}, \quad f_0 = \frac{f}{\omega_1\sqrt{\overline{k}_{PZT} + k_1}}, \quad \alpha = \theta\sqrt{\frac{\overline{c}_{PZT}}{\overline{k}_{PZT} + k_1}} = \frac{k_0}{\sqrt{1+\kappa}}.
\end{aligned}
\tag{6}
$$

Given a value of the dimensionless electromechanical coupling parameter α, the tuning of the shunt requires to determine the frequency δ and damping r ratios in Equation (5). Reference [5] derived the optimum values of these parameters (δ_{opt} and r_{opt}) which imposes exactly two equal peaks in the receptance function that are associated with the smallest possible vibration amplitude of the host structure (H_∞ optimization). The optimum resistance R and inductance L of the shunt circuit are calculated directly from r_{opt} and δ_{opt}:

$$
\begin{aligned}
L_{opt} &= \frac{1}{\delta_{opt}^2}\frac{m_1}{(1+\kappa)}\frac{s_{33}}{\epsilon_3^T}\left(\frac{l_0}{s_0}\right)^2, \\
R_{opt} &= \frac{r_{opt}}{\epsilon_3^T}\sqrt{\frac{m_1}{1+\kappa}}\sqrt{\frac{s_{33}}{1-{k_0}^2}\left(\frac{l_0}{s_0}\right)^3}.
\end{aligned}
\tag{7}
$$

3. LPTVA coupled to a nonlinear oscillator

To illustrate the detuning of the LPTVA in the presence of nonlinearity, a nonlinear term of order n is added in the equation governing the host oscillator:

$$
\begin{aligned}
\widetilde{x}'' + \widetilde{x} - \delta\alpha\widetilde{q} + k_n\widetilde{x}^n &= f_0 \sin\gamma\tau, \\
\widetilde{q}'' + r\delta^2\widetilde{q}' - \delta\alpha\widetilde{x} + \delta^2\widetilde{q} &= 0.
\end{aligned}
\tag{8}
$$

Figure 2 represents the response of the host structure coupled to a Duffing nonlinear term ($n = 3$) for different forcing amplitudes f_0. The frequency response of the coupled system is calculated using a path-following algorithm combining shooting and pseudo-arclength continuation [7]. The nonlinear coefficient k_3 is equal to $k_3 = 10^{-5}$, and α is chosen to be equal to 0.2. The linear tuning parameters $\delta_{opt} = 1.00017$ and $r_{opt} = 0.2491$ are computed according to the exact tuning rule proposed in [5]. For $f_0 = 2$ in Figure 2, the system behaves linearly, and two peaks of equal amplitude are obtained in accordance with linear theory. When the forcing amplitude is increased, the cubic nonlinearity of the host system is activated and is responsible for a substantial increase in the resonance frequency of the second peak. For $f_0 = 8$ and beyond, unequal peaks are observed.

Quintic nonlinear restoring force is also considered in the governing equation of the oscillating host by assuming $n = 5$. In this case, the nonlinear coefficient k_5 is equal to $k_3 = 10^{-8}$ with the same electromechanical coupling parameter ($\alpha = 0.2$). The frequency responses of the nonlinear primary system coupled to a LPTVA at different forcing amplitudes are shown in Figure 3 . Similar trends as the cubic nonlinear host are also observed and inequality in the resonant peaks are clearly seen at the forcing amplitude higher than $f_0 = 5$.
As a result, at high forcing amplitudes f_0, noticeable difference between the amplitudes of the resonances makes LPTVA inappropriate for the nonlinear oscillating host.

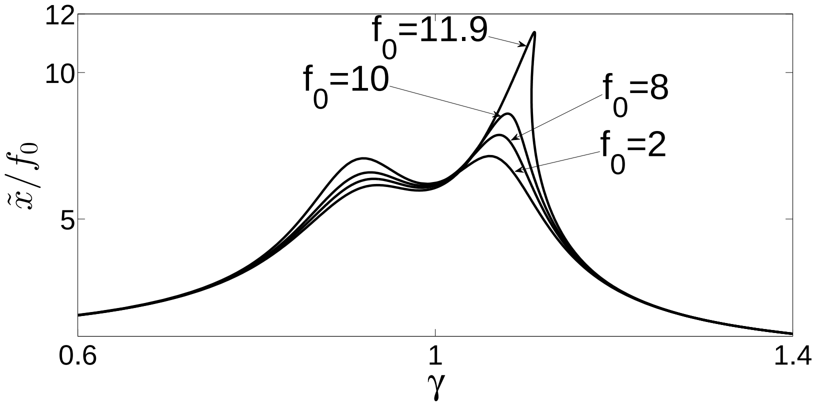

Figure 2. Frequency response of a nonlinear cubic oscillator with an attached LPTVA for different forcing amplitudes f_0 ($\alpha = 0.2$ and $k_3 = 10^{-5}$).

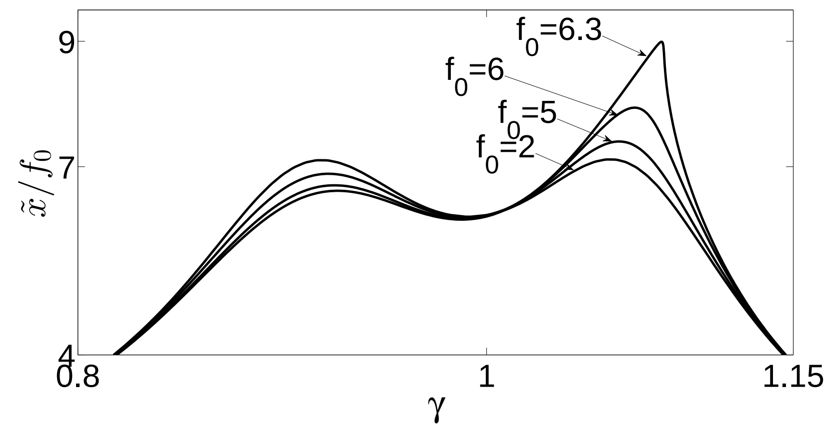

Figure 3. Frequency response of a nonlinear quintic oscillator with an attached LPTVA for different forcing amplitudes f_0 ($\alpha = 0.2$ and $k_5 = 10^{-8}$).

4. NPTVA coupled to a nonlinear oscillator

A NPTVA is now considered for suppressing the vibrations of the nonlinear oscillator. An unconventional feature of the NPTVA is that the mathematical form of its nonlinear restoring force is not imposed a priori, as it is the case for most existing nonlinear absorbers. The equations of motion are therefore:

$$\begin{aligned} \widetilde{x}'' + \widetilde{x} - \delta\alpha\widetilde{q} + k_n\tilde{x}^n &= f_0 \sin\gamma\tau, \\ \widetilde{q}'' + r\delta^2\widetilde{q}' - \delta\alpha\widetilde{x} + \delta^2\widetilde{q} + g(\widetilde{q}) &= 0, \end{aligned} \tag{9}$$

where $g(\widetilde{q})$ represents a nonlinear capacitor and is to be determined.

Reference [8], which examined the case of a nonlinear mechanical absorber coupled to a nonlinear mechanical structure, demonstrated that the absorber should be a 'mirror' of the host system for effective mitigation in a large range of forcing amplitudes. For instance, if the nonlinearity in the host system is quadratic or cubic, the absorber should possess a quadratic or a cubic spring, respectively. This mirror rule suggests that the NPTVA should be chosen such that the shunt is governed by an equation analogous to that of the host structure, thereby extending the principle of similarity described in [9, 10, 11, 12] to nonlinear systems. Following this result, the coupled host and NPTVA system should be written:

$$\begin{aligned} \widetilde{x}'' + \widetilde{x} - \delta\alpha\widetilde{q} + k_n\tilde{x}^n &= f_0 \sin\gamma\tau, \\ \widetilde{q}'' + r\delta^2\widetilde{q}' - \delta\alpha\widetilde{x} + \delta^2\widetilde{q} + \beta_n\widetilde{q}^n &= 0. \end{aligned} \tag{10}$$

The value of nonlinear coefficient β_n (or equivalently the nonlinear tuning parameter $m_n = \beta_n/k_n$) should be determined in a way that the frequency responce of the primary host realizes two equal peaks when the LPTVA starts to be detuned.
For instance, in case of cubic nonlinear term with $n = 3$ and at $f_0 = 8$, a value of $m_3 = 2.06$ was obtained for $k_3 = 10^{-5}$ by numerical iteration. Figure 4 illustrates the corresponding response of the host structure scaled by the forcing amplitude f_0. The result in this figure is remarkable for two reasons. First, not only equal peaks are obtained at $f_0 = 8$, but they are also observed at higher forcing amplitudes. This means that equal peaks can be maintained in a relatively large range of forcing amplitudes when a properly-tuned NPTVA is attached to a nonlinear oscillator. Second, the amplitudes of the resonance peaks in Figure 4 are barely affected by the forcing amplitude f_0, as if the coupled nonlinear system would obey the superposition principle.
A nonlinear oscillator with quintic spring and a quintic NPTVA was also studied ($i.e.\, n = 5$). For $k_5 = 10^{-8}$, the nonlinear coefficient $m_5 = \beta_5/k_5$ is found to be equal to 4.11 at $f_0 = 5$ which guarantees the equality of the amplitudes at the resonant peaks. Figure 5 confirms that qualitatively similar results are obtained for quintic nonlinearities, at different levels of the forcing amplitudes.

Figures 6 and 7 demonstrate the performance of the NPTVA against the LPTVA. The frequency response of the cubic and quintic oscillator are computed using numerical simulations with their corresponding nonlinear coefficient $m_3 = 2.06$ and $m_5 = 4.11$. Regardless of the

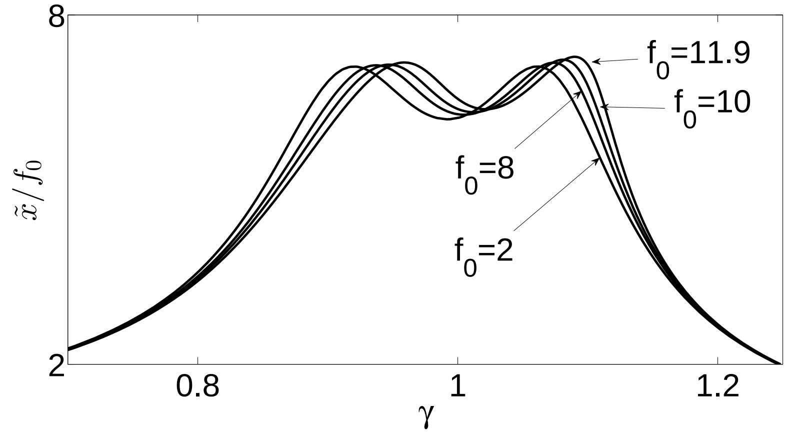

Figure 4. Frequency response of a nonlinear oscillator with an attached cubic NPTVA coupled to a cubic oscillator for different forcing amplitudes f_0 ($\alpha = 0.2$, $k_3 = 10^{-5}$ and $m_3 = 2.06$).

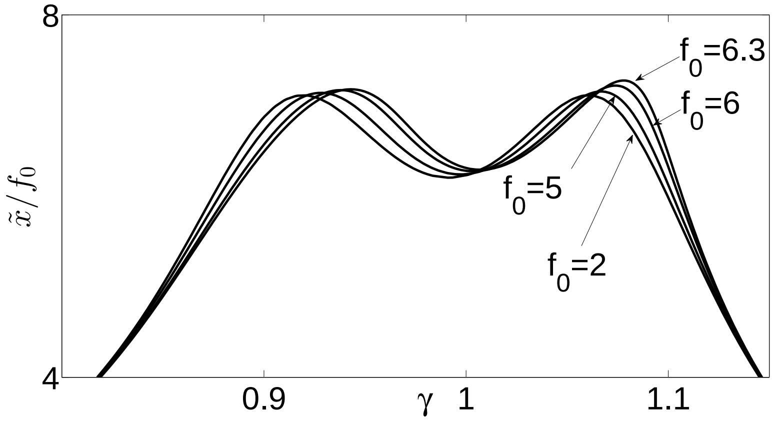

Figure 5. Frequency response of a nonlinear oscillator with an attached quintic NPTVA coupled to a quintic oscillator for different forcing amplitudes f_0 ($\alpha = 0.2$, $k_5 = 10^{-8}$ and $m_5 = 4.11$).

values of f_0, two peaks of almost equal amplitudes are obtained, which provides further evidence of the performance improvement brought by the NPTVA compared to the LPTVA.
For a more quantitative comparison between the performance of the LPTVA and the NPTVA, the amplitudes of the resonant peaks of the host oscillator are compared in Figure 8 as a function of f_0. Figures 8(a,b) reveal that, unlike the LPTVA, the amplitudes of the two resonance peaks of the host system with an attached NPTVA remain almost identical. In addition, an interesting observation is that these amplitudes seem linearly related to the forcing amplitude, meaning that considering a properly-tuned nonlinearity in a shunt circuit connected to an already nonlinear system can give rise to linear-like behaviors and operational domain of the forcing amplitudes. Figures 8(c,d) illustrate that the NPTVA performance is always superior to that of the LPTVA, which is an appealing feature of this device.

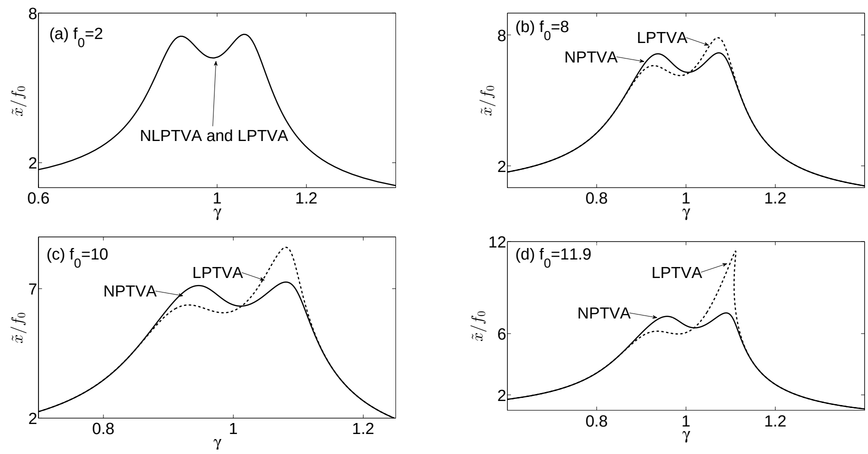

Figure 6. Frequency response of a cubic oscillator ($\alpha = 0.2$, $k_3 = 10^{-5}$) attached to a cubic NPTVA with $m_3 = 2.06$

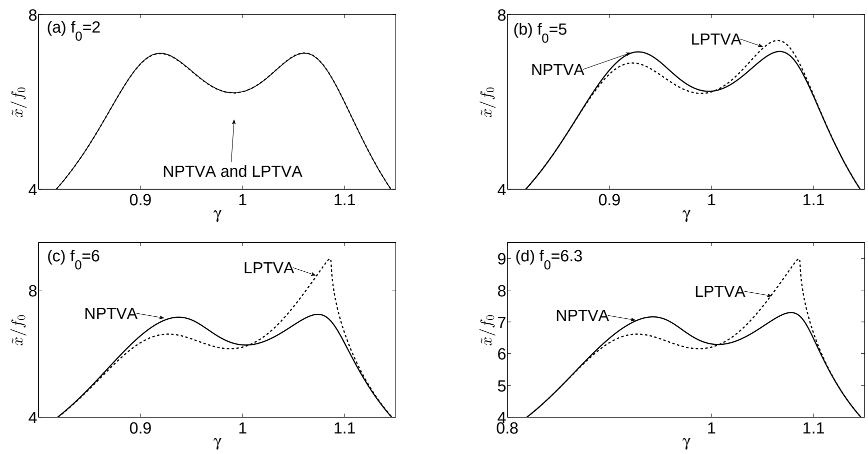

Figure 7. Frequency response of a quintic oscillator ($\alpha = 0.2$, $k_5 = 10^{-8}$) attached to a quintic NPTVA with $m_5 = 4.11$

5. Conclusion

This paper introduces a new nonlinear piezoelectric vibration absorber which aims the mitigation of a specific nonlinear resonance of a mechanical system. The tuning rule used for absorber design relies on a nonlinear principle of similarity, i.e., the nonlinear shunt should be an electrical analog of the host system, and allows us to extend Den Hartog's tuning method to nonlinear systems. Specifically, equal peaks can be maintained in a relatively large range of forcing amplitudes, and this, despite the variation of the resonance frequency of the host system. Compared to the linear piezoelectric vibration absorber, the nonlinear absorber improves mitigation performance and retains the linear-like behaviors in a wider range of the forcing amplitudes.

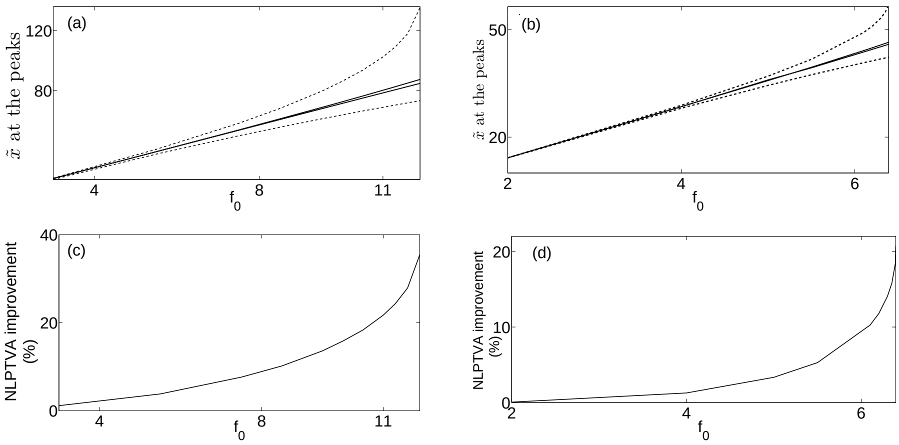

Figure 8. Peak amplitudes of the NPTVA (solid lines) against LPTVA (dashed lines) for (a) a Duffing nonlinear host with $k_3 = 1 \times 10^{-5}$ (b) a nonlinear host with quintic nonlinear terms with $k_5 = 1 \times 10^{-8}$ at $\alpha = 0.2$; The percentage of improvement brought by the NPTVA with respect to LPTVA for (c) the Duffing (cubic) nonlinear host , (d) the quintic nonlinear host.

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