7th ECCOMAS Thematic Conference on Smart Structures and Materials
SMART 2015
A.L. Araúo, C.A. Mota Soares, et al. (Editors)

DESIGN OF CONTROL CONCEPTS FOR A SMART BEAM STRUCTURE WITH REGARD TO SENSITIVITY ANALYSIS OF THE SYSTEM

Sushan Li, Steffen Ochs, Tobias Melz

research group System Reliability and Machine Acoustics SzM, TU Darmstadt
Magdalenenstr. 4, 64289 Darmstadt, Germany
sLi@szm.tu-darmstadt.de
ochs@szm.tu-darmstadt.de
melz@szm.tu-darmstadt.de

Keywords: Sensitivity analysis, Smart (beam) structures, Control concepts, Piezoelectric elements, Finite Element (FE) Model, Model order reduction (MOR).

Summary: *A smart structure is a structure that can reduce the structural vibration by integration of sensor(s), actuator(s), and controller(s). The sensor detects the vibration of the beam and transfers the signal to the controller. Then the controller computes the desired control signal and sends it to the actuator. The controller is designed to compensate the beam's vibration. A piezoelectric patch is often used as a sensor or an actuator in a smart structure, as in this research project. A smart structure with a properly designed controller can reduce the structural vibration without changing the structure's physical dimensions. As a smart structure has more components in comparison to a passive structure, it can contain more uncertainties. Therefore, it should be well analyzed to ensure its reliability and robustness. Sensitivity analysis is a method that can describe the system's behavior quantitatively. This paper explains how to build a numerical model of a smart beam structure and how to design the control concepts for it with regard to the sensitivity analysis of this system. To carry out the sensitivity analysis the parameters of the smart structure will be varied in a small deviation and thousands of variations can be simulated. The difficulties arise from the definitions of the parameters of the controller to meet the requirements of thousands of variations.*

1. INTRODUCTION

Nowadays lightweight materials are widely used in many machines in order to reduce the costs of production and power consumption. But this leads to a new problem: Under the same excitation, the structure made of the lightweight materials undergoes stronger vibrations than a structure made of conventional materials. A smart structure can solve this problem [1]. Through the integration of sensor(s) and actuator(s) with a properly designed control strategy, a smart structure can reduce the vibration of the lightweight structure. As a smart structure has more components, it contains more uncertainties in comparison to the passive structure. Therefore, a smart structure should be well analyzed to ensure its reliability and robustness. Sensitivity analysis is a method to quantitatively describe the relationship between the inputs and outputs of a structure [2]. One of the sensitivity analysis methods is the stochastic analysis of a numerical model, which predicts the structure's behavior by analysis of the simulation's results under thousands of random structural input parameters [2].

This paper uses a beam structure as a reference system to clarify how to design the control concepts for it with regard to the system's sensitivity analysis. The smart beam structure with its designed geometric and material's parameters is used as the reference in this project. During the sensitivity analysis the geometric and material's parameters are slightly varied according to their predefined variation. This means the control concept should be robust not only for the referenced beam structure but also for the beam structure with small variations.

The numerical model of a smart beam structure has at least two parts, one is the finite element (FE) model of the structure, and the other is its control strategy. Karagülle et al. [3] explained a way to build the numerical model of a smart beam structure including the control system by only use of the software ANSYS. The displacement of the beam's end in the time domain can indicate the performance of the control system under the instantaneous excitation. But this way is inconvenient for a frequency response analysis, which can directly give an impression of the vibration behavior of the beam in a wide frequency range. On the other hand, MATLAB is widely used for the design of a control strategy [4, 5]. But the sizes of the FE model matrices are too big, which makes the design of a control strategy almost impossible. Rudnyi and Korvink [6] point out that a model order reduction can solve these problems. It reduces the size of the structural matrices, which can be extracted from an FE model without carrying out a harmonic simulation. The reduced matrices can be used in the control system. Having this knowledge, the path used in this paper to build the numerical model of the smart beam structure is shown in Figure 1.

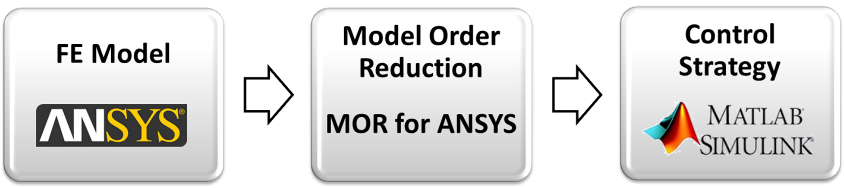


Figure 1. The process chain of the numerical model building.

There are many different kinds of control concepts that can be used to reduce the beam structure's vibration. Lead control (LC) is one of the popular active damping controllers, which can be used to compensate the beam structure's vibration [7]. The Linear Quadratic Regulator (LQR) and its extension, the Linear Quadratic Gaussian regulator (LQG), are very often discussed [8, 9] as a controller for a smart beam structure. Therefore, LC and LQR are chosen in this project for further analysis.

Chapter 2 introduces the smart beam structure and shows the way to build its FE model in the reduced state-space form. Chapter 3 focuses on the design process of the potential control concepts for the reference smart beam structure. Their performances are compared in Chapter 4 according to various criteria. The robustness of these control concepts is checked by varying some structural parameters of the smart beam structure according to a factorial experiment design, and the results are discussed in Chapter 5.

2. THE SMART BEAM STRUCTURE

A smart beam structure consists of a beam structure, at least a sensor, an actuator, and an appropriate control strategy (Figure 2). The smart beam structure used in this project is an aluminum beam, whose one side is clamped and whose other side is free. This beam is assumed to be an Euler-Bernoulli beam, therefore, its deformation is based on the basic equation of structural dynamics [10]. A vertical dynamic force at the free end of the beam acts as an excitation for the beam structure. Piezoelectric ceramic patches are widely used as sensor or actuator in a smart structure [3, 4, 11]. PIC 151, which is a type of the piezoelectric ceramic with a high permittivity, a high coupling factor, and a high piezoelectric charge constant, is chosen for this smart structure [12]. These two piezoelectric patches are collocated at the top and the bottom of the beam and act separately as actuator and sensor. The sensor detects the vibration of the beam and transfers the signal to the controller. Then the controller computes the desired control signal and sends it to the actuator. The controller is designed to compensate the beam's vibration. The dimensional and material data of the reference smart beam structure are listed in detail in Table 1.

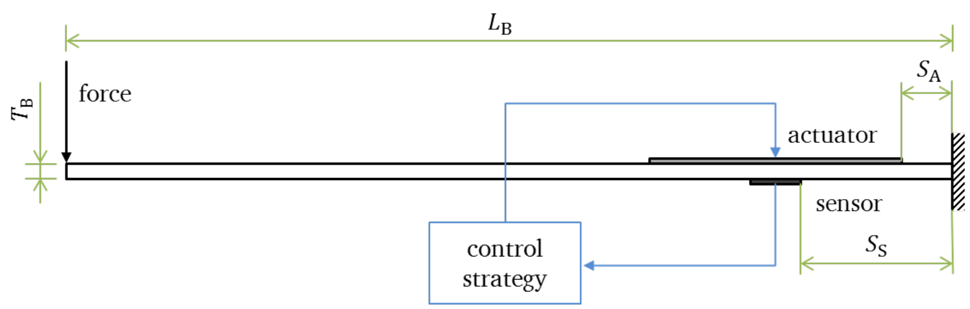


Figure 2. The smart beam structure.

	beam	actuator	sensor
length L in mm	200	50	10
width W in mm	40	30	10
thickness T in mm	3	1	1
position S_{A} in mm		10	
position S_{S} in mm			30
density ρ (kg/m^3)	2700	7800	7800
Young's modulus E (N/m^2)	$7 \cdot 10^{10}$		

Table 1. The dimensional and material's data of the reference smart beam structure.

2.1 Finite element model

In this project, the FE model of the smart beam structure is built by use of the software package ANSYS Workbench.

First the mechanic structure of the aluminum beam and the two piezoelectric patches is built in ANSYS. The type of contacts between the piezoelectric patches and the beam is defined as ideally bonded [13]. The element type for the beam is SOLID186, which is a three-dimensional structural SOLID element. SOLID226, which is an element type for coupled field components, is chosen for the piezoelectric patches [14]. Some pretests aimed to find out a proper size of the elements are done. By comparing the simulation results of the beam structures, which the size of the elements are set separately to be 0.002 m and 0.004 m, there are no differences.

Therefore, the size of the elements is determined to be 0.004 m. The structural damping is defined according to the Rayleigh damping, which is a mass- and stiffness-proportional damping

$$\mathbf{\Delta} = \alpha\mathbf{M} + \beta\mathbf{K}\,, \tag{1}$$

where Δ is the approximated structural damping, **M** is the structural mass matrix, **K** is the structural stiffness matrix, α is the mass-proportional damping coefficient, and β is the stiffness-proportional damping coefficient [15]. According to the ANSYS Help system[15], the mass damping α represents the friction damping and can be ignored in most situations. Therefore, in this case the mass damping is defined as $\alpha = 0$. In an experimental simulation it is measured that the whole structural damping ratio is about 5%. Then the corresponding stiffness value is defined as $\beta = 10^{-5}$.

After building the numerical model, the structural matrices **M** (the structural mass matrix), **K** (the structural stiffness matrix), **B** (the input matrix), and **C** (the output matrix) can be extracted to describe the dynamic behavior of the whole structure in form of differential equations [15].

2.2 Model order reduction

This FE model aims at the system's sensitivity analysis. According to the analysis' requirement it should be able to be built and simulated with more than thousands of small varied structural parameters' combination. Therefore, the duration of the model building and simulation should be short. Moreover, the size of the structural matrices should also be small for the control design. A good solution to meet both requirements is MOR. For this project, MOR for ANSYS [6] based on the Krylov subspace method is chosen. The difficulty of MOR lies in the definition of expansion points to overcome the singularity of the reduced matrices. According to the results of some pretests two expansion points are defined at $(-10, -10^5)$. The expanded dimension at each point can be purposely defined. In this case 6 dimensions are expanded at each point.

Then the reduced structural system can be described by

$$\begin{aligned} \mathbf{M_r}\ddot{\boldsymbol{q}} + \boldsymbol{\Delta}_\mathbf{r}\dot{\boldsymbol{q}} + \mathbf{K_r}\boldsymbol{q} &= \mathbf{B_r}\boldsymbol{u} \\ \boldsymbol{y} &= \mathbf{C_r}\boldsymbol{q}\,, \end{aligned} \tag{2}$$

where $\mathbf{M_r}$, $\boldsymbol{\Delta}_\mathbf{r}$, $\mathbf{K_r}$, $\mathbf{B_r}$, $\mathbf{C_r}$ are the reduced matrices of $\mathbf{M}$, $\boldsymbol{\Delta}$, $\mathbf{K}$, $\mathbf{B}$, and $\mathbf{C}$, respectively [6], $\boldsymbol{q}$ is the state vector, $\boldsymbol{u}$ is the input vector, and $\boldsymbol{y}$ is the output vector. For this smart beam structure, the input vector $\boldsymbol{u}$ is composed of the force at the beam's end $\boldsymbol{u_1}$ and the actuator's voltage $\boldsymbol{u_2}$. The output vector $\boldsymbol{y}$ includes the displacement of the beam's end $\boldsymbol{y_1}$ and the sensor's voltage $\boldsymbol{y_2}$. The vibration behavior of the beam structure based on the reduced matrices is checked by comparing it with that of the non-reduced matrices. The two curves in Figure 3 show that they are in excellent agreement.

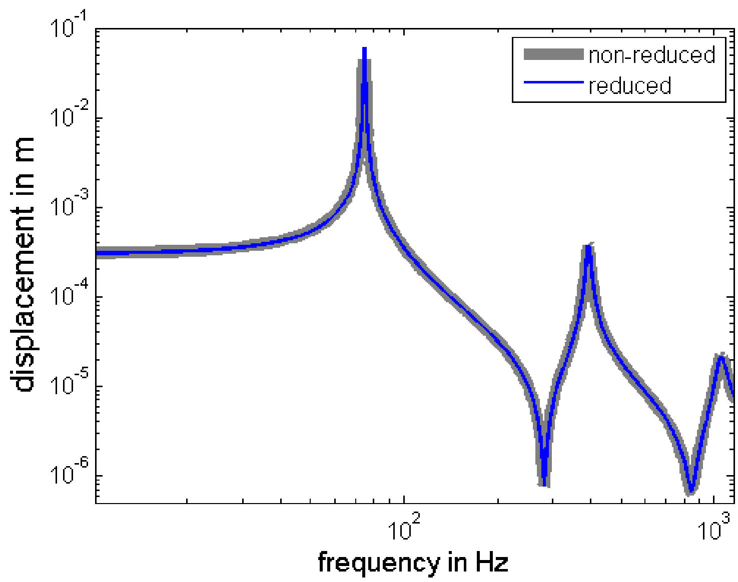


Figure 3. The displacement of the reference smart beam structure.

The differential equation can also be transformed to the state-space form

$$\begin{aligned}\dot{\boldsymbol{x}} &= \mathbf{A}_{\text{ss}}\boldsymbol{x} + \mathbf{B}_{\text{ss}}\boldsymbol{u} \\ \boldsymbol{y} &= \mathbf{C}_{\text{ss}}\boldsymbol{x} + \mathbf{D}_{\text{ss}}\boldsymbol{u}\ ,\end{aligned} \tag{3}$$

with the state vector $\boldsymbol{x} = \begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{bmatrix}$, the input vector $\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, the output vector $\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, the state matrix $\mathbf{A}_{\text{ss}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_{\mathbf{r}}^{-1}\mathbf{K}_{\mathbf{r}} & -\mathbf{M}_{\mathbf{r}}^{-1}\boldsymbol{\Delta}_{\mathbf{r}} \end{bmatrix}$, the input matrix $\mathbf{B}_{\text{ss}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_{\mathbf{r}} \end{bmatrix}$, the output matrix $\mathbf{C}_{\text{ss}} = \begin{bmatrix} \mathbf{C}_{\mathbf{r}} & \mathbf{0} \end{bmatrix}$, and the feedthrough matrix $\mathbf{D}_{\text{ss}} = \mathbf{0}$. The smart beam structure's control plant in state-space form is shown in Figure 4. The whole system is a multiple input and multiple output (MIMO) system.

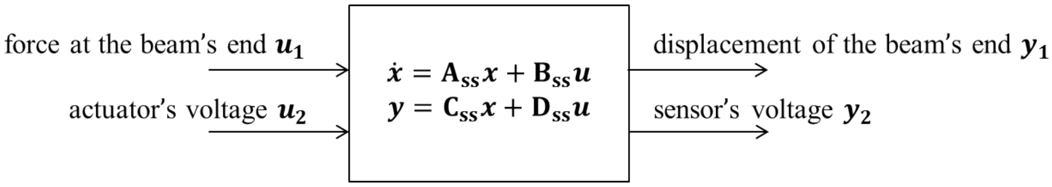


Figure 4. The smart beam structure's control plant in state-space form.

3. CONTROL CONCEPTS

Control concepts can be arranged in two groups. The first group consists of model-based controllers including the LQR. The control plant of the structure should be known before designing of the model-based controllers. The other group consists of non-model-based controllers, e.g., LC, which require little information about the structure but only the natural frequency [1]. The designing process of these two controllers and their technical parameters especially for the smart beam structure to reduce its vibration are explained in this chapter.

3.1 Linear Quadratic Regulator

An LQR can be designed based on the state-space model according to the principle of the state feedback (Figure 5).

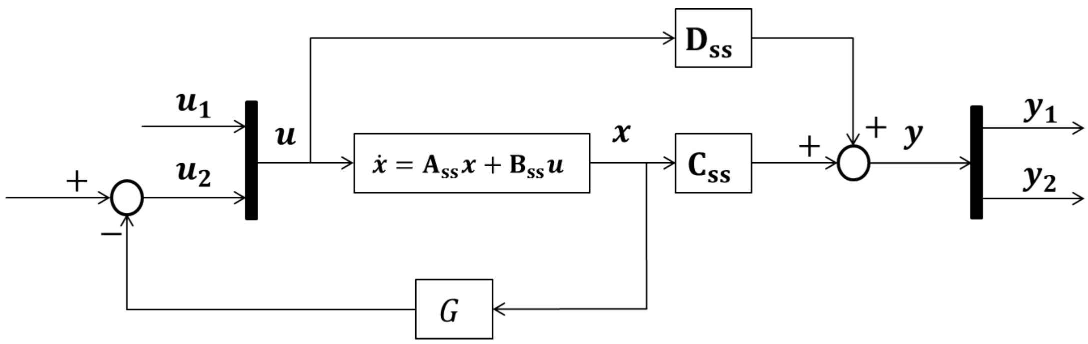


Figure 5. Block diagram of the LQR applied to the smart beam structure.

As an optimal control LQR seeks a linear state feedback with constant gain

$$\boldsymbol{u} = -G\boldsymbol{x} \tag{4}$$

to ensure the following quadratic cost function J is minimized

$$J = \int (\boldsymbol{x}^T\mathbf{Q}\boldsymbol{x} + \boldsymbol{u}^T\mathbf{R}\boldsymbol{u})\mathrm{d}t\,, \tag{5}$$

with suitably chosen matrices **Q** and **R**. The rule to choose **Q** and **R** is: **Q** must be positive semi-definite, while it implies that some of the states may be irrelevant for the design of the controller, meanwhile **R** must be positive definite as it expresses that any control has a cost. With suitable chosen matrices **Q** and **R**, the matrix **S** in the Riccati equation (6) can be calculated, and that means the optimal G in the LQR is also found out according to Equation (7)

$$\mathbf{A}_{\mathrm{ss}}^T\mathbf{S} + \mathbf{SA} - (\mathbf{SB}_{\mathrm{ss}})\mathbf{R}^{-1}(\mathbf{B}_{\mathrm{ss}}^T\mathbf{S}) + \mathbf{Q} = \mathbf{0} \tag{6}$$

$$\mathbf{G} = \mathbf{R}^{-1}\mathbf{B}_{\mathbf{ss}}^T\mathbf{S}\,. \tag{7}$$

The structure matrices $\mathbf{A}_{\mathbf{ss}}$ and $\mathbf{B}_{\mathbf{ss}}$ are exported from ANSYS via MOR for ANSYS. As the smart beam structure has two inputs, the force at the beam's end $\boldsymbol{u}_1$ and the actuator's voltage $\boldsymbol{u}_2$, so the matrix $\mathbf{B}_{\mathbf{ss}}$ is a matrix with two columns. According to the block diagram (Figure 5) this controller computes the desired control signal according to the sensor's signal. Hence, only the second input, which means not the whole matrix of $\mathbf{B}_{\mathbf{ss}}$ but just the second column of the matrix $\mathbf{B}_{\mathbf{ss}}$, is needed for the controller's design in Equations (6) and (7). As the actual output of the system is used as the control variable, **Q** is defined as $\mathbf{Q} = \mathbf{C}_{\mathbf{r}}^T\mathbf{C}_{\mathbf{r}}$. By carefully testing it is found out that by choosing $R = 10^4$ the reference smart beam's vibration can be compensated better than with other settings.

3.2 Lead Control

As already mentioned, the LC belongs to the non-model-based control concepts. A structure with a collocated, dual actuator/sensor pair can be actively damped with an LC [1]

$$H(s) = g\frac{s+z}{s+p} \tag{8}$$

with $p \gg z$.

Figure 6 shows the block diagramm of the LC. This controller produces a phase lead in the frequency band between the zero z and the pole p with an amplification g. As the result, all the modes $z < w_{\mathrm{i}} < p$ are actively damped. Therefore, the pole p must be set to be bigger than the zero z.

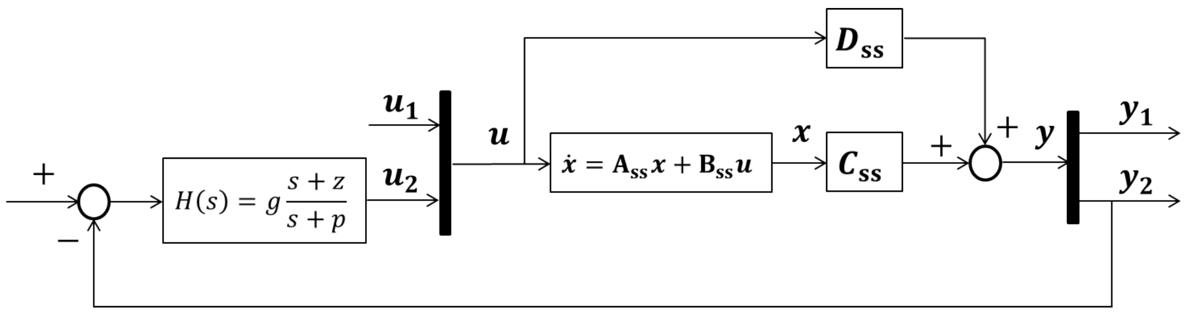


Figure 6. Block diagram of the LC applied to the smart beam structure.

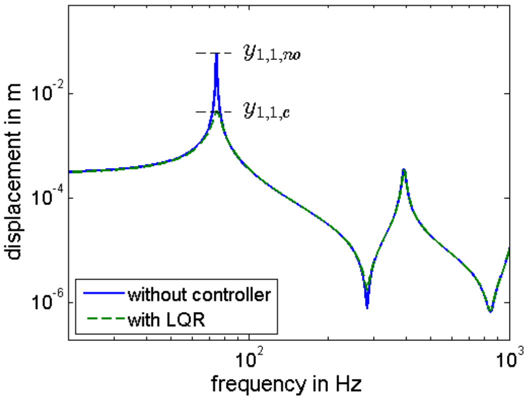
The MATLAB *Simulink Control Design Toolbox* is used in the project to find out the optimal setting of the parameters g, z, and p. By the trial moving the pole or the zero position in the toolbox, it is found out that, when $g = 3.85$, $z = 6$, and $p = 19664$, the vibration at the beam's end is optimally compensated.

4. COMPARISON FOR THE REFERENCE SMARTN BEAM STRUCTURE

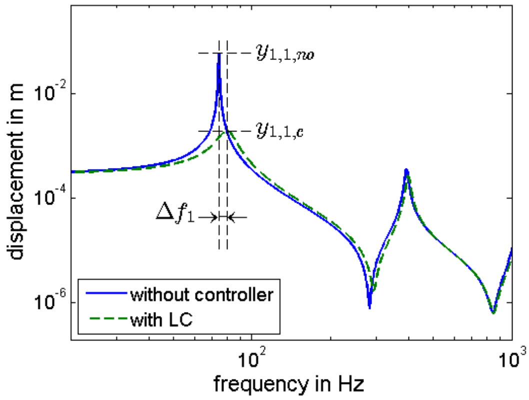
In this chapter the performances of these two controllers for the reference smart beam structure are compared in the frequency domain according to the Bode-diagram and also in the time domain by checking its step response.

4.1 Bode diagram amplitude gain

The Bode diagram's magnitude plot expresses the amplitude gain, which in this project is the displacement of the beam's end in meters. The frequency response of the smart structure with and without the controller is illustrated in the same Bode diagram (Figure 7). By comparing the two lines it can be directly determined, if the controller can compensate the vibration at the beam's end. From Figure 7 it can be observed that no matter with which controller, the first peak of the solid line is always sharper than the peak of the dashed line. That means both controllers perform well and the vibration of the beam's end at the first resonance frequency can be compensated.



(a) LQR.



(b) LC.

Figure 7. Bode diagram of the reference smart beam structure.

Two criteria are used in this project to compare the performances of the two controllers. The first criterion is the vibration reduction percentage at the first resonance frequency

$$\Delta y_{1,1} = \frac{(y_{1,1,\text{no}} - y_{1,1,\text{c}})}{y_{1,1,\text{no}}} \times 100\% \,, \tag{9}$$

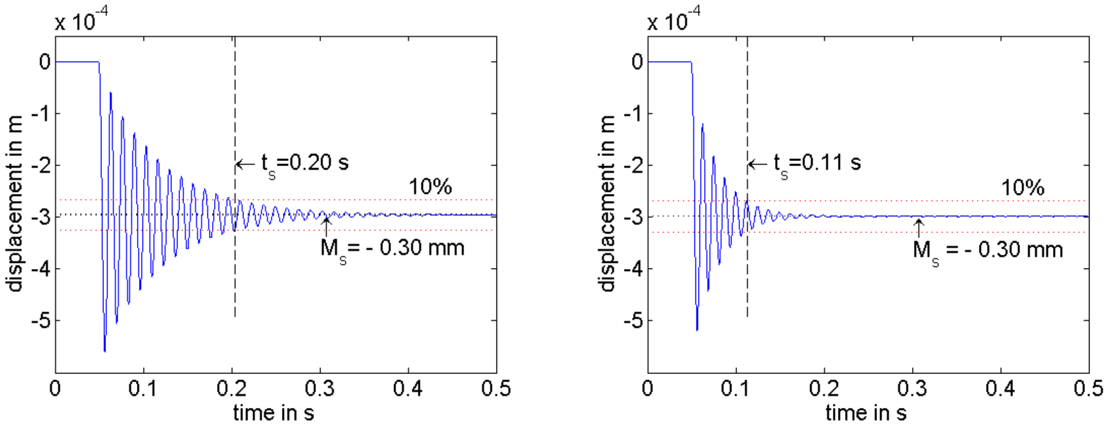
where $y_{1,1,no}$ is the displacement of the beam's end (y_1) at the first resonance frequency f_1 without a controller, and $y_{1,1,\text{c}}$ is the displacement of the beam's end (y_1) at the first resonance frequency f_1 with a controller (see also Figure 7(a)). The second criterion is the offset of the first resonance frequency Δf_1 (see also Figure 7(b)).

The smart beam structure with LQR can reduce 92.3% of the vibration (corresponding to 22.3 dB) at the first resonance frequency f_1. This vibration reduction percentage is 96.7% (corresponding to 29.7 dB) when the structure is connected with the LC. But the vibrations at the second f_2 and the third resonance frequencies f_3 are almost the same as those without controller. LC as an active damping controller that can change the structural vibration's behavior. In this case, the first resonance frequency f_1 of the reference smart beam structure with LC is shifted by $\Delta f_1 = 5.3$ Hz.

4.2 Step response

The step response of the smart beam structure with various controllers is checked to ensure if the smart structure is stable under a step force at the beam's end $u_1 = 1$ N (Figure 4). The settling time t_s and the response's final value M_s of these two controllers are compared. The settling time t_s presents the duration until the system is stable. The response's final value M_s

indicates the accuracy of the controller. The step response of the reference smart beam structure with both controllers is illustrated in Figure 8.



(a) LQR. (b) LC.

Figure 8. Step response of the reference smart beam structure.

From Figure 8 it can be determined that both controllers lead to a stable state. By comparing the settling time t_s of both controllers it can be found that the LC compensates the vibration faster than the LQR. The structure with LC needs only 0.11 s to settle the vibration in the range of $\pm 10\%$ of the final value M_s. But the structure with LQR needs 0.20 s, almost twice as long as the structure with LC. The response's final values M_s of both structures are almost identical.

All the compared data are listed in Table 2. The Δf_1 describes the offset of the first resonance frequency, but it is not a critical point to judge the controllers. Therefore, by the following robustness analysis of the controllers only the other three criteria are used to compare the performance of the two controllers.

	LQR	LC
$\Delta y_{1,1}$ in %	92.3	96.7
Δf_1 in Hz	0.4	5.3
t_s in s	0.20	0.11
M_s in mm	−0.30	−0.30

Table 2. Comparison of the controllers according to the criteria.

5. THE CONTROLLERS' ROBUSTNESS ANALYSIS

As the design of the control concepts is regarding to a stochastic simulation for sensitivity analysis of the smart beam structure, the robustness of the control concepts must be checked to ensure that the controller does not only work for the reference structure but also for the structures with small parameter variations. The robustness analysis is done according to a factorial experiment design.

5.1 The factorial experiment

In a stochastic simulation the geometric or material's parameters of the smart beam structure are randomly varied in a predetermined range. It is not feasible to check if the controller is working properly for all these varied structures. Instead of checking for every simulation combination in a stochastic simulation a statistic simulation is carried out by combining the minimum ($-$), the midpoint (0), and the maximum ($+$) of each varied parameter. But if all the geometric or material's parameters of the smart beam structure are varied, the simulation combinations are still too many to carry out. Han [16] did a sensitivity analysis of a very similar smart beam structure based on its analytical model. According to his sensitivity analysis result, the beam's length L_{B}, the beam's thickness T_{B}, and the actuator's position S_{A} have more influence on the beam's vibration than the other parameters. Hence, these three parameters are chosen as the designed factors to check the controllers' robustness and the other parameters are held constant as for the reference smart beam structure (Table 1). Table 3 shows the design factors' three varied levels. Therefore, $3^3 = 27$ simulation combinations (SC) are simulated in this experiment (Table 4).

	$(-)$	(0)	$(+)$
L_{B} in m	0.1950	0.2000	0.2050
T_{B} in m	0.0025	0.0030	0.0035
S_{A} in m	0.0080	0.0100	0.0120

Table 3. The design factors' values at each level.

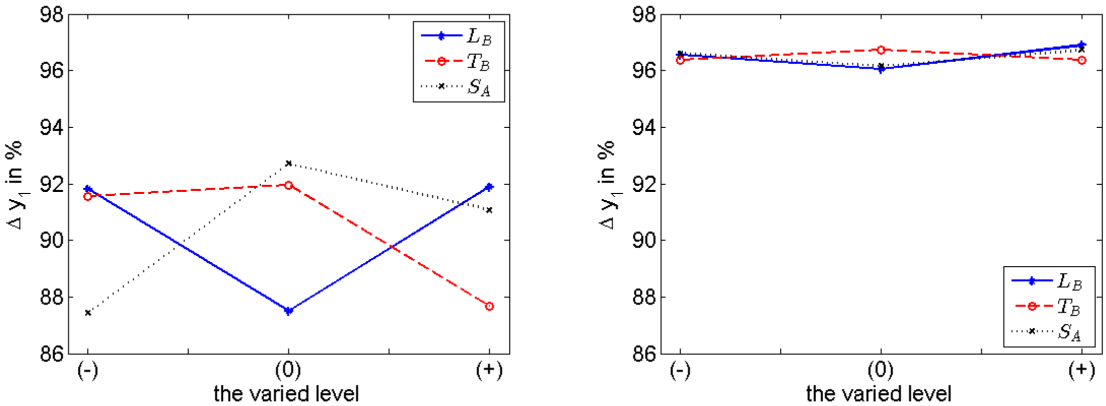
SC	L_{B}	T_{B}	S_{A}	SC	L_{B}	T_{B}	S_{A}	SC	L_{B}	T_{B}	S_{A}
1	$(-)$	$(-)$	$(-)$	10	$(-)$	$(-)$	(0)	19	$(-)$	$(-)$	$(+)$
2	(0)	$(-)$	$(-)$	11	(0)	$(-)$	(0)	20	(0)	$(-)$	$(+)$
3	$(+)$	$(-)$	$(-)$	12	$(+)$	$(-)$	(0)	21	$(+)$	$(-)$	$(+)$
4	$(-)$	(0)	$(-)$	13	$(-)$	(0)	(0)	22	$(-)$	(0)	$(+)$
5	(0)	(0)	$(-)$	14	(0)	(0)	(0)	23	(0)	(0)	$(+)$
6	$(+)$	(0)	$(-)$	15	$(+)$	(0)	(0)	24	$(+)$	(0)	$(+)$
7	$(-)$	$(+)$	$(-)$	16	$(-)$	$(+)$	(0)	25	$(-)$	$(+)$	$(+)$
8	(0)	$(+)$	$(-)$	17	(0)	$(+)$	(0)	26	(0)	$(+)$	$(+)$
9	$(+)$	$(+)$	$(-)$	18	$(+)$	$(+)$	(0)	27	$(+)$	$(+)$	$(+)$

Table 4. The level of each parameter of the 27 simulation combinations (SC).

5.2 Results and discussion

According to the full factorial experiments' plan in Table 4, 27 smart beam structures are simulated. Each structure is successively connected with the two controllers without changing the controllers' parameters. The influences of the three beam's parameters and the controllers are compared in this subsection according to the three criteria $\Delta y_{1,1}$, t_{s}, and M_{s}.

From Table 4 it can be found that there are 9 SCs (SC 1, 4, 7, 10, 13, 16, 19, 22, and 25) that are set at the low level of L_{B}. Then the average influence of L_{B} at the low level on the criterion $\Delta y_{1,1}$ is the average of the $\Delta y_{1,1}$ of these 9 SCs. Similarly the average of the influence of the other parameters T_{B} and S_{A} at other levels can be calculated. The results are plotted in Figure 9. It shows the varying tendencies of the criterion $\Delta y_{1,1}$ by changing the levels of parameter L_{B}.



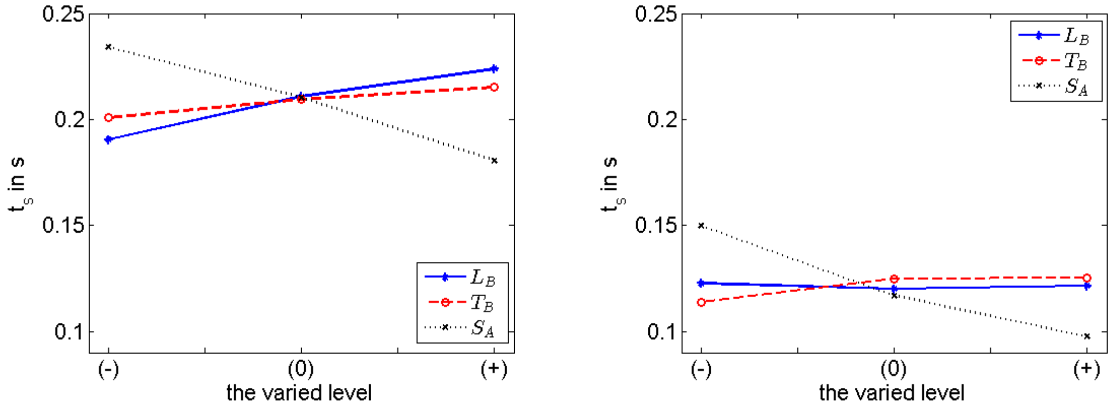
(a) LQR

(b) LC

Figure 9. The average of vibration reduction percentage $\Delta y_{1,1}$ at the first resonance frequency f_1 on different levels of L_{B}, T_{B}, S_{A}.

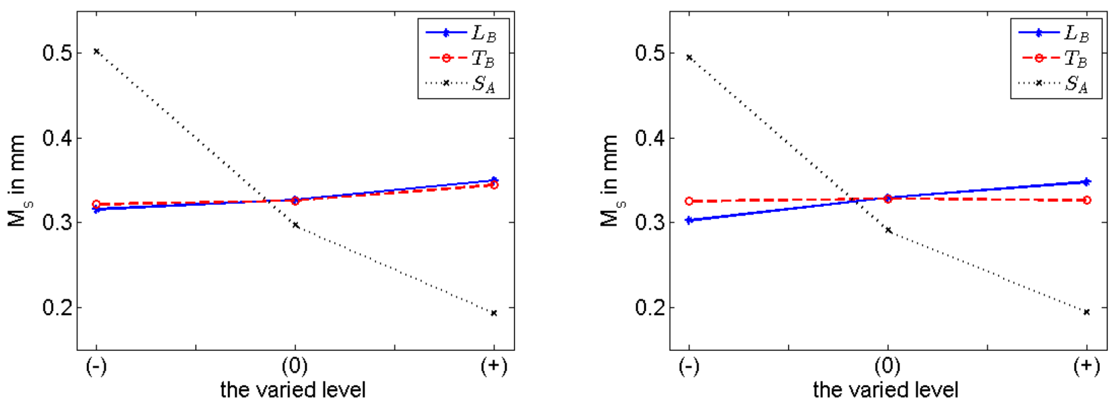
From Figure 9 it can be observed that the varying tendencies of the criterion $\Delta y_{1,1}$ are not linear. The minimum of the average of the criterion $\Delta y_{1,1}$ on each level of each parameter with both controllers is still larger than 87%, which means that both controllers can compensate the beam's vibration very well for all the SCs (Table 4). But comparing Figures 9(a) and 9(b) shows that the LC controller has a greater compensatory effect with a smaller variance than the LQR controller.

In a similar way the average of the other two criteria, t_{s} and M_{s}, of the SCs, which the three beam's parameters at each level is calculated and plotted in Figures 10 and 11.



(a) LQR (b) LC

Figure 10. The average of the step response time t_s on different levels of L_B, T_B, S_A.



(a) LQR (b) LC

Figure 11. The average of the step response final value M_s on different levels of L_B, T_B, S_A.

By comparing the results of the criteria t_s and M_s (Figures 10 and 11) it can be found that the position of the actuator S_A has more influence than the beam's length L_B or the beam's thickness T_B on the settling time of the step response t_s and the step response's final value M_s. The varying tendencies of t_s and M_s by changing the L_B and T_B are the same: The values of t_s and M_s are enlarged when the L_B or T_B are varied from the low level to the high level. In contrast, the values of t_s and M_s are diminished when the S_A is varied from the low level to the high level. In general no matter on which level, no matter which controller is used, the maximum of the t_s is smaller than 0.27 s and the maximum of the M_s is smaller than 0.56 mm, which means both controllers are stable and robust in this variation range. However, by detailed comparison between Figures 10(a) and 10(b) it can be confirmed that the LC controller answers the step excitation faster than the LQR controller. It needs about half the time the LQR controller needs.

Based on the numerical simulations' results of these 27 simulation points it can be concluded that the LC controller with higher compensation speed and better compensation ability is better than the LQR controller .

6. SUMMARY

This paper explains the numerical model building process of a smart beam structure. Based on the smart beam structure, two different controllers are designed in order to reduce the beam's vibration. The performances of both controllers are compared in the frequency domain according to the Bode diagram and also in the time domain by checking the step response. By the comparison of these two controllers for the reference smart beam structure it is found out that the LC with higher compensation speed and better compensation's ability is better than the LQR. As the controller's design aims for the sensitivity analysis of the smart structure, the controller should be robust when the structure is slightly varied. A numerical experiment is done to check the robustness of these two controllers by varying three parameters of the beam's structure in three levels and it is found out that both controllers are robust. By comparing the three criteria $\Delta y_{1,1}$, t_{s}, and M_{s} it is found out that the LC controller has a greater compensatory effect with a smaller variance than the LQR controller. Moreover, all the results discussed in this paper are numerical simulation results and they should be validated in experimental simulations. The LQR controller is designed based on a complete state's feedback. But the complete state vector cannot be measured in the experimental simulation. Therefore, by using the LQR controller in the experimental simulation an observer is needed to estimate the complete state vector. In conclusion, the LC controller performs better for the smart beam structure and it is recommended to be used for the following sensitivity analysis of the smart structure.

7. ACKNOWLEDGMENT

The authors would like to thank the German Research Foundation (DFG) for funding the project "Stochastische Simulationstechniken zur Bewertung der Sensitivität und Zuverlässigkeit adaptronischer Struktursysteme" (grant no.: HA 1634/35-1).

References

[1] A. Preumont. *Vibration Control of Active Structures: An Intoduction*. Springer, 3rd edition, 2011.

[2] A. Saltelli et al. *Global Sensitivity Analysis: The Primer*. John Wiley & Sons, Ltd, 2008.

[3] H. Karagülle et al. Analysis of active vibration control in smart structures by ANSYS. *Smart Materials and Structures*, 13(4):661–667, 2004.

[4] S. X. Xu and T. S. Koko. Finite element analysis and design of actively controlled piezoelectric smart structures. *Finite Elements in Analysis and Design*, 40(3):241–262, 2004.

[5] M. Kurch et al. A framework for numerical modeling and simulation of shunt damping technology. *The Sixteenth International Congress on Sound and Vibration (ICSV16)*, 2009.

[6] E. B. Rudnyi and J. G. Korvink. Model order reduction for large scale engineering models developed in ANSYS. *Lecture Notes in Computer Science*, 3732:349–356, 2006.

[7] W. K. Al-Ashtari. The deflection control of a thin cantilever beam by using a piezoelectric actuator / sensor. *The 1st regional conference of Eng. Sci. Nahrain University College of Engineering Journal (NUCEJ) spatial Issue*, 11:43–51, 2008.

[8] N. D. Zorić et al. Optimal vibration control of smart composite beams with optimal size and location of piezoelectric sensing and actuation. *Journal of Intelligent Material Systems and Structures*, 4:499–526, 2012.

[9] G. E. Stavroulakis et al. Design and robust optimal control of smart beams with application on vibrations suppression. *Advances in Engineering Software*, 36(11–12):806 – 813, 2005.

[10] E. N. Strømmen. *Structural Dynamics*. Springer, 2014.

[11] S. Peng et al. Modeling of a micro-cantilevered piezo-actuator considering the buffer layer and electrodes. *Journal of Micromechanics and Microengineering*, 22(6):065005, 2012.

[12] Piezoelectric Materials. Website.
http://piceramic.com/products/piezoelectric-materials.html; accessed April 24,2015.

[13] A. R. De Faria and S. F. M. De Almeida. Modeling of actively damped beams with piezoelectric actuators with finite stiffness bond. *Journal of Intelligent Material Systems and Structures*, 7(6):677–688, 1996.

[14] ANSYS® Academic Research, Release 12.1. *Help system: Element reference*, 2009.

[15] ANSYS® Academic Research, Release 14.0. *Help system: ANSYS Mechanical APDL theory reference*, 2012.

[16] S.-O. Han. *Varianzbasierte Sensitivitätsanalyse als Beitrag zur Bewertung der Zuverlässigkeit adaptronischer Struktursysteme (En.: Variance-based sensitivity analysis of smart structural dynamical systems)*. PhD thesis, TU Darmstadt, 2011.