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STATISTICAL ANALYSIS FOR PIEZO-BASED STRUCTURAL DAMAGE DETECTION USING ENHANCED NONLINEAR CRACK-WAVE INTERACTIONS

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Summary: *The paper presents statistical analysis of damage index used to detect structural damage in the recently developed enhanced nonlinear crack-wave interaction technique. Considered damage detection technique combines Lamb waves propagation with nonlinear acoustics. Low-frequency excitation is used to modulate propagating Lamb waves in the presence of fatigue cracks. Analysis of these modulations is used to detect the presence of damage. This method is extremely sensitive to any nonlinearity source present in an inspected object. It is, however, also sensitive to measurement noise. Therefore, currently it is necessary to perform high number of measurements in order to minimize the influence of noise on the results. A possible solution to that problem is the use of statistical resampling techniques. This paper presents a method of measurement data resampling in order to artificially increment the population size and improve the quality of estimates.*

1. INTRODUCTION

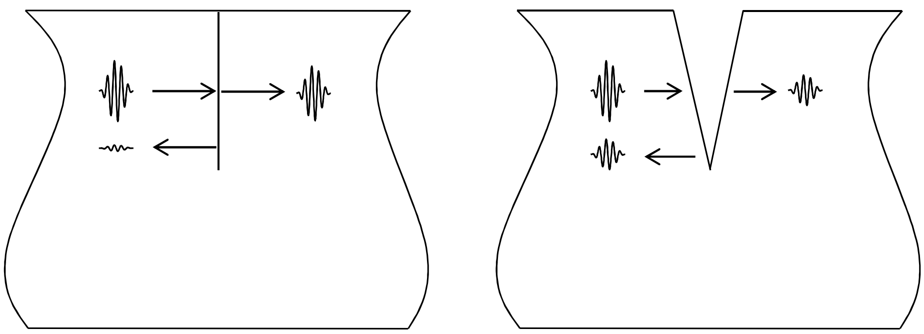
Various damage detection methods based on ultrasonic wave propagation have been developed for the last few decades [1, 2]. Within this group of methods there are the techniques based on nonlinear vibration and acoustic phenomena that gain an increasing attention in the scientific community [2, 3, 4]. This is mainly due to the fact that the nonlinear damage detection methods are usually much more sensitive to detect small damage severities than their linear counterparts [5].

The nonlinear vibro-acoustic modulation technique [8, 9, 10, 11, 12] is one of the most widely used approaches. The method allows for crack detection in metals [12, 13, 14] and impact damage detection in composites [15, 16, 17, 18]. There are two major drawbacks associated with the method. Firstly, damage location is limited and often not possible. Although a few attempts have been made to perform imaging s of damage-related nonlinearities, these studies are still limited.

Secondly, reliable damage indices - that indicate damage-related nonlinearities - are required. This is mainly due to the fact that nonlinear effects can be often associated with boundaries, material behaviour or measureing chain. The paper aims to address the second problem. The objective is to present a method of measurement data resampling in order to artificially increment the population size and improve the quality of estimates.

2. DESCRIPTION OF PROBLEM

The proposed damage detection technique utilizes the combination of a high frequency Lamb wave packet along with a low frequency harmonic excitation for detection of fatigue cracks. The main idea is to use a low frequency harmonic excitation to perturb fatigue crack, and probe that behaviour with a high frequency Lamb wave packet. Conceptually similar approaches for locating isolated nonlinear scattering sources, that utilize low frequency pumping wave and high frequency burst of probing wave, have been already proposed in the literature [6, 7] with the difference in the experimental setup and signal processing. In the proposed approach we observe that the behaviour of Lamb wave packet that propagates through the material is altered when interacting with the perturbed crack. If we synchronize the high frequency wave packets with low frequency harmonic excitation in such a way that the wave packet crosses the crack when it is in compression (Figure 1a) and in tension (Figure 1b) we can extract useful diagnostic information. When crack is in tension the crack faces will separate or partly separate and a greater part of an incident wave packet will be reflected. In contrast when the crack is in compression the incident wave packet can travel through the crack.



(a) (b)

Figure 1. Crack states: (a) compressed and (b) tensioned.

The responses should be, therefore, measured in these two configurations and the difference between them should be calculated. It is expected that in the presence of nonlinearity (i.e.

fatigue crack) the difference of those signals will be non-zero. This non-zero difference can be used to detect the presence and the location of a fatigue crack. It is expected that in case of experimental measurements, obtained responses will be of small amplitudes and signal-to-noise ratio will be relatively low. Therefore, it is required to repeat experiments a number of times to obtain more confidence concerning the results [19].

3. MATHEMATICAL MODEL

Mathematical model which describes considered situation will be presented in this section. There are two families of signals, C and D. Each family consists n signals from only one dynamic state, respectively close and open state. It is assumed that n is even. Each element of family C is random process and can be written as

$$X_i(t) = C(t) + \epsilon_{i,t}, \tag{1}$$

analogously, each element of family D can be written as

$$Y_i(t) = D(t) + \epsilon_{n+i,t} \tag{2}$$

where t is time, i is the index of signal (from 1 to n), $C(t)$ and $D(t)$ are deterministic functions and $\epsilon_{i,t}$ are independent and identically distributed zero-mean random variables. Values of $\epsilon_{i,t}$ can be understood as measurement noise and functions C and D are responses in a perfect situation without any measurement errors. The second index t in $\epsilon_{i,t}$ is fixed, so can be omitted to make calculations more lucid, hence $\epsilon_{n+i} = \epsilon_{n+i,t}$.

4. DOES C(t) EQUAL D(t)?

The main idea of the test is to analyse differences between collected signals. Based on differences between elements from the family C, $n/2$ random variables is constructed in the following way.

$$\begin{aligned} A_i &= X_{2i-1}(t) - X_{2i}(t) \\ &= C(t) + \epsilon_{2i-1} - C(t) - \epsilon_{2i} \\ &= \epsilon_{2i-1} - \epsilon_{2i} \end{aligned} \tag{3}$$

Analogously, next $n/2$ random variables is created for the family D.

$$\begin{aligned} A_{n/2+i} &= Y_{2i-1}(t) - Y_{2i}(t) \\ &= D(t) + \epsilon_{n+2i-1} - D(t) - \epsilon_{n+2i} \\ &= \epsilon_{n+2i-1} - \epsilon_{n+2i} \end{aligned} \tag{4}$$

This process produces n random variables which are linked with differences of pairs drawn from the same state. Afterwards n random variables $B_1, B_2, \ldots, B_n$ are created. Those are based on differences between different families, hence:

$$\begin{aligned} B_i &= X_i(t) - Y_i(t) \\ &= C(t) + \epsilon_i - D(t) - \epsilon_{n+i} \\ &= \epsilon_i - \epsilon_{n+i} + C(t) - D(t) \end{aligned} \tag{5}$$

Of course A_i and B_i depend on t, but since it is fixed, it is omitted. Ordinary testing if $C(t)$ equals $D(t)$ isn't plausible due to the relatively big measurement errors. One can eliminate this inconvenience by averaging difference between $C(t)$ and $D(t)$ for all t. Unfortunately this approach causes a problem as well, because sign of $C(t) - D(t)$ varies. To resolve this issue the mean of the squares A_i and B_i are considered.

$$\frac{1}{n}\sum_{i=1}^{n} A_i^2 = \frac{1}{n}\sum_{i=1}^{2n} \epsilon_i^2 - \frac{2}{n}\sum_{i=1}^{n} \epsilon_{2i-1}\epsilon_{2i} \tag{6}$$

$$\begin{aligned} \frac{1}{n}\sum_{i=1}^{n} B_i^2 =& \frac{1}{n}\sum_{i=1}^{2n} \epsilon_i^2 - \frac{2}{n}\sum_{i=1}^{n} \epsilon_i\epsilon_{n+i} + \frac{2(C(t)-D(t))}{n}\sum_{i=1}^{n}(\epsilon_i - \epsilon_{n+i}) \\ &+ (C(t)-D(t))^2 \end{aligned} \tag{7}$$

Hence the difference $Z(t)$ between $\frac{1}{n}\sum_{i=1}^{n} B_i^2$ and $\frac{1}{n}\sum_{i=1}^{n} A_i^2$ is

$$\begin{aligned} Z(t) =& \frac{2}{n}\sum_{i=1}^{n} \epsilon_{2i-1}\epsilon_{2i} - \frac{2}{n}\sum_{i=1}^{n} \epsilon_i\epsilon_{n+i} + \frac{2(C(t)-D(t))}{n}\sum_{i=1}^{n}(\epsilon_i - \epsilon_{n+i}) \\ &+ (C(t)-D(t))^2 \end{aligned} \tag{8}$$

Since expected value of random variable is linear one can calculate expected values separately for $\epsilon_{2i-1}\epsilon_{2i}$, $\epsilon_i\epsilon_{n+i}$ and $\epsilon_i - \epsilon_{n+i}$. All of them are zero, so the average of $Z(t)$ is just $(C(t) - D(t))^2$. Hence an estimator which is based on the same idea is unbiased. Since $(C(t) - D(t))^2$ is always positive, the problem with the sign of $(C(t) - D(t))$ is resolved. One can average all $Z(t)$ to get better approximation.

5. ESTIMATOR $Z_2(t)$

Currently it is necessary to perform high number of measurements in order to minimize the influence of noise on the results. In this section possible solution will be presented. The main idea is to build $Z(t)$ in a slightly different way to make it more reliable.

Assuming the same model as in previous section, one can construct $Z_2(t)$ in expanded way. Based on differences between elements from the family C, $\frac{n(n-1)}{2}$ random variables is constructed.

$$\begin{aligned} A_{i,j} &= X_i(t) - X_j(t) \\ &= C(t) + \epsilon_i - C(t) - \epsilon_j \\ &= \epsilon_i - \epsilon_j \end{aligned} \tag{9}$$

for $1 \leq i < j \leq n$. In analogous way, next $\frac{n(n-1)}{2}$ random variables is created for family D.

$$\begin{aligned} A_{n/2+i,n/2+j} &= Y_i(t) - Y_j(t) \\ &= D(t) + \epsilon_{n+i} - D(t) - \epsilon_{n+j} \\ &= \epsilon_{n+i} - \epsilon_{n+j} \end{aligned} \tag{10}$$

for $1 \leq i < j \leq n$. This process produces $n(n-1)$ random variables which are linked with the differences of pairs drawn from the same state. Afterwards n^2 random variables is created. Those are based on differences between different families, hence:

$$\begin{aligned} B_{i,j} &= X_i(t) - Y_j(t) \\ &= C(t) + \epsilon_i - D(t) - \epsilon_{n+j} \\ &= \epsilon_i - \epsilon_{n+j} + C(t) - D(t) \end{aligned} \tag{11}$$

for $1 \leq i, j \leq n$. To make calculation more lucid only $B_{i,j}$ for $i \neq j$ will be take into account. So eventually $n(n-1)$ variables of B kind is considered. As before, the average of A and B has to be calculated, the results are respectively equal:

$$A = \frac{1}{n}\sum_{i=1}^{2n} \epsilon_i^2 - \frac{2}{n(n-1)} \sum_{1 \leq i < j \leq n} (\epsilon_i \epsilon_j + \epsilon_{n+i}\epsilon_{n+j}) \tag{12}$$

$$\begin{aligned} B =& \frac{1}{n}\sum_{i=1}^{2n} \epsilon_i^2 - \frac{2}{n(n-1)} \sum_{1 \leq i \neq j \leq n} \epsilon_i \epsilon_{n+j} \\ &+ \frac{2(C(t) - D(t))}{n(n-1)} \sum_{1 \leq i \neq j \leq n} (\epsilon_i - \epsilon_{n+j}) + (C(t) - D(t))^2 \end{aligned} \tag{13}$$

and the difference between them is $Z_2(t)$. The same arguments as in case of $Z(t)$ prove that expected value of $Z_2(t)$ is also $(C(t) - D(t))^2$.

A question arises, is there a gain from using $Z_2(t)$ instead of $Z(t)$? To answer this question one can compare variance of $Z(t)$ and $Z_2(t)$.

6. VARIANCE OF $Z(t)$ AND $Z_2(t)$ ESTIMATORS

In this section comparison of variance of $Z(t)$ and $Z_2(t)$ will be provided.

$$\begin{aligned} Var(Z(t)) =& \mathbf{E}\Bigg(\Bigg(\frac{2}{n}\sum_{i=1}^{n} \epsilon_{2i-1}\epsilon_{2i} - \frac{2}{n}\sum_{i=1}^{n} \epsilon_i \epsilon_{n+i} \\ &+ \frac{2(C(t) - D(t))}{n} \sum_{i=1}^{n} (\epsilon_i - \epsilon_{n+i}) + (C(t) - D(t))^2 - \mathbf{E}(Z(t))\Bigg)^2\Bigg) \end{aligned} \tag{14}$$

Since $\mathbf{E}(Z(t)) = (C(t) - D(t))^2$ and $\frac{2}{n}$ is constant, variance of $Z(t)$ equals

$$Z(t) = \frac{4}{n^2}\mathbf{E}\left(\left(\sum_{i=1}^{n}\epsilon_{2i-1}\epsilon_{2i} - \sum_{i=1}^{n}\epsilon_i\epsilon_{n+i} + (C(t)-D(t))\sum_{i=1}^{n}(\epsilon_i - \epsilon_{n+i})\right)^2\right). \tag{15}$$

After multiplication one obtains expressions of following kinds:

- $(\epsilon_{2i-1}\epsilon_{2i})^2$, for $1 \le i \le n$,
- $(\epsilon_i\epsilon_{n+i})^2$, for $1 \le i \le n$,
- ϵ_i^2, for $1 \le i \le 2n$,
- $\epsilon_i\epsilon_{n+i}$, for $1 \le i \le n$,
- $\epsilon_{2i-1}\epsilon_{2i}\epsilon_{2j-1}\epsilon_{2j}$, for $1 \le i \le n$, $1 \le j \le n$, but $i \ne j$,
- $\epsilon_i\epsilon_{n+i}\epsilon_j\epsilon_{n+j}$, for $1 \le i \le n$, $1 \le j \le n$, but $i \ne j$,
- $\epsilon_i\epsilon_{n+j}$, for $1 \le i \le n$, $1 \le j \le n$,
- $\epsilon_{2i-1}\epsilon_{2i}\epsilon_j\epsilon_{n+j}$, for $1 \le i \le n$, $1 \le j \le n$,
- $\epsilon_{2i-1}\epsilon_{2i}\epsilon_j$, for $1 \le i \le n$, $1 \le j \le 2n$,
- $\epsilon_i\epsilon_{n+i}\epsilon_j$, for $1 \le i \le n$, $1 \le j \le 2n$.

Since ϵ_i are independent and zero-mean only first three kinds of random variables have non-zero mean. To obtain $Var(Z(t))$ one has to count how often they occur.

- $(\epsilon_{2i-1}\epsilon_{2i})^2$ occurs once for each $1 \le i \le n$,
- $(\epsilon_i\epsilon_{n+i})^2$ occurs once for each $1 \le i \le n$,
- ϵ_i^2 occurs once for each $1 \le i \le 2n$,

Hence

$$\begin{aligned} Var(Z(t)) =& \frac{4}{n^2}\mathbf{E}\left(\sum_{i=1}^{n}(\epsilon_{2i-1}\epsilon_{2i})^2\right) + \frac{4}{n^2}\mathbf{E}\left(\sum_{i=1}^{n}(\epsilon_i\epsilon_{n+i})^2\right) \\ &+ \frac{4(C(t)-D(t))^2}{n^2}\mathbf{E}\left(\sum_{i=1}^{2n}\epsilon_i^2\right). \end{aligned} \tag{16}$$

Therefore

$$Var(Z(t)) = \frac{8}{n}\alpha^2 + \frac{8(C(t)-D(t))^2}{n}\sigma^2, \tag{17}$$

where α^2 is variance of $\epsilon_i\epsilon_j$ $(i \neq j)$ and σ^2 is variance of ϵ_i. Analogously, variance of $Z_2(t)$ equals

$$\begin{aligned} Z_2(t) =& \mathbf{E}\Bigg(\Bigg(\frac{2}{n(n-1)}\sum_{1\leq i<j\leq n}(\epsilon_i\epsilon_j + \epsilon_{n+i}\epsilon_{n+j}) - \frac{2}{n(n-1)}\sum_{1\leq i\neq j\leq n}\epsilon_i\epsilon_{n+j} \\ &+ \frac{2(C(t)-D(t))}{n(n-1)}\sum_{1\leq i\neq j\leq n}(\epsilon_i - \epsilon_{n+j})\Bigg)^2\Bigg). \end{aligned} \tag{18}$$

One can reformulate (18) into

$$\begin{aligned} Z_2(t) =& \frac{4}{n^2(n-1)^2}\mathbf{E}\Bigg(\Bigg(\sum_{1\leq i<j\leq n}\epsilon_i\epsilon_j + \sum_{1\leq i<j\leq n}\epsilon_{n+i}\epsilon_{n+j} - \sum_{1\leq i\neq j\leq n}\epsilon_i\epsilon_{n+j} \\ &+ (C(t)-D(t))\sum_{1\leq i\neq j\leq n}(\epsilon_i - \epsilon_{n+j})\Bigg)^2\Bigg). \end{aligned} \tag{19}$$

After multiplication one obtains expressions of following kinds:

- $(\epsilon_i\epsilon_j)^2$, for $1 \leq i \leq n$, $1 \leq j \leq n$,
- $(\epsilon_{i+n}\epsilon_{j+n})^2$, for $1 \leq i \leq n$, $1 \leq j \leq n$,
- $(\epsilon_i\epsilon_{j+n})^2$, for $1 \leq i \leq n$, $1 \leq j \leq n$, but $i \neq j$
- ϵ_i^2, for $1 \leq i \leq 2n$,
- $\epsilon_i\epsilon_{n+j}$, for $1 \leq i \leq n$, $1 \leq j \leq n$,
- $\epsilon_i\epsilon_j\epsilon_k\epsilon_l$, for $1 \leq i \leq n$, $1 \leq j \leq n$, $1 \leq k \leq n$, $1 \leq l \leq n$, but $i \neq k$ or $j \neq l$,
- $\epsilon_i\epsilon_j\epsilon_k$, for $1 \leq i \leq n$, $1 \leq j \leq n$, $1 \leq k \leq 2n$.

Only first four kinds of random variables listed above have non-zero mean. To obtain $Var(Z_2(t))$ one has to count how often they occur.

- $(\epsilon_i\epsilon_j)^2$ occurs once for each $1 \leq i < j \leq n$,
- $(\epsilon_{i+n}\epsilon_{j+n})^2$ occurs once for each $1 \leq i < j \leq n$,
- $(\epsilon_i\epsilon_{j+n})^2$ occurs once for each $1 \leq i \leq n$, $1 \leq j \leq n$, but $i \neq j$

- ϵ_i^2 occurs $(n-1)$ times for each $1 \leq i \leq 2n$,

Once everything is sum up together $Var(Z_2(t))$ equals

$$\begin{aligned} Var(Z_2(t)) = & \frac{4}{n^2(n-1)^2}\mathbf{E}\left(\sum_{\substack{1\leq i<j\leq 2n \\ i\neq j+n}} (\epsilon_{2i-1}\epsilon_{2i})^2\right) \\ & + \frac{4(C(t)-D(t))^2}{n^2(n-1)^2}\mathbf{E}\left(\sum_{i=1}^{2n}(n-1)\epsilon_i^2\right). \end{aligned} \tag{20}$$

Using the same designation as before, equation (20) transforms into,

$$Var(Z_2(t)) = \frac{4}{n^2(n-1)^2}\sum_{\substack{1\leq i<j\leq 2n \\ i\neq j+n}} \alpha^2 + \frac{4(C(t)-D(t))^2}{n^2(n-1)^2}\sum_{i=1}^{2n}(n-1)\sigma^2. \tag{21}$$

Hence, finally

$$\begin{aligned} Var(Z_2(t)) &= \frac{8}{n(n-1)}\alpha^2 + \frac{8(C(t)-D(t))^2}{n(n-1)}\sigma^2 \\ &= \frac{Var(Z(t))}{n-1}. \end{aligned} \tag{22}$$

Above calculations show that variance of $Z_2(t)$ is $(n-1)$ times smaller than variance of $Z(t)$.

7. CONCLUSIONS

A new estimator $Z(t)$ for detection of nonlinear sources, such as fatigue crack damages, has been presented. This estimator is based on comparison of responses from two dynamic states, i.e. when the crack is opened and closed. The main advantage of the proposed estimator is the lack of necessity for baseline measurements representing undamaged condition and lack of sensitivity to temperature variations, as these measurements can be taken in short time apart. It is expected that resulting estimator $Z(t)$ will be zero, however in practise, due to measurement noises it never is. Therefore, confidence interval should be constructed, which tells whether value of $Z(t)$ is zero or not. Aforementioned confidence interval is linearly connected to the standard deviation, and this is a square root of variance. It means that lower the variance, there is more confidence concerning obtained results. In case of utilisation of estimator $Z_2(t)$ confidence interval is much smaller than in case of the estimator $Z(t)$, as variance is $(n-1)$ times smaller. Calculation of this confidence interval is not an easy task and it requires application of the bootstrap technique, which is a future step of this work.

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