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EXPERIMENTAL INVESTIGATIONS TO ENHANCE THE BUCKLING LOAD OF SLENDER BEAMS

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Summary: *This paper aims to enhance the buckling load of beam-type structures by active control. For this purpose, a cantilever beam loaded by a compressive force is investigated experimentally. The compressive force is introduced by a cable, which is fixed at the tip of the beam and directed through the beam's foundation. The structural displacements are measured by means of laser displacement sensors and strain gauges. For increasing the structural stability, discrete piezoelectric patches are applied for actuation. Different kinds of constant feedback control approaches are studied. The influence on the buckling and the active control limits are demonstrated. The experimental and numerical results agree very well. Furthermore, an analytical formulation based on the Bernoulli-Euler beam theory is presented. As the system represents a non-conservative system, a dynamic stability analysis is performed.*

1. INTRODUCTION

Due to their capability to withstand relatively large axial loads, slender beam-type structures are a common member in design, e.g., of light-weight structures. Depending on the load characteristics, constraints and geometry, however, there are different ways such a structure may lose stability, e.g., buckling beyond critical compressive forces, cf. Timoshenko and Gere [1]. The loss of stability may even cause the structure to collapse in the worst case.

In practice, high efforts are spent to avoid these states of instability and prevent buckling from the beginning. Obvious approaches to enhance the stability of a structure are methods to increase its rigidity passively, e.g., by means of modification of design, material, etc. Nowadays, these methods may not satisfy the challenging design criteria of aerospace, automotive, mechanical, civil and even medical engineering like volume and weight restrictions or flexibility demands. Instead, it may be advantageous to increase the load capacity by active control strategies. As piezoelectric actuators have been proved their effectiveness in many applications, cf., Preumont [2], they are utilized in the present contribution.

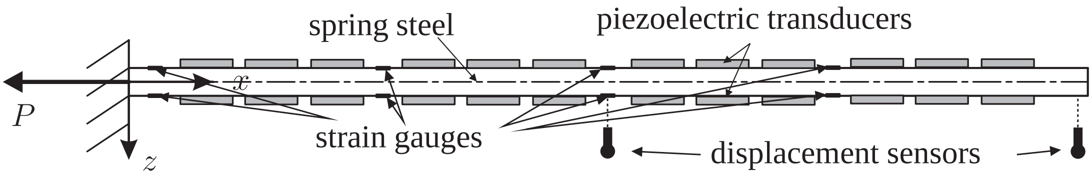
In the literature, only few contributions can be found, which are concerned with experimental investigations on buckling control. Firstly, different piezoelectric material strips applied to simply supported beams are actuated by means of feedback control in Thompson and Loughlan [3]. As control error a displacement sensor is used. A maximum increase of a factor of 1.371 could be achieved. Berlin [4] studied simply supported beams, again. Here, the sensor signals of five pairs of strain gauges are feed back to the same number of piezoelectric patches by a proportional-integral-derivative controller. By tuning the controller manually for every different load case, an increase of factor 5.6 of the uncontrolled buckling load is reported. In Chase and Yim [5] an optimal control algorithm based on a numerical state-space model is designed with eight pairs of piezoelectric patches, which are applied to a simply supported beam. Buckling loads up to a factor of 2.9 are stated with the help of the control system. Analytical formulation, numerical simulations and experimental results are firstly presented in Zenz and Humer [6]. In the experimental setup, twelve pairs of piezoelectric patches increase the buckling load with the help of a displacement sensor and a proportional feedback controller. A factor of 2.05 is presented. Beside experimental studies, numerical investigations are the topic of, e.g, Meressi and Paden [7], Fridmann and Abramovich [8] and Zenz and Humer [9]. All aforementioned publications utilize the bending moment for controlling the structural stability. Normal forces are not applicable for increasing the buckling load, which could be clarified in the work of Zehetner and Irschik [10].

The present work experimentally investigates the stability control of a slender cantilever beam subjected to a compressive force. The compressive force is introduced by tensioning a cable, which is fixed at the free end of the beam and guided through the foundation of the cantilever. In order to control the measured tip deflection, discrete piezoelectric patches are used for actuation. Different proportional feedback control approaches aim to enhance the critical load at which buckling occurs.

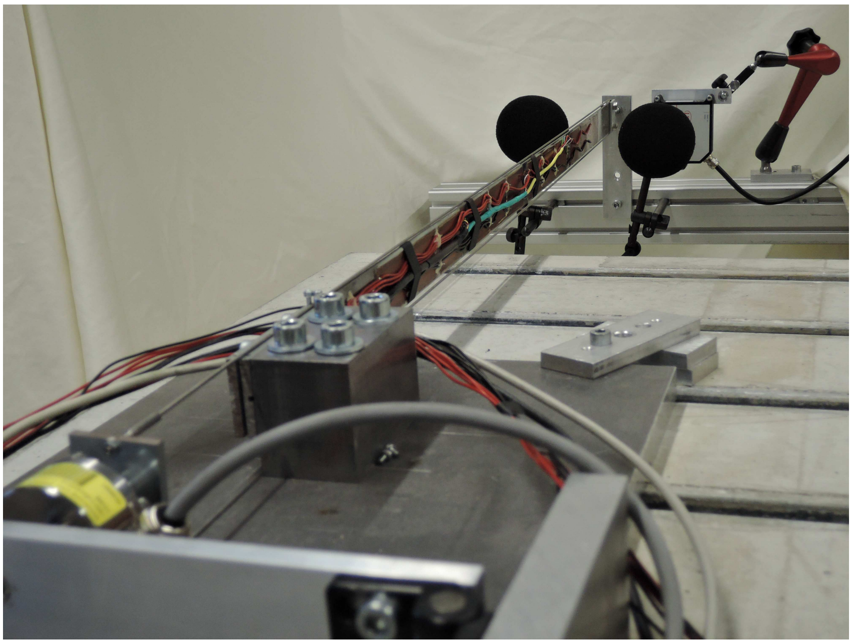
The paper is structured as follows: In a first step, the investigated experimental setup is presented. Subsequently, the equations of motion of the considered setup are derived. The consistent analytical formulation, which is based on the Bernoulli-Euler assumptions, includes discrete sensing and actuation as well as controller design. In a second step, a numerical model is built up, which, again, includes the used sensors, actuators and the control elements. Finally, the experimental results are presented and compared to the numerical ones. Different control approaches, i.e. uniform control, discretely weighted actuation based on the shape control approach and modal control, are studied in order to derive a maximum increase of the buckling load. The behaviour of real world structures, e.g., imperfections, sensor noise, discrete measurements, etc. is emphasized.

2. INVESTIGATED SETUP

The setup under consideration consists of a cantilever beam loaded by a compressive force, cf. Fig. 1. Unlike conventional problems, the compressive force changes its orientation during buckling as it is imposed by tensioning a steel cable, which passes through a fixed point at the beam's root. On a substrate layer made of spring steel, 24 discrete piezoelectric patches are applied on the top and bottom surfaces of the cantilever by means of an adhesive layer. Using type DuraAct 15-A patches, the piezoelectric material is enclosed in an isolator cover. The patches are electrically connected in four groups of three piezoelectric patches each. The distance between each active material in a group is 0.013 m, between each group 0.031 m and between the clamping and the first patch 0.099 m. Further geometric and material parameters can be found in Table 1.



(a)



(b)

Figure 1. Column with load through a fixed point, 24 piezoelectric patches, strain gauges and two laser displacement sensors: a) schematic and b) experimental setup.

For measuring the deflection of the beam, two laser displacement sensors are placed at the middle and at the tip of the beam. Furthermore, four strain sensors are applied between each patch group at each side of the beam, starting next to the clamping. Conventional strain gauges of type Micro-measurements CEA-06-125WT-120 in half bridge mode are used as sensors. The half bridge mode avoids inaccurate measurements due to temperature drifts. In order to derive the bending strain only, the sensor data of one strain gauge is subtracted from the data of the

	beam (steel)	PZT
length l, l_p / m	1.02	0.05
width b, b_p / m	0.04	0.03
thickness h, h_p / m	0.002	0.0007
Young's modulus E, E_p / Nm^{-2}	2.1×10^{11}	6.5×10^{10}
density $\rho,\ \rho_p$ / kgm^{-3}	7900	7600
piezoelectric coefficient d_{31} / AsN^{-1}	0	-1.71×10^{-10}

Table 1. Geometric and material properties

collocated applied sensor. The applied compressive force is measured by a force sensor. The measurements are processed and the controllers are implemented within a dSpace system. The control output of the dSpace system is amplified by factor of 100 in order to exploit the voltage range and consequently the deflection range of the piezoelectric patches. To avoid irreversible depolarization of the piezoelectric material, the outputs are limited to ± 200 V.

3. ANALYTICAL CONSIDERATIONS

In this section, an analytical formulation for determining the critical load of an initially straight and slender beam compressed by a load through a fixed point is presented. The formulation includes discrete piezoelectric actuators, discrete displacement sensors and feedback control. For the formulation using strain sensors instead of displacement sensors, see Zenz and Humer [9].

In the following, it is assumed that the Bernoulli-Euler assumptions hold. Furthermore, the longitudinal axis of the undeformed configuration coincides with x-axis of the fixed Cartesian frame, cf. Fig. 1(a), and the deformations are assumed to take place in the xz-plane only. The critical loads can be determined by solving the eigenvalue problem of the linearized problem. Due to the feedback control, the considered setup represents a non-conservative system, as the externally imposed loads depend on the displacement field and on its derivatives. Consequently, a dynamic criterion must be used to derive all states of stability, those of divergent as well as flutter instability, cf., Leipholz [11] and Timoshenko and Gere [1] for more details.

The equation of motion for small deformations of a beam subjected to an compressive force reads:

$$\frac{\partial^2 M_j}{\partial x^2} - P\frac{\partial^2 w_j}{\partial x^2} - \mu_j \frac{\partial^2 w_j}{\partial t^2} = 0, \tag{1}$$

where $w_j(x,t)$ denotes the lateral deflection, $M_j(x,t)$ is the bending moment about the y-axis, and $\mu_j = \rho_j A_j$ the beam's mass per unit length. In order to take into account the changing cross section due to the discrete piezoelectric patches, the beam is segmented such that the properties of each section j of the n segments can be included. In the considered framework, which neglects shear deformation and shortening of the beam and regards small deflections only, the curvature can be approximated by $\kappa(x,t) \approx -\partial^2 w/\partial x^2$. Therefore, following relation between bending moment and the beam's curvature via the bending stiffness can be found:

$$M_j(x,t) = -(EI)_j \frac{\partial^2 w_j}{\partial x^2} + M_j^a. \tag{2}$$

The additional term $M^a(t)$ respects the influence of the piezoelectric bending actuation of the beam. In the following, only a transverse component of the electric field $\mathcal{E}_z$ is considered. This electric field is considered to be constant over the piezoelectric thickness layer $h_{p,j}$ and proportional to the applied voltage $v_j(t)$, which leads to the correlation $\mathcal{E}_z = \mp v_j/h_{p,j}$. Assuming furthermore a perfectly bonded patch, with the plane of symmetry in the xy-plane and polarized in z-direction, the equation of the actuation moment is determined by

$$M_j^a = 2 \int\limits_{h/2}^{h/2+h_{p,j}} E_{p,j} d_{31,j} \mathcal{E}_z z b_{p,j} \, \mathrm{d}z \approx \mp E_{p,j} d_{31,j} b_{p,j} (h + h_{p,j}) v_j, \tag{3}$$

In the above relation E_p denotes Young's modulus of the piezoelectric material, $d_{31,j}$ the piezoelectric coefficient and $\mathcal{E}_z$ the electric field. The height of the substrate and width of the piezoelectric actuator are h and $b_{p,j}$. Inserting the constitutive relation (2) into the equation of motion (1) leads to the well-known fourth-order differential equation:

$$(EI)_j \frac{\partial^4 w_j}{\partial x^4} + P \frac{\partial^2 w_j}{\partial x^2} + \mu_j \frac{\partial^2 w_j}{\partial t^2} = 0. \tag{4}$$

As the actuation moments M_j^a remain constant over the length, it enters the boundary and necessary continuity condition of the considered configuration only.

Separating the above equation into the product of a spatial $f_j(x)$ and a time-harmonic part $e^{i\omega_j t}$ due to its linearity, leads to a fourth-order ordinary differential equation, which can be commonly solved by the help of an general solution of the spatial part, cf., e.g., Zenz and Humer [6].

Anyways, the general solution needs to be determined from the boundary conditions. At the clamped end of the structure $x = 0$ the rotation of the cross-section are prohibited kinematically. At the free end $x = l$, $j = n$, the bending moment must vanish while the shear force $Q_n(l) = M_n'(l)$ needs to satisfy the condition $Q_n(l) = P w_n'(l) - \frac{P w_n(l)}{l}$, due to configuration of a load through the fix point at the beam's foundation, when assuming small deformations, see Timoshenko and Gere [1]. In terms of deflection $w_j(l)$, the boundary conditions read

$$\begin{aligned} x = 0: &\quad w_1(0) = 0, \quad w_1'(0) = 0, \\ x = l: &\quad w_n''(l) - \frac{M_j^a}{(EI)_j} = 0, \quad w_n'''(l) + k_n^2 w_n'(l) - \frac{k_n^2 w_n(l)}{l} = 0. \end{aligned} \tag{5}$$

As a set of n differential equations are considered, continuity conditions for each segment $x = l_j$ are needed:

$$\begin{aligned} w_j(l_j) &= w_{j+1}(l_j), & w_j'(l_j) &= w_{j+1}'(l_j), \\ M_j &= M_{j+1}, & Q_j &= Q_{j+1}, \end{aligned} \tag{6}$$

where l_j denotes the distance from the clamped end in the straight configuration up to j, with consequently $l_n = l$, the length of the entire cantilever. For segment without piezoelectric material, the actuation moment is set to zero.

So far, the piezoelectric transducers are activated by some kind of applied voltage. Here, this voltage depends on some kind of measurement, which is manipulated by a feedback control algorithm. In this experimental setup, the deflection w_j is measured at the tip l of the beam. Using a proportional gain feedback control, the applied voltage v_j of Eq. (3) is given as

$$v_j = \pm \frac{g w_n(l)}{E_{p,j} d_{31,j} b_{p,j} (h + h_{p,j})}, \tag{7}$$

where g denotes the gain value of the proportional controller. For more details on the analytical formulation and its validation, see Zenz and Humer [6].

4. NUMERICAL SIMULATIONS

In this section, the finite element model is briefly presented and the characteristics of the uncontrolled numerical simulation are compared to the experimental ones. The mechanical, sensory, actuatorical as well as control parts of the setup are implemented within the framework of the multibody and finite system code HOTINT [12]. Within this framework, a planar beam finite element based on the absolute coordinate formulation (ANCF) is used for modelling the smart beams, cf. Gerstmayr and Irschik [13] for details on the formulation.

The numerical model is set up according to Fig. 1(a). Based on a convergence study, showing a convergence rate of O(n_{el}^4), a number of $n_{el} = 25$ elements, each element with its adequate material parameters, is already sufficient to compute the solution accurately. During simulation, the axial compressive force P is increased gradually from 0 N up to a maximum of 1000 N. In order to avoid instable equilibrium paths, a small lateral force is introduced at the tip of the beam.

Before investigating the controlled configurations, the accuracy of the numerical setup is validated by the experimental results of the uncontrolled setup. For this purpose the eigenfrequencies, transfer function, deflection of the beam for certain actuations and natural buckling load are compared. The results agree very well, see Fig. 2 for the comparison of the transfer functions, even though the numerical model does not include the adhesive layer, weight of the electric wiring for actuation and measurements and the fixation of the cable for the compressive force at the tip of the beam. The buckling load of the numerical model with 72.9 N deviates from the experimental setup with 73.5 N by a factor of only 0.8%. Also the comparisons of different voltage levels applied to the piezoelectric patches agreed with a maximum achievable deflection of ± 7 mm.

5. RESULTS

In this section, the experimental results are presented and compared to the numerical ones. For evaluating the influence of the active feedback control approach, the load-deflection curves

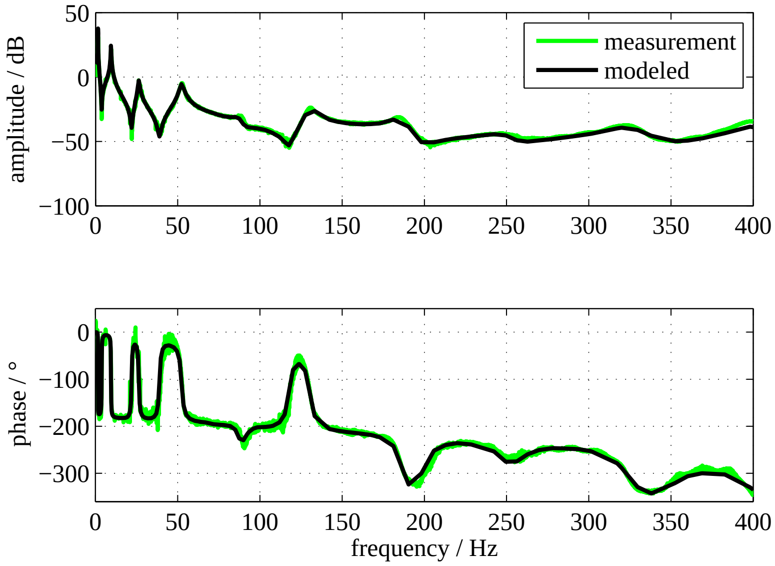


Figure 2. Comparison of experimental and numerical results.

with the applied compressive load and the tip deflection of the controlled case are compared to the natural buckling load of the setup. While an analytical solution determines the buckling load by a single bifurcation point, the load-deflection curve of experimental and numerical investigations are changing gradually, due to imperfections. Consequently, a condition is defined in order to determine the bucking load. For the numerical models, the point with the maximal curvature on the load-deflection curve determines the buckling load. Similar to the numerical model with a small lateral force, experimental setups are slightly bent in the unloaded configuration due to several imperfections and have therefore a predominant deflection direction. When gradually loading the beam with a compressive force, the configuration exhibits a natural equilibrium path before the critical load and the same natural as well as a secondary complementary equilibrium path, cf., Singer [14]. This secondary path is only accessed, if the buckled beam is manually deflected in the opposite predominant direction. In the following, the buckling load of the experimental setup is defined as the force at which the beam persists on the secondary equilibrium path. Any decrease of the compressive force would consequently lead to a swing back to the natural path. All following results are obtained by different proportional control approaches using the same preceding experimental setup, cf. Fig. 1.

Besides increasing the stability limit of a structure, feedback control can also cause a decrease of the maximum bearable buckling load. Such a decrease of the buckling load can be obtained by positive feedback control and may be advantageous for, e.g., applications, which require both, flexible and stiff states of a structure. In the following, however, only the enhancement of the buckling load by negative feedback – denoted with negative gain values – will be presented.

5.1 Uniform control approach

In a first approach, the same output of one proportional controller, is applied reversely to all piezoelectric patches on both sides. The controller voltage is calculated from the error of the displacement sensor of the beam's tip to the non-deflected configuration. The influence of different gain values can be found in Fig. 3. By playing with the gain values, a maximum increase up to 130 N could be achieved for a gain value at approximately $g = -0.318$. Further increase of the gain value does not increase the buckling load but results to states of a flutter behavior around 130 N. Such flutter behavior may occur in non-conservative systems. In contrast to divergent instability, where the stability limit is determined by a frequency becoming zero for a certain compressive force, i.e., the buckling load, flutter instability is characterized by two frequencies coinciding exactly at a certain compressive force intensity. At this buckling load, the frequency has a double root, which would change to a complex solution for higher compressive load intensities and therefore to increasing oscillations. The investigations on the stability of the control system could confirm the observations. Furthermore, the numerical simulations show the same results with comparable load-displacement curves, cf. Fig. 3(b). The smaller deformation can be attributed to the neglected and broadly unknown imperfections of the experimental setup.

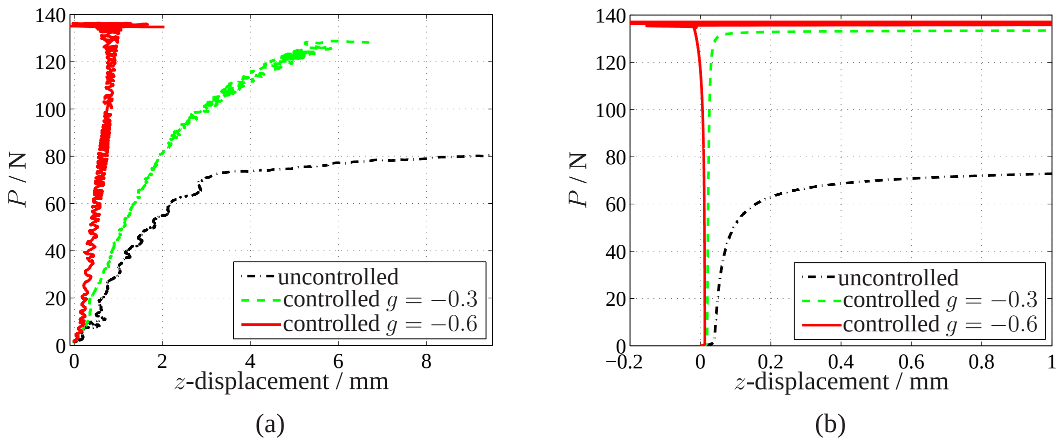


Figure 3. Load-displacement curves of uniform control approach: a) Experimental and b) simulation results for different gain values g.

5.2 Weighted control approach

In a second approach, it is aimed to nullify not only the tip deflection as in the previous section, but to compensate the deformation along the entire axis of the beam. With the approach of shape control, cf. Irschik [15], a compensation of flexible vibrations can be achieved by applying spatially distributed piezoelectric eigenstrains. Following the considerations of Huber [16] for discretely distributed piezoelectric patches, corresponding weighting coefficients for discretely applying the voltage to the four groups of the piezoelectric patches can be found. This weighting coefficients are calculated by consulting the first modal strain function, cf. Tzou and Hollkamp [17],

$$k_i = \frac{M_i^a}{M_1^a} = \frac{\int_{x_i}^{x_{i+1}} w_1''(x)dx}{\int_{x_1}^{x_2} w_1''(x)dx}. \tag{8}$$

The modal strain function is derived from the first eigenmode $w_1(x)$. In contrast to conventional setups, the compressive load causes load dependent and consequently changing eigenfrequencies, cf. Fig 4 for different eigenmodes due to compressive force P, scaled with the Euler load P_E. Consequently, the influence on the buckling load of diverse shapes calculated on the basis of different first modes is studied. As in the previous setup, the output of one proportional feedback algorithm, which is calculated from the tip deflection error, is used as feedback. The maximum increase could be obtained by using a weighting function at a force of $P = 70$ N, which is close to the uncontrolled buckling mode. The calculated weighting factors of $p_{1,1} = 1$, $p_{2,1} = 1.457$, $p_{3,1} = 1.457$ and $p_{4,1} = 1.004$ could increase the buckling load to 151 N, cf. Fig. 5. Here, the first index denotes the patch group, beginning at the clamping and the second index identifies the addressed mode. Again, the simulation results agree very well with the experimental outcome.

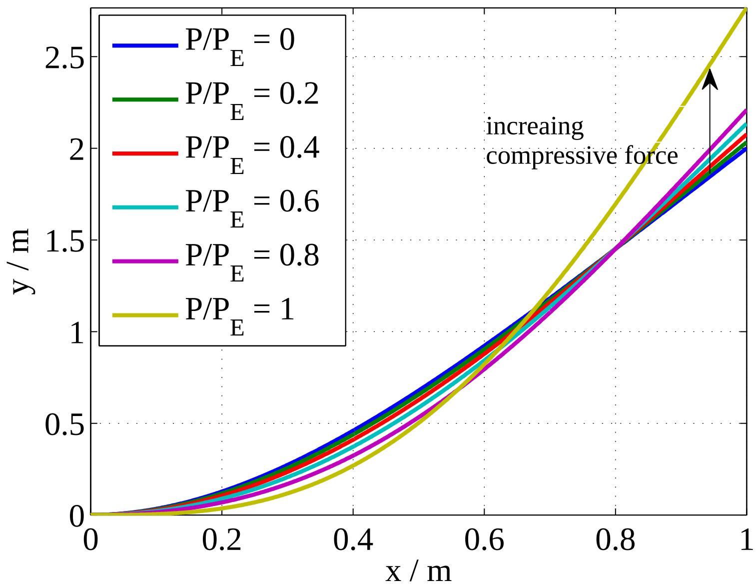


Figure 4. Influence of compressive force on the first eigenmode.

So far, the proposed approaches to enhance the critical load cannot increase the buckling load over the second critical load $P_2 = 4P_1$. An explanation of this behavior can be that controlling the tip of the beam corresponds to a beam with one side clamped and the other pinned. The appropriate Euler load of such a configuration is determined by $P_E = \pi^2 EI/(0.699l)^2$

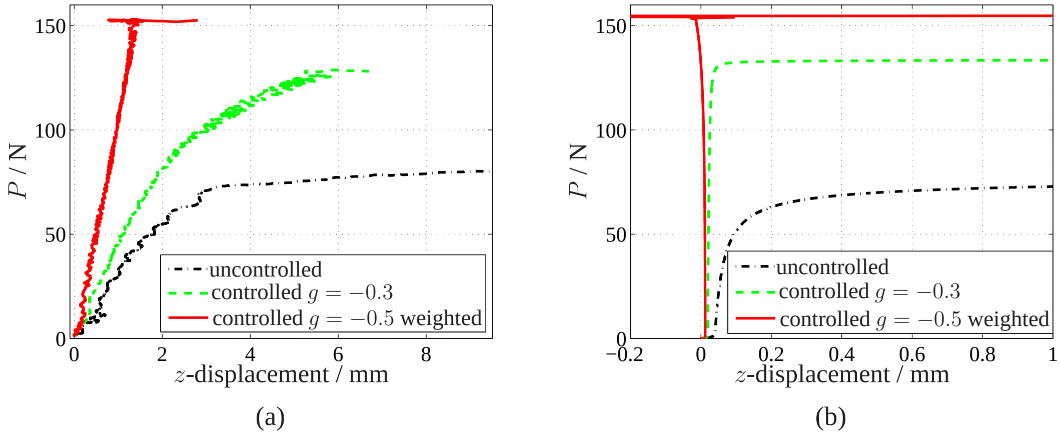


Figure 5. Load-displacement curves including uniform and a weighted control strategy: a) Experimental and b) simulation results.

instead of $P_E = \pi^2 EI/l^2$ for the uncontrolled configuration. Hence, the new configuration exhibits a 2.05 higher buckling load, which is the same factor derived from the maximal increase of using the active control. As the buckling mode of the controlled configuration changes, the used control approach is not applicable any more. In order to overcome this buckling control limit, a control approach has to be found, which can handle the changing mode shapes.

5.3 Modal control approach

To overcome the limitations of the previous approaches, a modal control approach is considered. The aim of modal control is to control each mode by one controller individually, cf. Hanson and Snyder [18] and Inman [19]. The particular modes are obtained by an appropriate transformation of the sensors and actuators. Consequently, each modal sensor information is manipulated by its individual controller and applied to the modal actuators.

In the present approach, two displacement sensors are used in order to get the information of a possibly occurring second mode. In typical setups, the sensors utilize symmetries and modal nodes for filtering the proper modal information. As in our configuration, a quasi-static behavior of an axially compressed cantilever is considered, there are no symmetries nor nodes occurring. However, it is assumed that for small deformations of the beam, the sensors can be located such, that a constant relation can be obtained for the measured deflection when the beam is controlled, as long as only the first mode appears. Consequently, mainly a second mode will be observed if the measurements are subtracted appropriately.

In our configuration, the same control loop as in the weighted configuration is used for controlling the first mode. The second mode is identified by using a second deflection sensor, which is applied in the middle of the beam. The constant relation of the two deflection measurements is estimated by a factor of 4.8. As for the first mode, the influence of different second eigenmodes is studied in order to calculate an appropriate weighting function. The maximum experimental increase could be achieved by using the eigenmode shape at $P = 80$ N, with the factors $p_{1,2} = 1$, $p_{2,2} = -0.841$, $p_{3,2} = -1.408$ and $p_{4,2} = -1.131$. The influence of the modal control approach with the proportional controllers $g_1 = -0.5$ and $g = -0.2$ for the first and second control loop, respectively, are shown in Fig. 6(a). The overall control amount in terms of voltage applied at the piezoelectric patches are presented in Fig 6(b).

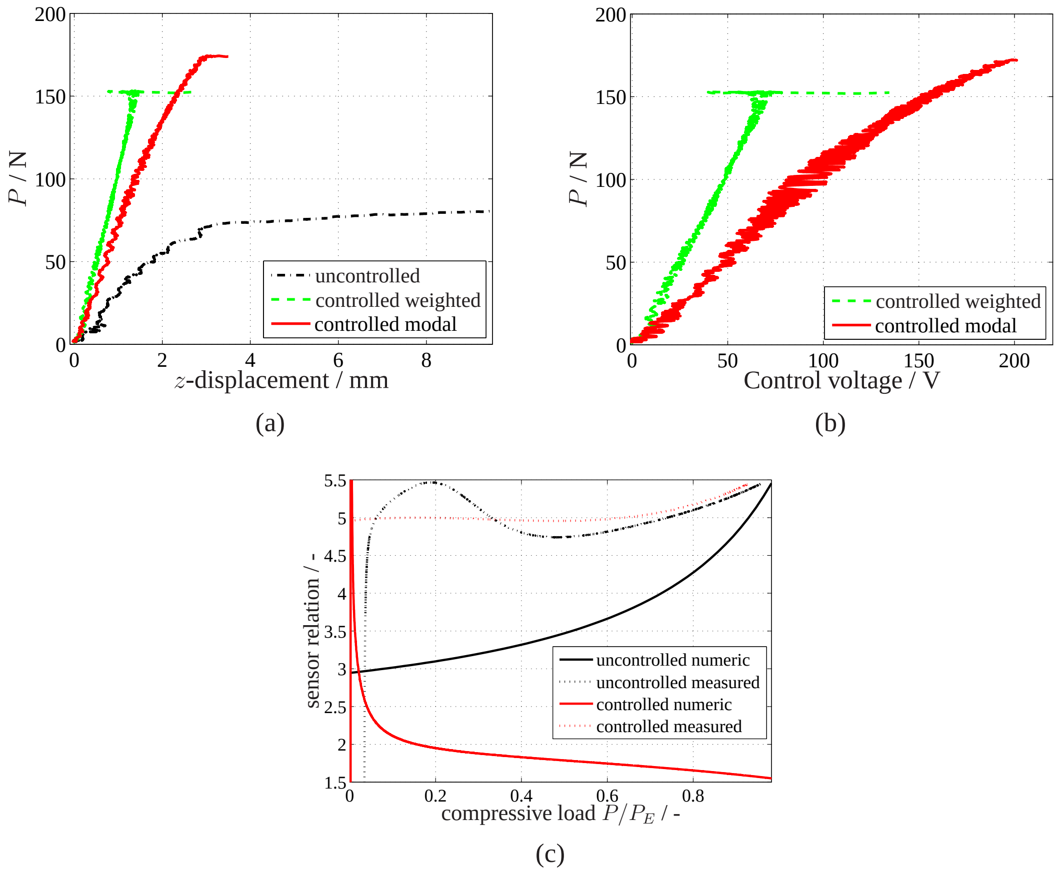


Figure 6. Modal control approach compared to a) Experimental load-displacement curves, b) control amount and c) displacement ratio of the sensors.

While the numerical simulation could shift the buckling load to the second critical load P_2, only a buckling load of 174 N could be achieved in the experimental setup. In this context, the ratio of the displacement measurements of the two displacement sensors are regarded, see Fig. 6(c). While the ratio is almost constant with a ratio of five for compressive loads smaller than $P/P_E = 0.6$, the ratio changes for higher values. As this change increases the applied control voltage to the patches, the experimental increase of the buckling load is limited by the maximal voltage of ± 200 V. In case of the simulation, such a voltage limit is not introduced. However, numerical and experimental results show a high variance. So far, no explanation for this phenomena has been found.

5.4 Control using strain gauges

Instead of using displacement sensors, also strain gauges can be used for measurement, cf. Berlin [4] and Chase and Yim [5]. However, in the considered setup the used strain gauges are inappropriate due to the signal-to-noise ratio, especially when two collocated strain gauges are used in order to obtain the bending strain only. For the purpose of controlling the buckling load of a beam, the feedback control has to be able to influence even minimal deflections. For these small deflections and consequently small strains, the noise of the measurements is too high. In order to derive appropriate measurements also for small strains, special strain gauges have to be applied, e.g., MEMS-based strain gauges as used in Chase and Yim [5].

6. CONCLUSIONS

In the present work, the influence of active feedback control on the bucking bending load of a beam subjected to a compressive load is investigated. For this purpose, a cantilever compressed by a load through a fix point is considered. The feedback control loop consist of displacements sensors, constant gain feedback approaches and 12 discretely distributed piezoelectric patches along each side of the beam. Three different control approaches, characterized by uniform actuation, weighted actuation and weighted modal actuation, are investigated.

After describing the general experimental setup, an analytical formulation based on the Bernoulli-Euler assumptions is presented. The formulation takes into account the influence of the axial compressive force and the entire feedback control, including discretely distributed piezoelectric actuators, discrete sensors and the proportional controller on the beam.

Subsequently, the experimental results using different control approaches are presented and compared to a numerical model, which is implemented within the multibody and finite element system code HOTINT. In a first approach, all piezoelectric patches are actuated in parallel – mirror inverted on one side in order to gain a maximum bending moment – based on the sensor information of the tip deflection, which is manipulated by one proportional controller. With this configuration, the natural buckling load of $P_1 = 73.5$ N could be increased to $P = 130$ N. The numerical results agree very well with the experimental ones. In a second approach, it is aimed to compensate the entire deflections along the beam's axis. Therefore, the actuation of piezoelectric patches is weighted, following the theory of shape control, according to their position. Using again a displacement sensor at the tip and one proportional controller, the buckling load could be enhanced by factor 2.05 to $P = 150$ N. The numerical model showed the same results. Finally, a modal control approach is studied, in order to control the second eigenform. Hence, modal sensors and actuator are used. Due to the voltage limit, the increase of the buckling load is restricted to a factor of 2.37. Again, the numerical results agree very well, but could achieve higher buckling loads by means of a larger voltage range.

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