

A Bayesian Grid Method PCA-based for Mobile Robots Localization in Unstructured Environments

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Abstract—This paper presents the experimental validation of a new method for mobile robot global self-localization in unstructured environments, i.e. that does not need any beacons or other artifacts structuring the environment. The method resorts to a PCA-based positioning sensor, filtered in a Bayesian probabilistic grid and combined with linear Kalman filters to estimate the global pose of mobile robots. In the implemented system, the information of the environment is captured only with onboard sensors installed in a differential drive robot: encoders, compass, and 2D depth sensor pointed to the ceiling. The use of PCA in a Bayesian probabilistic grid allows to fuse the highly compressed PCA database information, obtained with the low computational effort, in an environment where repetitive scenarios can occur. To avoid the negative impact in the localization estimate caused by the corrupted data existing in the 2D depth sensor, an extension to the classic PCA algorithm is suggested. Thus, the proposed method allows the self-localization of mobile robots in indoor environments with bounded accuracy and working in a wide range of illumination conditions.

I. INTRODUCTION

The problem of localization has been a great challenge to the scientific community in the area of mobile robotics; see [11], [3] and the references therein. Typically, the localization problem consists in the robot knowing what is its position in a map, based on the knowledge that it can obtain from the environment, through sensors. However, since usually the robot may capture similar observations about the environment at different points of the map, the fusion of the sensors with the robot motion model, through a Bayes filter, is common in localization systems, allowing the global localization of the robot [28], [10].

In [7] a Linear Parameter Varying (LPV) model of a Dubins car kinematic model being used in a classical Kalman filter (KF) has been developed and experimentally validated. However, as KF (or EKF that is the current practice in mobile robot localization) represents the estimate about the robot position through a Gaussian function, the probabilistic function is unimodal and, therefore, it is unable to represent the belief of being in different localizations at same time [10].

An alternative that allows the tracking of multiple possible localizations is the implementation of a multimodal probabilistic function to represent the probability about the robot localization in the Bayesian filter, using a discretization of the environment in a map. Several systems use topological information about landmarks with propose of reducing the number of states and, hence, to obtain a more compact representation of the environment [27], [23], [9].

The PCA was used in [22] for terrain reference navigation of underwater vehicles. In [7], [8], it was used for a self-localization system with a high compression ratio of the environment and low computational efforts [16]. These advantages are due the conversion of the map database into

an orthogonal space, allowing to obtain a new database with a high compression ratio, when compared with the amount of captured data. Moreover, the PCA allows to develop localization systems that do not depend on any predefined structure [17], [2], i.e. does not need to identify any specific features about the environment.

The use of vision systems to obtain information about the environment is common in robot localization due to the large amount of information that can be extracted from the RGB image [25], [24], [18], [13]. Although the common solution in the development of the localization systems is to install cameras to look around the mobile robot to obtain its position [26], [13], [17], some robots use a single camera looking upward [15], [12], [29]. The use of vision from the ceiling has the advantage that images can be considered static and without scaling. This solution of a ceiling vision is also successfully implemented in [7], [6], which approach is related to the work presented in this paper.

However, a general handicap of vision sensors is the low robustness working in different environment lighting conditions, decreasing the robustness of the mobile robot localization systems. To avoid this problem, the use of time-to-flight sensors is implemented in same localization systems [19], [1]. Moreover, the time-of-flight has the advantage that allows the capture of depth images with a grid of depth information from the field of view. Recently, due to its low price and a straightforward way to be connected with a computer, the Kinect - device with a RGB and a depth camera developed to video games by PrimeSense and Microsoft - becomes very popular in mobile robotics community, creating some interesting mobile robots applications [4], [14]. Kinect depth images are obtained by a structured-light 3D scanner.

A common problem in depth sensors, including the Kinect depth sensor, is the existence of missing data in signals, caused by IR beams that are not well reflected, not returning to the depth sensor receiver. In [20], [21], a method using the Principal Component Analysis (PCA) methodology to avoid the problem of missing data in signals is presented and its performance is compared with other state-of-the-art algorithms, concluding that PCA overperforms other algorithms for recovery of corrupted signals. This method is experimentally validated in self-localization system of mobile robots with corrupted depth images captured by Kinect installed onboard, pointing upwards to the ceiling [6].

In the present paper, an adaptation of the self-localization method proposed in [6], considering a Bayesian filter in the PCA position sensor is implemented in a grid map and experimentally validated.

This paper is organized as follows: Section II presents the mobile robot platform and the motivation for the use of Kinect in the proposed localization system; in the Section III, the

proposed method that fuses the principal component analysis algorithm for signals corrupted with missing data in a Bayesian filter to obtain the mobile robot position is detailed. For the method validation purpose, Section IV presents experimental results of the proposed method; Finally, some conclusions and topics about future work are presented in Section V.

II. MODEL PLATFORM

The experimental validation of the positioning system proposed in this paper is performed resorting to a low cost mobile robotic platform [5], with the configuration of a Dubins car. A Microsoft Kinect is installed on the platform, pointing upwards to the ceiling, and a digital compass, located on the extension arm (robot rear part) to avoid the motors magnetic interference (see Fig. 1).

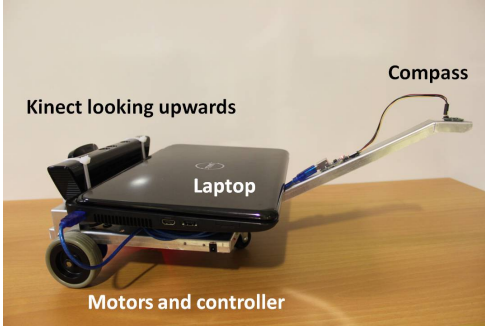


Fig. 1. Mobile platform equipped with kinect sensor and compass

The Kinect includes a RGB camera with a VGA resolution (640×480 pixels) using 8 bits and a 2D depth sensor (640×480 pixels) with 11 bits of resolution. Once the robot moves in an environment indoors in buildings with some information (e.g. building-related systems such as HVAC, electrical and security systems, etc.), it is possible to use the signals captured by a Kinect looking upward (RGB image, depth map or both) by an algorithm that can provide mobile robot global position in the environment.

Due to limitations found in image-based mobile robot localization approaches, regarding illumination changes, and aiming the development of an efficient self-localization solution that can work in places with variation on the level of illumination, only the Kinect depth signal is used, resorting to an adaptation to the method proposed in [22], [8], [7] to the problem at hand.

III. PCA-BASED LOCALIZATION WITH A PROBABILISTIC BAYESIAN GRID

PCA [16] is a methodology based on the Karhunen-Loève (KL) transformation which is often used in applications that need data compression, like image and voice processing, data mining, exploratory data analysis and pattern recognition. The data reduction is obtained through the use of a database eigenspace approximation by the best fit eigenvectors. This technique makes the PCA an algorithm that has a high compression ratio and requires reduced computational resources. The PCA algorithm is successfully used as the mobile robot's position sensor in [6], [8].

The PCA eigenspace is created based on a set of M stochastic signals $\mathbf{x}_i \in \mathbb{R}^N$, $i = 1, \dots, M$ acquired by a Kinect depth sensor installed onboard the mobile robot, considering an area with N mosaics in two dimensional space, $N = N_x N_y$, where N_x and N_y are the number of mosaics in x and y axis, respectively.

In the common PCA-based approaches, the eigenspace of the set of acquired data is characterized by the corresponding mean $\mathbf{m}_x = \frac{1}{M} \sum_{i=1}^M \mathbf{x}_i$ and covariance $\mathbf{R}_{xx} = \frac{1}{M-1} \sum_{i=1}^M (\mathbf{x}_i - \mathbf{m}_x)(\mathbf{x}_i - \mathbf{m}_x)^T$.

However, the existence of missing data in signal \mathbf{x}_i corrupts the PCA mean value computation creating an orthogonal space with erroneous data. To avoid the negative impact of the sensor signals missing data in PCA-based approaches performance, an extension to this methodology, shown in [6] is used which is not detailed here.

Considering the mean ensemble and covariance of the PCA database computed, the decomposition into the orthogonal space follows the PCA algorithm classical approach, i.e. $\mathbf{v} = \mathbf{U}^T(\mathbf{x} - \mathbf{m}_x)$. The matrix $\mathbf{U} = [\mathbf{u}_1 \mathbf{u}_2 \dots \mathbf{u}_N]$ should be composed by the N orthogonal column vectors of the basis, verifying the eigenvalue problem:

$$\mathbf{R}_{xx} \mathbf{u}_j = \lambda_j \mathbf{u}_j, \quad j = 1, \dots, N, \quad (1)$$

Assuming that the eigenvalues are ordered, i.e. $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_N$, the choice of the first $n \ll N$ principal components leads to stochastic signals approximation given by the ratio on the covariances associated with the components, i.e. $\sum_n \lambda_n / \sum_N \lambda_N$.

During the mission, the signal \mathbf{x} is decomposed into the orthogonal space considering only the non-corrupted data. Thus, before the projection of the depth image into the orthogonal space, the mean substitution should be followed, i.e. all j^{th} component of the signal \mathbf{x}_i with corrupted data should be replaced by the corresponding mean value $\mathbf{m}_x(j)$. This method removes the effect of the corrupted data in its decomposition in the orthogonal space $\mathbf{v} = \mathbf{U}^T(\mathbf{x} - \mathbf{m}_x)$.

The robot position is obtained by finding in all PCA eigenspace, the mosaic i whose eigenvector \mathbf{v}_i is nearest to the acquired signal \mathbf{v} decomposed into the orthogonal space:

$$r_{PCA} = \min_i \|\mathbf{v} - \mathbf{v}_i\|_2, \quad i = 1, \dots, N; \quad (2)$$

Given the mosaic i that verifies this condition, its center coordinates $[x_i y_i]^T$ are selected as the robot position, obtained by the PCA-based sensor.

However, when the robot moves inside a large area or in scenery with periodic information, the information becomes often ambiguous, resulting frequently in wrong values and consequently, the robot can quickly lose its position. This problem occurs because the use of the PCA for localization as expressed by equation (2) resorts to an unimodal position algorithm, which estimates the position based on the data stored in the database with the eigenvector closest to the acquired data at that instant. To solve this problem, a Markov localization (ML) algorithm is applied, integrating the images eigenvectors distance in a Bayesian probabilistic grid.

A. Bayesian position sensor with PCA-based probabilistic observations

One common method to provide a global localization in robotics is the ML algorithm, that is a straightforward application of a Bayes filter, often used in robot navigation [28], [10]. The ML provides the robot localization through a probabilistic grid map of the environment, that represents the belief of the robot about its position. This algorithm applies the Markov assumption, where knowledge of the previous state and current inputs is enough to predict the probability of the current state. Thus, ML can estimate the states \mathbf{x} in instant k based only in instant $k - 1$. The Markov assumption make ML a multi-modal algorithm allowing the track of multiple possible positions, disambiguate the periodic information, and find the right robot position.

During an experiment, when the robot is moving, the probability about the robot states is continuously updated with the robot motion prediction and consequent observation, following the Markov algorithm in discrete instant k , where $\mathbf{x}(k-1)$ is the state vector in the previous iteration, $\mathbf{u}_a(k)$ the controller action, $\mathbf{z}(k)$ the observation in the current iteration and \mathbf{m} the grip map of the environment (Fig. 2).

Markov localization ($\mathbf{x}(k-1), \mathbf{u}_a(k), \mathbf{z}(k), \mathbf{m}$)
for $i = 1$ to N do
 $\bar{P}(\mathbf{x}_i(k)) = \sum_{i=1}^N p(\mathbf{x}_i(k) | \mathbf{u}_a(k), \mathbf{x}_i(k-1)) P(\mathbf{x}_i(k-1))$
 $P(\mathbf{x}_i(k)) = p(\mathbf{z}(k) | \mathbf{x}_i(k), \mathbf{m}) \bar{P}(\mathbf{x}_i(k))$
return $P(\mathbf{x}(k))$

Fig. 2. Markov localization algorithm

The correction of the prediction step is performed considering that the observation $\mathbf{z}(k)$ is given by the signal captured by the Kinect sensor and processed by the missing data correction:

$$P(\mathbf{x}_i(k)) = p(\mathbf{z}(k) | \mathbf{x}_i(k), \mathbf{m}) \bar{P}(\mathbf{x}_i(k)), i = 1, \dots, N \quad (3)$$

The belief (here referred more generally as a probability) of the robot being in any position of the map is obtained by the distance between the captured image and the corresponding image into the PCA eigenspace:

$$p(\mathbf{z}(k) | \mathbf{x}_i(k), \mathbf{m}) = (1 - \eta \|\mathbf{v} - \mathbf{v}_i\|_2), i = 1, \dots, N \quad (4)$$

where η is a normalization factor, which ensures $\sum p(\mathbf{z}(k) | \mathbf{x}_i(k), \mathbf{m}) = 1$, \mathbf{v} the eigenvector of the captured image, \mathbf{v}_i the eigenvector of the i image into the eigenspace and $p(\mathbf{z}(k) | \mathbf{x}_i(k), \mathbf{m})$ the probability to observe $\mathbf{z}(k)$ in the state $\mathbf{x}_i(k)$ considering the map \mathbf{m} .

Finally, the robot position given by the PCA-based position sensor with ML is obtained finding the mosaic that has the maximum probability calculated in (3):

$$[\hat{x} \ \hat{y}]^T = \arg \max_{\mathbf{x}_i(k)} (P(\mathbf{x}_i(k))); \quad (5)$$

IV. EXPERIMENTAL RESULTS

A. Experimental Setup

To create the ceiling image database, a set of 1115 snapshots with depth images have been captured at pre-specified grid locations, with the robot in the same attitude, along a grid map with a distance of 0.3 m (in x and y axis) in an area of $18.9 \text{ m} \times 9.6 \text{ m}$ (Figs. 3–5). The captured depth images are cropped with a circular mark allowing the rotation and comparison of captured depth images when the robot is in the same position, but with different attitude, during a mission. In order to compress the amount of data, the depth images are sampled with a compression ratio of $100 : 1$ and converted into a vector that will be added to PCA eigenspace. Therefore, analyzing the corresponding PCA eigenvalues and selecting the components that explain the images variability in an excess of 85% , leads to an image database of 9 eigenvectors. This, correspond to a reduction of 99.9% in the memory resources when compared with the capacity needed to store the captured database, and 95% when compared with the size after the subsample. In [8], [7], the authors followed a similar approach using a RGB camera, but the method revealed to be sensitive to illumination conditions.

During an experiment, it is possible to estimate the robot's attitude and position, as well as the angular motion speed and the robot's angular slippage, using only the signals obtained by the onboard sensors (Kinect, compass and encoders), through a self-localization sensor based in two KF and one PCA algorithm, with an architecture as detailed in Fig. 4.

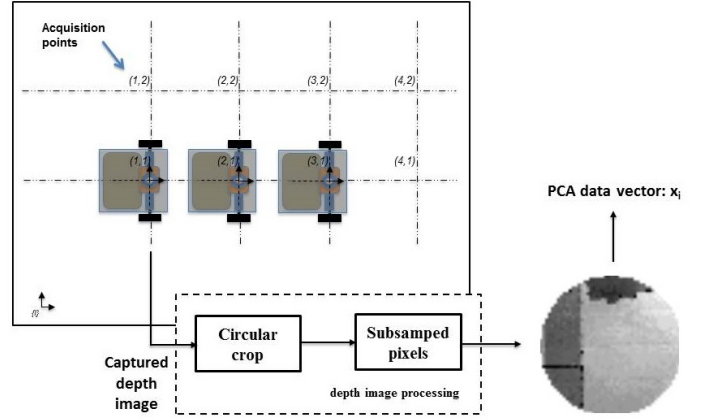


Fig. 3. Grid map and depth image processing to create a PCA eigenspace

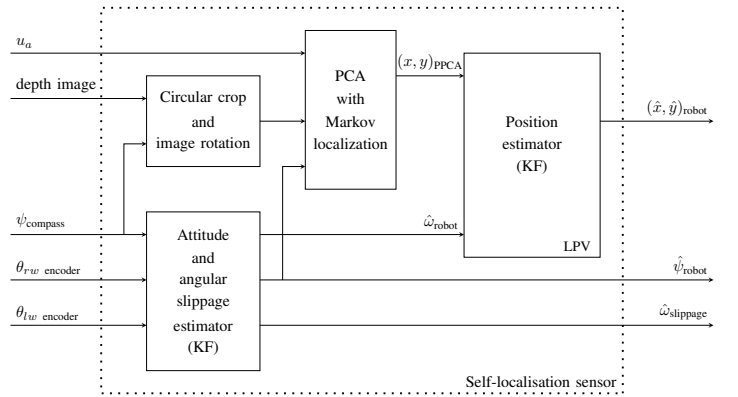


Fig. 4. Architecture of the self-localization sensor

The following notation is used in Fig. 4: u_a - controller action; $\psi_{compass}$ - orientation angle given by the compass; $\theta_{rw_encoder}/\theta_{lw_encoder}$ - angle given by the encoder of the right/left wheel; $(x, y)_{PPCA}$ - position coordinate given by the probability grid; $(\hat{x}, \hat{y})_{robot}$ - estimated robot coordinates in the world referential; $\hat{\omega}_{robot}$ - estimated angular speed; $\hat{\omega}_{slippage}$ - estimated differential slippage.

Detailing the architecture of the self-localization sensor presented in Fig. 4, the KF depicted on the left of the figure implements the attitude optimal estimator model that is responsible to estimate the mobile robot attitude and the angular slippage. Once all acquired depth images for the PCA database are taken with the same orientation and compressed with a circular crop (Fig. 3), during a mission, the acquired depth images must be rotated to zero degrees of attitude, using the compass angle, and compressed with the same circular crop. The position estimator (on the right of the figure) implements a Linear Parameter-Varying (LPV) model as a function of the estimated angular speed in a KF, fusing it with the position obtained by the PCA with Markov localization algorithm.

Resorting to this architecture, it is possible to estimate the position, attitude and angular slippage of the mobile robot with global stable error dynamics. For more details about this self-localization architecture see technical report [8].

B. Bayesian Motion Prediction

The method detailed in Fig. 4 applies the ML algorithm to compute the position of the robot in a probabilistic grid, considering the position of the mosaics captured to create the PCA database. Thus, the state vector of the Bayesian



Fig. 5. Ceiling view of the environment with periodic elements

filter is the position of obtained by the PCA with the ML algorithm: $\mathbf{x}_i = [x_{PPCA} \ y_{PPCA}]^T$. As presented in Section III, the filtering step of the Bayesian filter is computed through a probability grid based on the PCA algorithm applied to the captured depth image. With the propose of computing automatically the probability of the state transition in the prediction step, and assuming that the robot moves in the direction given by the attitude estimator, the following method is applied.

Considering a grid with N_x and N_y mosaics in x and y axis, respectively, the probability map with $N_x \times N_y$ is created and initialized with the same probability in all states $P(\mathbf{x}(0)) = \frac{1}{N}$, where $N = N_x \times N_y$.

During a mission, following the classical Markov localization algorithm, the robot position probability map is updated in a prediction step, considering the belief of the robot reaching a new state $\mathbf{x}_i(k)$ with the control $\mathbf{u}_a(k)$. This probability is obtained by the sum of all possible ways to the robot to reach the mentioned state.

$$\bar{P}(\mathbf{x}_i(k)) = \sum_{i=1}^N p(\mathbf{x}_i(k)|\mathbf{u}_a(k), \mathbf{x}_i(k-1))P(\mathbf{x}_i(k-1)), \quad i = 1, \dots, N \quad (6)$$

Once that the states are stored in a grid, the probability map computation in the prediction step is performed considering the geometry of the grid and the speed and attitude of the robot, as shown in Fig. 6.

Analyzing the probability state transition of Fig. 6 and considering that the robot is moving with a speed u in a direction with an attitude ψ , the images of eigenspace are captured with a distance d between mosaics and the localizer has been computed in a digital processor with a sampling time T , being the probability states transition is given by:

$$\bar{P}(\mathbf{x}_i(k)) = \eta \sum_{l=1}^9 p_l P(\mathbf{x}_l(k-1)) \quad (7)$$

where η is a normalizer, which ensures $\sum \bar{P}(\mathbf{x}_i(k)) = 1$. Thus, considering that the robot is moving in a direction ψ , the possible way that is proposed in this paper to compute the

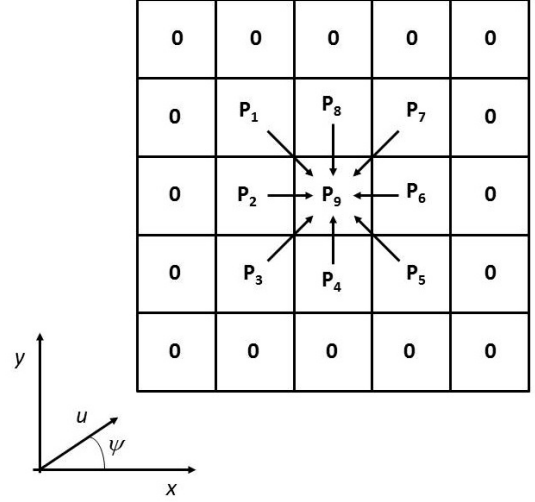


Fig. 6. Probabilities in prediction step

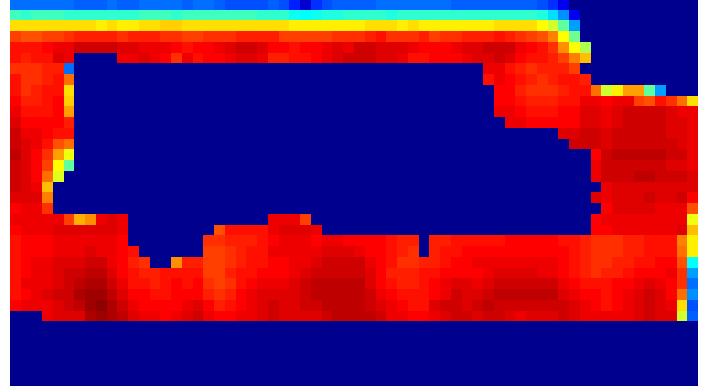


Fig. 7. Localization belief grid after 1 observation

state transition probability from each 8 neighbors ($l = 1 \dots 8$) is:

$$p_l = \begin{cases} (\cos(\psi + \tan^{-1} \frac{y_l - y_9}{x_9 - x_l}))^q & \text{if } p_l > 0 \\ 0 & \text{if } p_l \leq 0 \end{cases} \quad (8)$$

$$p_l = \eta \cdot p_l (1 - p_9) \quad (9)$$

with η a normalization factor, which ensure $\sum p_l = 1$, and p_9 is the probability that the robot is kept in the same state.

Once $p_i < 1, i = 1 \dots 8$, the exponent q allows to increase the probability between the states in the same direction of the robot attitude, reducing the probability between the states whose transition is caused by angular motion uncertainty (diagonal direction in state transition).

Finally, the new predict states probability map $\mathbf{x}_i(k)$ is obtained:

$$\bar{P}(\mathbf{x}_i(k)) = \eta (\sum_{l=1}^8 (p_l \frac{u \cdot T}{d} P(\mathbf{x}_l(k-1))) + p_9 (1 - \frac{u \cdot T}{d}) \cdot P(\mathbf{x}_9(k-1))) \quad (10)$$

C. Results for 2D localization

To test the mobile robot self-localization performance of the proposed approach, several tests have been performed in an environment with repeatability along a predefined path with 93 m, combining both straight lines and curves and traveling

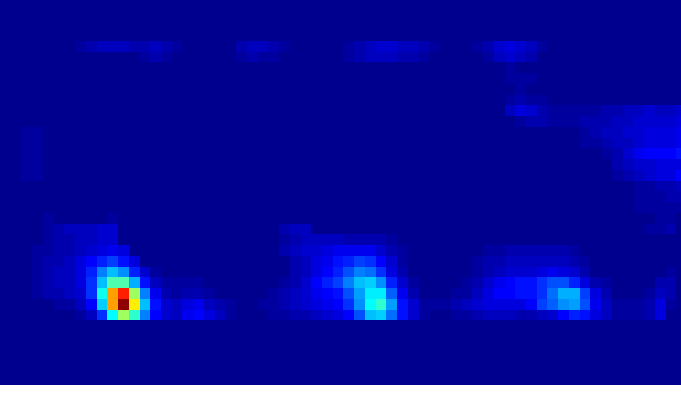


Fig. 8. Localization belief grid after 30 observations

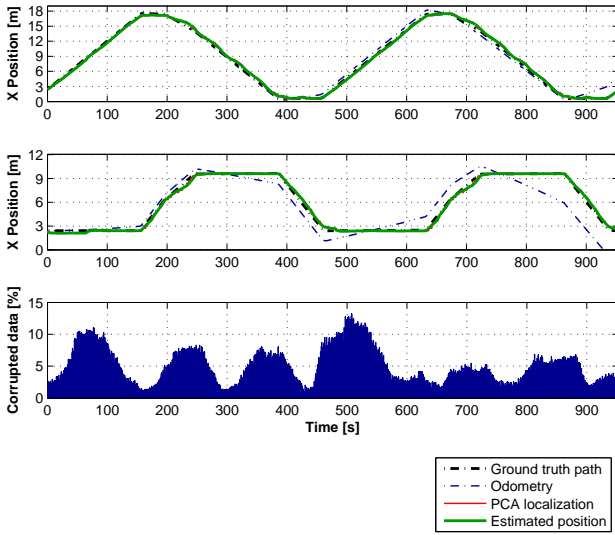


Fig. 9. Estimated position along time

two laps inside the mapped area. During the experiment, the robot is moving with $0.1 \text{ m} \cdot \text{s}^{-1}$ robot speed and 2.5 Hz of sampling frequency.

Figures 7–8 show the probability grid performed by ML with a filtering step based on the PCA algorithm, where the "hot colors" (dark red) represent high localization belief and "cold colors" (blue) the low localization belief. The blue areas in the center, bottom and top in Fig. 7 represent unmapped areas which do not exist in the PCA eigenspace and, so, have null probability.

Figure 8 shows that ML is able to quickly disambiguate possible repeatability of the scenery and find the right position. Figure 8 shows that after 30 samplings (12 seconds), the position sensor has a high belief about the right position of the robot, which can be validated with the results presented in Fig. 9.

Figures 9 and 10 present the results of the self-localization sensor, comparing the position obtained by the ML with PCA, the KF position estimator and the real path of the robot, measured in the ground. The results of Fig. 9 show that the ML with PCA approach is able to achieve an accurate position of the robot, allowing good performance on the global self-position sensor. Notice that the grid with depth images has a 0.3 m accuracy whereas the position measured is far less than

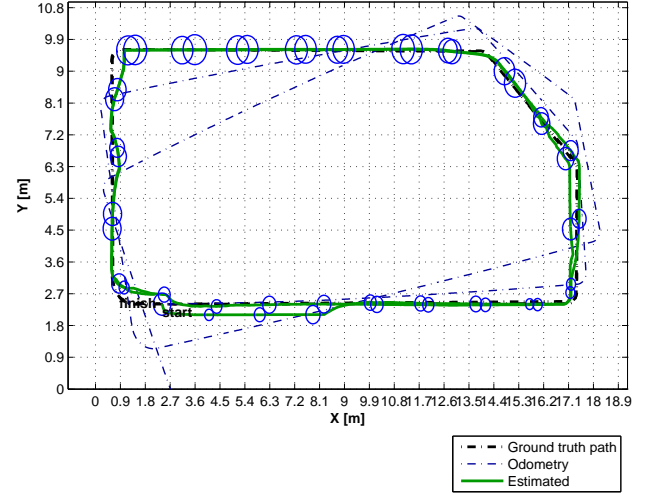


Fig. 10. Map with estimated position considering a ground truth path

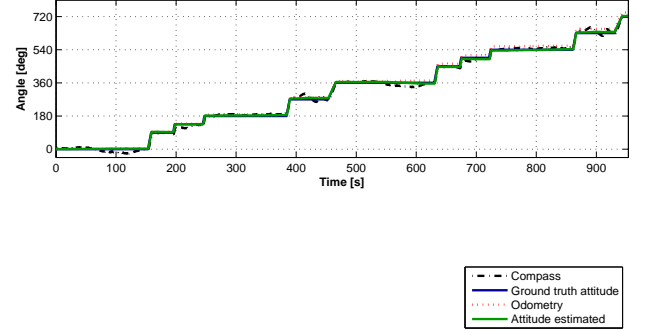


Fig. 11. Estimated attitude along time

1 meter close to the ground truth.

Figure 10 shows that the self-localization system estimates the path correctly while the path estimated by odometry diverges completely from the ground truth path. The blue circles represents the position uncertainty obtained by the Kalman filter. As it is possible to observe, the uncertainty increases when the robot are in the top of the Fig 10. This happen because in this area the ceiling has less information. Nevertheless, Fig 9 and Fig 10 show that the robot is still able to estimate the position of the robot, even when uncertainty increases.

Analyzing the results of the attitude estimator in Fig. 11, it is possible to observe that the estimated attitude is very close to the ground truth, allowing to conclude that this Kalman filter provides results with good accuracy. For more details about Kalman filters design, see technical report [8].

Finally, analyzing the distribution of the estimated position error in Fig. 12 can be concluded that it is approximately Gaussian with a mean close to zero. The distribution is non-zero mean Gaussian because the trajectory is not random and due to the finite resolution of the PCA probabilistic grid (0.3 m). However the mean error in both axis is very close to zero being $\bar{e}_x = 5.6 \times 10^{-2} \text{ m}$ and $\bar{e}_y = -1.0 \times 10^{-2} \text{ m}$, thus giving an positive indication for the validation of the Gaussian error assumption of the Kalman Filter design.

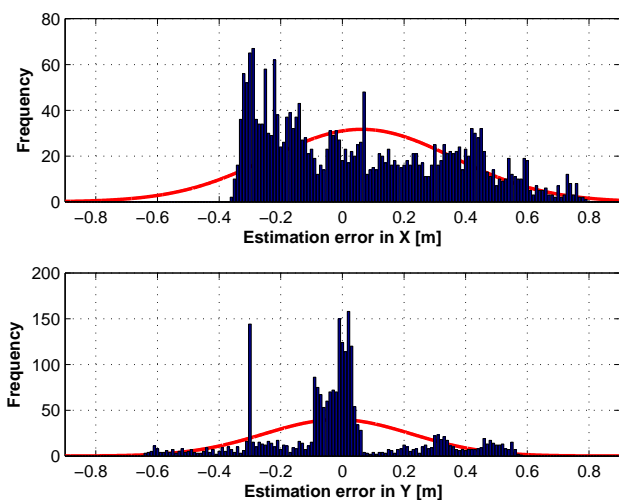


Fig. 12. Distribution of the estimated position error

V. CONCLUSIONS

The existence of scenarios with periodic elements or very similar in different points of the environment is usual and it can induce the position systems to a erroneous localization of mobile robots. In this paper a Bayesian method using a grid map with an observation belief based on PCA algorithm has been developed and experimentally validated. The proposed self-localization system is based on depth images of a Kinect sensor installed onboard of a mobile robot, looking upwards to the ceiling.

The integration of the Bayesian PCA-based position sensor with a linear Kalman filter allows to obtain an suboptimal and globally stable localization of the mobile robot. The sub-optimality is due to the non linear PCA sensor. However the results show that the localization error distribution is approximate Gaussian thus validating the Kalman filter synthesis assumption.

In the future, the proposed localization method will be implemented in a path following control approach, where the self-localization system will be integrated in a control close loop performing different tasks like obstacle avoidance and human-robot interaction.

ACKNOWLEDGMENTS

This work was supported by FCT, through IDMEC, under LAETA Pest-OE/EME/LA0022 and partially supported by the project PRODUTECH-PTI (Proj. 3904) under the program COMPETE/QREN/FEDER.

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